MEIOG MIDTERM 2 – SOLUTIONS

① a) $\underline{F} = \frac{d\underline{G}}{dt} \Rightarrow \int_{t_1}^{t_2} \underline{F}\,dt = \int_{\underline{G}(t_1)}^{\underline{G}(t_2)} d\underline{G} \Rightarrow \underline{G}(t_2) = \underline{G}(t_1) + \underline{I}$

$\underline{I} = \int_{t_1}^{t_2} \underline{F}\,dt$

b) Before:

$\underline{V}_0 = 1.25\,\underline{i}$ m/s

After:

$\underline{V} = (3.75)(\cos 60^\circ)\,\underline{i} + (3.75)(\sin 60^\circ)\,\underline{j}$

$= 1.875\,\underline{i} + 3.2476\,\underline{j}$

$$\int_0^{0.4} F\underline{e}\,dt = m\underline{V} - m\underline{V}_0$$

$(0.4)\,F\underline{e} = (0.2)(1.875)\,\underline{i} + (0.2)(3.2476)\,\underline{j} - (0.2)(1.25)\,\underline{i}$

$(0.4)\,F\underline{e} = 0.125\,\underline{i} + 0.6495\,\underline{j}$

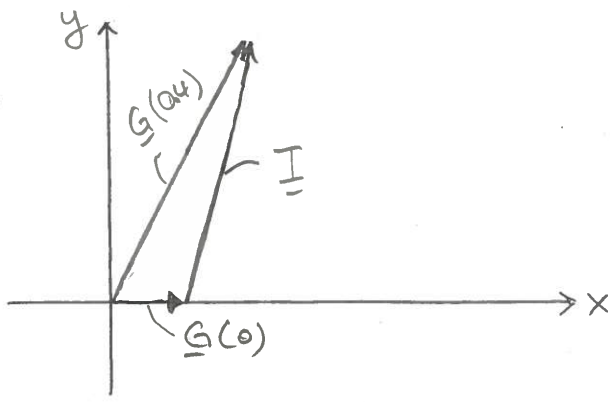
$$\frac{0.4\,F\cos\phi}{0.4\,F\sin\phi} = \frac{0.125}{0.6495} \Rightarrow \tan\phi = 5.1962$$

$\phi = 79.1^\circ$

$F = 1.6536$ N

$\underline{e} = 0.189\,\underline{i} + 0.982\,\underline{j}$

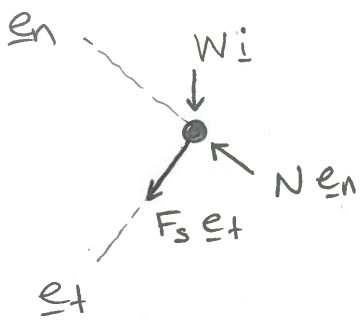
c)



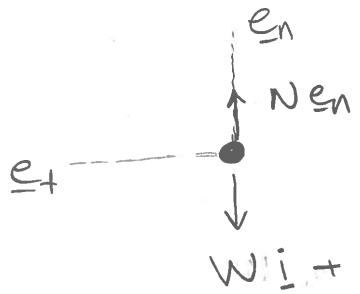
(2)

At D:

a)



At B:



b) N does no work, gravity + spring are potential
$\therefore T + V_e + V_g = \text{const}$

$$\dot{T} = P = \underline{F}\cdot\underline{V} = \underline{N}\cdot\underline{V} + \underbrace{W\underline{i}\cdot\underline{V}}_{-\dot{V}_g} + \underbrace{\underline{F}_s\cdot\underline{V}}_{-\dot{V}_e}$$

$$\dot{T} + \dot{V}_g + \dot{V}_e = 0 \Rightarrow \boxed{T + V_g + V_e = \text{const.}}$$

c) $\cancel{T_D}^{\,0} + (V_g)_D + (V_e)_D = T_B + \cancel{(V_g)_B}^{\,0} + (V_e)_B$

$$Wr(1-\cos 60^\circ) + \frac{1}{2}k\left(6\sqrt{3}-6\right)^2 = \frac{1}{2}mv_B^2 + \cancel{\frac{1}{2}k(6-6)^2}^{\,0}$$

$$(9)(0.5)(1-0.5) + \frac{1}{2}(24)(0.134) = \frac{1}{2}\left(\frac{9}{32.2}\right)v_B^2$$

$$2.25 + 1.608 = 0.14\,v_B^2 \Rightarrow \boxed{v_B = 5.254\ \text{ft/s}}$$

d) $\underline{F} = m\underline{a}$ at B:

$$W\underline{i} + F_s\underline{i} - N\underline{i} = ma_t\underline{e}_t + ma_n\underline{e}_n$$

$(\underline{i} = -\underline{e}_n)$ $$-W\underline{e}_n - F_s\underline{e}_n + N\underline{e}_n = ma_t\underline{e}_t + ma_n\underline{e}_n$$

$\underline{e}_t \Rightarrow$ $\boxed{a_t = 0}$

$\underline{e}_n \Rightarrow$ $$-W - F_s + N = ma_n = m\frac{V_B^2}{r}$$

$$N = W + \cancel{F_s}^{\,0} + m\frac{V_B^2}{r}$$

$$= 9 + \left(\frac{9}{32.2}\right)\left(\frac{5.254^2}{0.5}\right)$$

$$= \underline{\underline{24.432 \text{ lb}}}$$

e) Oscillation between A-B-D for all $t \geq 0$

(3) $V = -\frac{GMm}{r}$

a) $T + V = E = \text{const}$ ✓

$$\frac{1}{2} m v_p^2 - \frac{GMm}{r_p} = \frac{1}{2} m v_a^2 - \frac{GMm}{r_a}$$

$$v_p^2 = v_a^2 + 2GM\left(\frac{1}{r_p} - \frac{1}{r_a}\right)$$

b) $\underline{M}^\circ = \underline{\dot{H}}^\circ = 0 \Rightarrow \underline{+1}^\circ = \text{const.}$ ✓

$$\underline{H}^\circ = \underline{r}_a \times m\underline{v}_a = m r_a v_a \underline{k} = m r_p v_p \underline{k}$$

$$\Rightarrow r_a v_a = r_p v_p$$

c) $v_a = \left(\frac{r_p}{r_a}\right) v_p$, $r_p = 3694$ km , $r_a = 13503$ km

$$v_p^2 = \left(\frac{r_p}{r_a}\right)^2 v_p^2 + 2GM\left(\frac{1}{r_p} - \frac{1}{r_a}\right)$$

$$v_p^2\left(1 - \left(\frac{3694}{13503}\right)^2\right) = 2(6.673\times10^{-11})(0.642\times10^{24})\left(\frac{1}{3694\times10^3} - \frac{1}{13503\times10^3}\right)$$

$$0.9252\, v_p^2 = 1.6849 \times 10^7$$

$$v_p = 4267.5 \text{ m/s}$$

$$v_a = 1167.5 \text{ m/s}$$