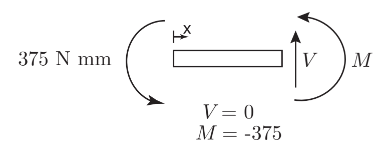
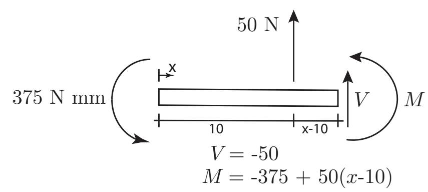
UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
Summer 2016

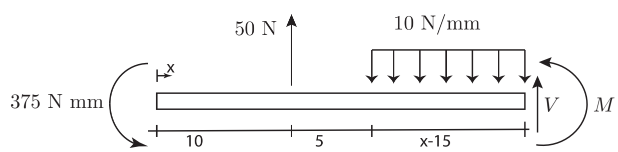
Structural Engineering,
Mechanics and Materials
Professor: S. Govindjee

# CE W30 / ME W85
## Midterm Exam 2
## Solutions

1. There are two ways to solve this problem. One is to make section cuts as indicated and determine the needed expressions for $V(x)$ and $M(x)$ in various sections of the beam using force and moment equilibrium.




$V$ = -50 + 10($x$-15)
$M$ = -375 + 50($x$-10) - 10($x$-15)($x$-15)/2

The other way to do this is to note that the distributed load

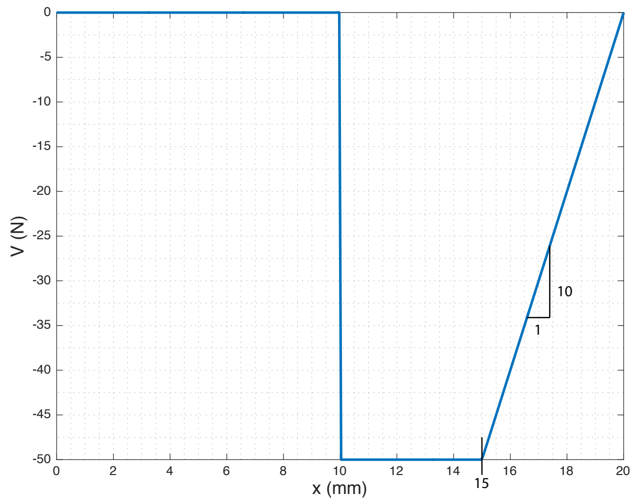
$$q(x) = 50\delta(x-10) - 10H(x-15) \tag{1}$$

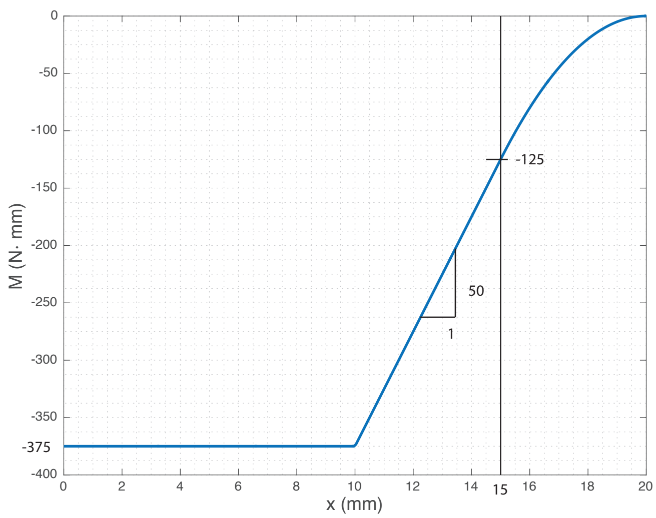
and that

$$V(x) = V(0) - \int_0^x q(x)\,dx = 0 - 50H(x-10) + 10\langle x-15\rangle \tag{2}$$

$$M(x) = M(0) - \int V(x)\,dx = -375 + 50\langle x-10\rangle - 5\langle x-15\rangle^2 \tag{3}$$

