EECS 16A   Designing Information Devices and Systems I
Fall 2016   Babak Ayazifar, Vladimir Stojanovic   Midterm 1

Exam location: 145 Dwinelle, last SID# 2

PRINT your student ID: _____

PRINT AND SIGN your name: _____, _____ _____
    (last)   (first)   (signature)

PRINT your Unix account login: ee16a-_____

PRINT your discussion section and GSI (the one you attend): _____

Name and SID of the person to your left: _____

Name and SID of the person to your right: _____

Name and SID of the person in front of you: _____

Name and SID of the person behind you: _____

Section 0: Pre-exam questions (2 points)

1. **What do you like to do in your free time? (1 pt)**

2. **Label the current in any direction through this resistor, using the element name as the subscript. Follow this convention throughout the exam. (1 pt)**


R_3
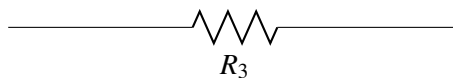
Do not turn this page until the proctor tells you to do so. You can work on Section 0 above before time starts.

Section 1 (54 points)

3. **Resistor Combinations (10 points)**

   (a) **(4 points)** What is the equivalent resistance between nodes A and B? Write your solution in terms of R_1, R_2, R_3, and R_4. Use the parallel operator $\|$ wherever applicable. What is the equivalent resistance if R_4 approaches infinity?
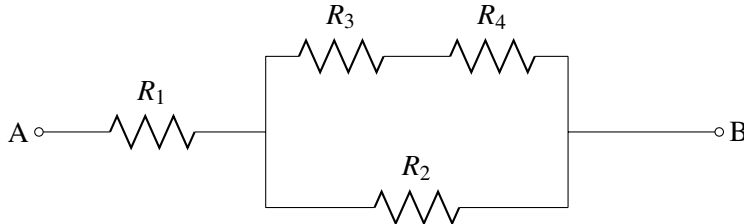
   **Solutions:** First we combine R_3 and R_4 in series: $R_3 + R_4$.

   Then we see that $R_3 + R_4$ is in parallel with R_2, resulting in $(R_3 + R_4)\|R_2$.

   Finally, we see that the R_2, R_3, R_4 block is in series with R_1, giving us

   $R_{eq} = R_1 + (R_3 + R_4)\|R_2$.

   When R_4 approaches infinity, the branch becomes an open circuit. Thus $(R_3 + R_4)\|R_2$ becomes just R_2. The equivalent resistance is then just $R_1 + R_2$.

   (b) **(6 points)** What is the current i_2 through R_2? Write your solution in terms of R_1, R_2, R_3, R_4, R_5, and V_s. Use the parallel operator $\|$ in your expression wherever applicable.
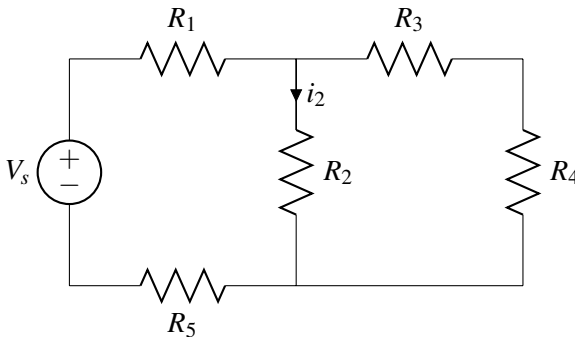
   Solutions: First, we find the current running through R_1, i_1.

   Notice how the circuit is the same combination of the resistors from part a in series with R_5:

   $i_1 = \frac{Vs}{R_{eq} + R_5}$

   We can view this as a current divider problem, and thus use the current divider formula:

   $i_2 = i_1 \frac{(R_3 + R_4)\|R_2}{R_2}$

   Substituting for i_1:

