UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
Summer 2016

Structural Engineering, Mechanics and Materials
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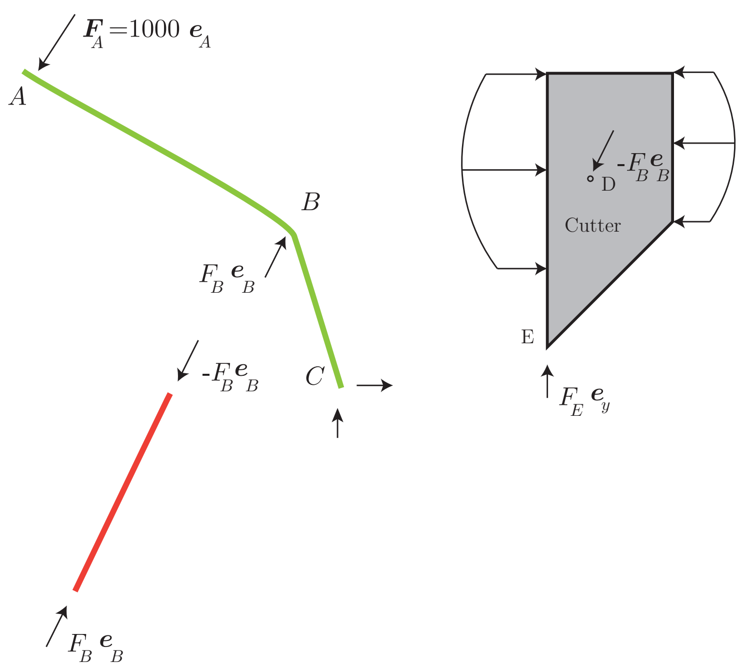
CE W30 / ME W85
Midterm Exam 1
Solutions

1. (a) Draw a free-body diagram for each rigid body:



Note that $\boldsymbol{e}_A = -\frac{1}{2}\boldsymbol{e}_x - \frac{\sqrt{3}}{2}\boldsymbol{e}_y$ and $\boldsymbol{e}_B = \frac{1}{\sqrt{5}}\boldsymbol{e}_x + \frac{2}{\sqrt{5}}\boldsymbol{e}_y$.

Start with moment equilibrium for the lever-arm about C, which gives

$$
\begin{aligned}
0 &= \boldsymbol{r}_{CA} \times 1000\boldsymbol{e}_A + \boldsymbol{r}_{CB} \times F_B\boldsymbol{e}_B && (1)\\
0 &= \left[-\left(25 + 300\frac{\sqrt{3}}{2}\right)\boldsymbol{e}_x + 200\boldsymbol{e}_y\right] \times 1000\left[-\frac{1}{2}\boldsymbol{e}_x - \frac{\sqrt{3}}{2}\boldsymbol{e}_y\right] \\
&\quad + [-25\boldsymbol{e}_x + 50\boldsymbol{e}_y] \times F_B\left[\frac{1}{\sqrt{5}}\boldsymbol{e}_x + \frac{2}{\sqrt{5}}\boldsymbol{e}_y\right] && (2)\\
0 &= 1000\left(\left(25 + 300\frac{\sqrt{3}}{2}\right)\frac{\sqrt{3}}{2} + 200\frac{1}{2}\right) - F_B\left(25\frac{2}{\sqrt{5}} + 50\frac{1}{\sqrt{5}}\right) && (3)
\end{aligned}
$$

Solve this for F_B

$$F_B = 7.75 \text{ kN} \qquad (4)$$

and note that the linkage is a two force member. Now, apply force equilibrium to the cutter:

$$F_E\boldsymbol{e}_y - F_B\boldsymbol{e}_B = \boldsymbol{0}\,; \qquad (5)$$

Take the y-component of the force equilibrium to find:

$$
\begin{aligned}
F_E - F_B\boldsymbol{e}_y \cdot \boldsymbol{e}_B &= 0 && (6)\\
F_E = F_B e_{By} &= 7.75\frac{2}{\sqrt{5}} = \underline{\underline{6.93 \text{ kN}}} && (7)
\end{aligned}
$$

(b) Impose equilibrium on the cutter

$$
\begin{aligned}
\{\boldsymbol{0}, \boldsymbol{0}\} &= \{\boldsymbol{R}, \boldsymbol{M}_R^{(D)}\}_{\text{guides}} + \{F_E\boldsymbol{e}_y, -F_E \cdot 20\boldsymbol{e}_z\} + \{-F_B\boldsymbol{e}_B, \boldsymbol{0}\} && (8)\\
\{\boldsymbol{R}, \boldsymbol{M}_R^{(D)}\}_{\text{guides}} &= -\{F_E\boldsymbol{e}_y, -F_E \cdot 20\boldsymbol{e}_z\} - \{-F_B\boldsymbol{e}_B, \boldsymbol{0}\} && (9)\\
\{\boldsymbol{R}, \boldsymbol{M}_R^{(D)}\}_{\text{guides}} &= \underline{\underline{\{3.47\boldsymbol{e}_x \text{ kN}, 133\boldsymbol{e}_z \text{ kN} \cdot \text{mm}\}}} && (10)
\end{aligned}
$$

Grading Rubric: Free body diagrams (FBDs) 20pts. Determination of the internal force in the two-force linkage 10pts. Determination of the cutting force 10pts. Determination of the necessary net force and moment from the guides 10pts.