The end result, either way is:




*Grading Rubric: Method 1: Free body diagrams 20pts; Vertical Equilibrium 10pts; Moment Equilibrium 10pts; Graph of $V(x)$ 5pts; Graph of $M(x)$ 5pts. Method 2: Distributed load expression 20pts; Integration with BCs to get $V(x)$ 10pts; Integration with BCs to get $M(x)$ 10pts; Graph of $V(x)$ 5pts; Graph of $M(x)$ 5pts.*

2. Find the rotation field:

$$GJ\phi'' = -t_o(1 + z^2/L^2) \quad (4)$$
$$GJ\phi' = -t_o(z + z^3/3L^2) + C \quad (5)$$
$$GJ\phi = -t_o(z^2/2 + z^4/12L^2) + Cz + D \quad (6)$$

Apply the boundary condtions:

$$\phi(0) = 0 \Rightarrow \qquad D = 0 \quad (7)$$
$$\phi(L) = 0 \Rightarrow \qquad C = t_o(L^2/2L + L^4/12L^2L) = 7t_oL/12 \quad (8)$$

Apply the given condition and solve for $t_o$:

$$\delta R/L = \frac{t_o}{GJ}\left[-\left(\frac{3^2L^2}{4^2\cdot 2} + \frac{3^4L^4}{4^4\cdot 12L^2}\right) + \frac{7L\cdot 3L}{12\cdot 4}\right] \quad (9)$$
$$t_o = \frac{\delta RGJ}{L^3}\frac{1}{\frac{21}{48} - \frac{9}{32} - \frac{81}{12\cdot 256}} \quad (10)$$
$$= 7.70\frac{\delta RGJ}{L^3} = 3.85\frac{\delta\pi R^5 G}{L^3} \quad (11)$$

*Grading Rubric: Starting ODE and its integration 15pts; Boundary conditions and constants of integration 9pts; Derivation and value of* $t_o$ *6pts.*

3. The state of stress in the bar is

$$\boldsymbol{\sigma} = \begin{bmatrix} \frac{P}{A} & 0 \\ 0 & 0 \end{bmatrix} \quad (12)$$

The traction on the Aluminum-Epoxy interface is

$$\boldsymbol{t} = \boldsymbol{\sigma}^T\boldsymbol{n} = \begin{bmatrix} \frac{P}{A} & 0 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} \frac{P}{A}\cos\theta \\ 0 \end{pmatrix} \quad (13)$$

The normal stress on the interface is $\sigma = \boldsymbol{t}\cdot\boldsymbol{n}$ and the shear stress on the interface is $\tau = \boldsymbol{t}\cdot\boldsymbol{s}$:

$$\sigma = \begin{pmatrix} \frac{P}{A}\cos\theta \\ 0 \end{pmatrix}\cdot\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \frac{P}{A}\cos^2\theta = \frac{P}{2A}(1+\cos 2\theta) \quad (14)$$
$$\tau = \begin{pmatrix} \frac{P}{A}\cos\theta \\ 0 \end{pmatrix}\cdot\begin{pmatrix} \sin\theta \\ -\cos\theta \end{pmatrix} = \frac{P}{A}\cos\theta\sin\theta = \frac{P}{2A}(\sin 2\theta) \quad (15)$$

The largest normal stress happens at $\theta = 0$; the largest shear stress happens at $\theta = \pi/4$. Thus $P/A \leq \sigma_{\text{max}}$ and $P/2A \leq \tau_{\text{max}}$, which implies

$$P \leq 20 \cdot 10 = 200 \text{ N} \tag{16}$$

$$P \leq 40 \cdot 4 = 160 \text{ N} \tag{17}$$

Taking the more restrictive of the two conditions indicates that $P \leq 160$ N.

*Grading Rubric: Overall state of stress 5pts; Determination of interface traction 5pts; Decomposition into normal and shear components 5pts; Application of conditions 5pts.*