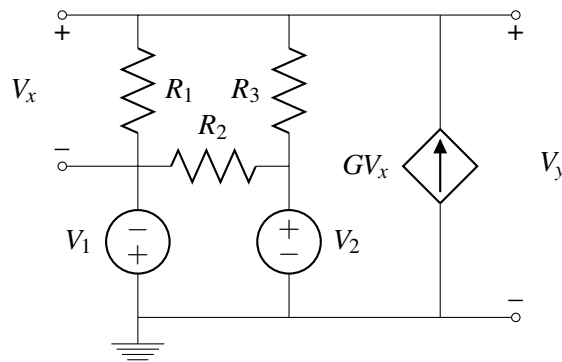 $i_2 = \frac{Vs}{R_{eq} + R_5} * \frac{(R3 + R4)\|R2}{R2}$

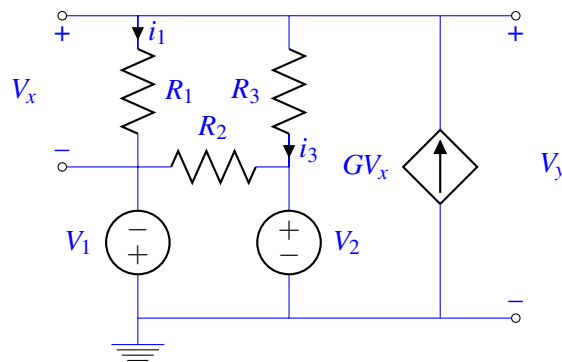 $i_2 = \frac{Vs}{R_5 + R_1 + (R_3 + R_4)\|(R_2)} * \frac{(R3 + R4)\|R2}{R2}$

4. **Take Node of the Voltage Sources (8 points)**

Use nodal analysis to solve for the voltages V_x and V_y. Use the following values for numerical calculations. **Note the polarity on the voltage sources. Solutions that solve the circuit without using nodal analysis will not be awarded full credit.**

$$V_1 = 5 \text{ V} \quad R_1 = 10 \ \Omega$$
$$V_2 = 5 \text{ V} \quad R_2 = 50 \ \Omega$$
$$G = \tfrac{1}{4} \text{ S} \quad R_3 = 40 \ \Omega$$

Solutions:

Set up a nodal equation setting the sum of currents out of our single node V_y to zero:
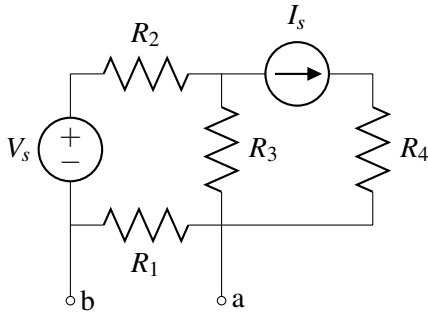
$$i_1 + i_3 - GV_x = 0$$

$$\frac{V_y + 5}{10} + \frac{V_y - 5}{40} - \frac{1}{4}(V_y + 5) = 0$$

$$4(V_y + 5) + (V_y - 5) - 10(V_y + 5) = 0$$

$$4V_y + 20 + V_y - 5 - 10V_y - 50 = 0$$

$$-5V_y - 35 = 0$$

$$V_y = -7$$

$$V_x = V_y + 5 = -7 + 5 = -2$$

Our solution:

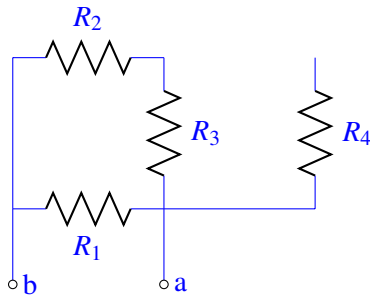$$V_x = -2\text{V}$$

$$V_y = -7\text{V}$$

5. **Thévenin and Norton Equivalence (10 points)**

(a) **(4 points)** Redraw the circuit with all sources nulled, then calculate R_{th} between terminals a and b.

   Solutions:

   When we null the voltage source, it becomes a short circuit. When we null the current source, it becomes an open circuit. So with the nulled sources, our circuit looks like this:

   To calculate R_{th}, we simplify our circuit with nulled sources down into a single resistor, from the perspective of R_1. This leaves us with R_2 and R_3 in parallel with R_1. So,

$$R_{th} = (R_2 + R_3) \parallel R_1$$

(b) **(6 points)** Find the Thévenin voltage between the terminals a and b. Hint: superposition may be useful.