2. Net force

$$R = \int_0^L q_o + q_1 \frac{x}{L} + q_2 \left(\frac{x}{L}\right)^2 \, dx \tag{11}$$

$$\boldsymbol{R} = \underline{\underline{\left[q_o L + q_1 \frac{L}{2} + q_2 \frac{L}{3} \right] \boldsymbol{e}_y}} \tag{12}$$

acting at x_c where

$$0 = \int_0^L (x - x_c) \cdot \left(q_o + q_1 \frac{x}{L} + q_2 \left(\frac{x}{L}\right)^2 \right) \, dx \tag{13}$$

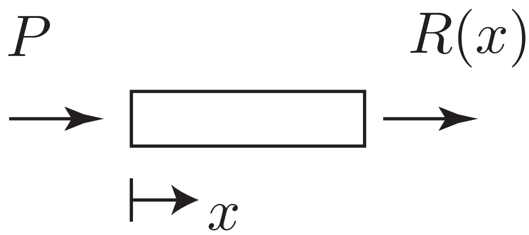
$$x_c = \frac{\int_0^L x \cdot \left(q_o + q_1 \frac{x}{L} + q_2 \left(\frac{x}{L}\right)^2 \right) \, dx}{\int_0^L q_o + q_1 \frac{x}{L} + q_2 \left(\frac{x}{L}\right)^2 \, dx} \tag{14}$$

$$x_c = \underline{\underline{\frac{q_o L/2 + q_1 L/3 + q_2 L/4}{q_o + q_1/2 + q_2/3}}} \tag{15}$$

Note x_c is chosen at the point where the resultant moment is zero.

Grading Rubric: Determination of total force 20pts. Determination of necessary point of application for equivalency.

3. (a) Make a section cut at any location and look at the left piece.



This shows that:

$$R(x) = \underline{\underline{-P}} \tag{16}$$

$$\sigma(x) = \underline{\underline{-P/A}} \tag{17}$$

$$\varepsilon(x) = \underline{\underline{-P/EA + \left(\alpha_o + \alpha_1 \frac{x}{L}\right) \Delta T}} \tag{18}$$

$$u(x) = -Px/EA + \left(\alpha_o x + \alpha_1 \frac{x^2}{2L}\right) \Delta T + C \tag{19}$$

Use the boundary condition $u(L) = 0$ to find C:

$$C = PL/AE - \left(\alpha_o L + \alpha_1 \frac{L}{2}\right) \Delta T \tag{20}$$

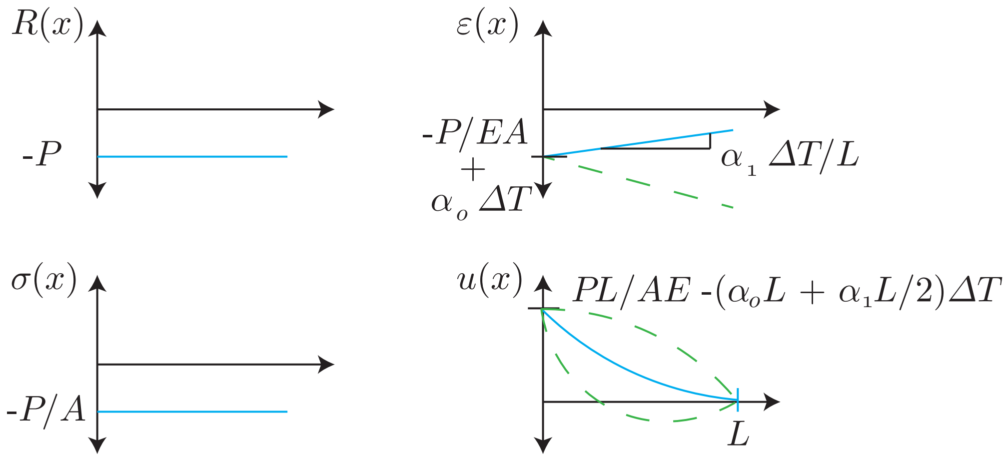
$$\underline{\underline{u(x) = P(L-x)/EA + \left(\alpha_o (x - L) + \alpha_1 \frac{L}{2} \left(\frac{x^2}{L^2} - 1\right)\right) \Delta T}} \tag{21}$$

Alternately, one could have used a differential equation method using the thermal form of the second-order governing ODE:

$$(AEu'(x) - AE\Delta T \alpha(x))' + b(x) = 0 \tag{22}$$

where $b(x) = 0$ and the boundary conditions are $u(L) = 0$ and $R(0) = AEu'(0) - AE\Delta T\alpha(0) = -P$. Double integration and application of the boundary conditions results in the same expression for $u(x)$. One then find $\varepsilon(x)$ as the first derivative of $u(x)$, $\sigma(x)$ is found from $\varepsilon(x) = \sigma(x)/E + \alpha(x)\Delta T$, and $R(x)$ is found from $A\sigma(x)$.

Part (b)



Dashed curves show alternate acceptable answers (among various options depending on the sign and magnitude of the given constants).

Grading Rubric: Determination of internal force field 4pts. Determination of stress field 3pts. Determination of strain field 5pts. Determination of displacement field 3pts. Sketches 5pts.