   Solutions:

   We can either determine V_{oc} directly, or find I_{sc} and solve for V_{th} using R_{th} from the previous part. Here, we will determine V_{oc} with superposition.

   With only V_S on, we have a simple voltage divider. So

$$V_{1_V} = V_S \frac{R_1}{R_1 + R_2 + R_3}$$

   With only I_S on, we have a bit more work to do. We see a current divider, with R_3 in parallel with $R_2 + R_1$. So the current through resistor R_1 is
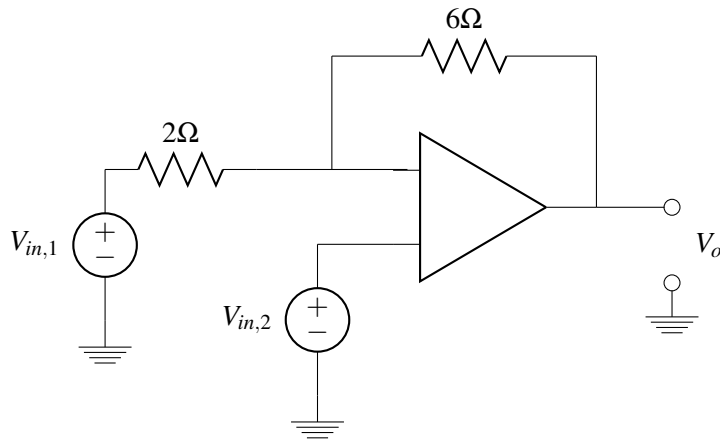
$$I_1 = I_S \frac{R_3}{R_1 + R_2 + R_3}$$

   And the voltage drop V_1 across R_1 is $V_1 = R_1 I_1 = R_1 I_S \frac{R_3}{R_1 + R_2 + R_3}$

   So

$$V_{th} = V_{oc} = V_{1_V} + V_{1_I} = R_1 I_S \frac{R_3}{R_1 + R_2 + R_3} + V_S \frac{R_1}{R_1 + R_2 + R_3}$$

6. Superposition and Op-Amps (8 points)

(a) **(2 points)** For the circuit above, label the polarity of the terminals of the op-amp such that there is negative feedback.

Solutions: The top terminal should be the inverting input, and the bottom terminal should be the non-inverting input

(b) **(6 points)** Find V_o in terms of $V_{in,1}$ and $V_{in,2}$. (**Hint**: Assume the op-amp is in negative feedback, then use superposition and golden rules)

Solutions: Considering just the effect of $V_{in,1}$, we have

$$V_o = -V_{in,1}\frac{6\Omega}{2\Omega} = -3V_{in,1}$$

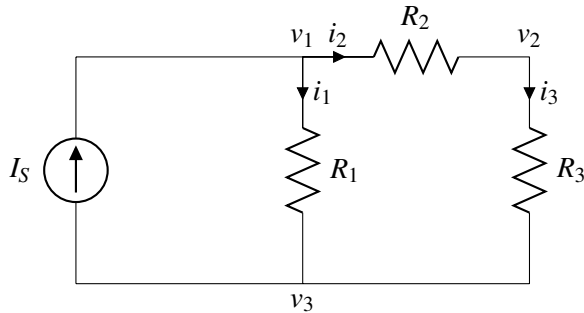Now, considering just the effect of the voltage source V_s, we find

$$V_o = V_{in,2} + \frac{6\Omega}{2\Omega}V_{in,2} = 4V_{in,2}$$

Thus,

$$V_o = 4V_{in,2} - 3V_{in,1}$$

7. 16A-Spice (10 points)

(a) **(5 points)** Write down the incidence matrix, $\mathbf{F}$ of this circuit. Let node v_n correspond to column n in your incidence matrix, and current i_m correspond to row m.

Solutions:

The incidence matrix is:
$$\mathbf{F} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

(b) **(5 points)** Determine the rank of $\mathbf{F}$, and a basis for the null space of $\mathbf{F}$.
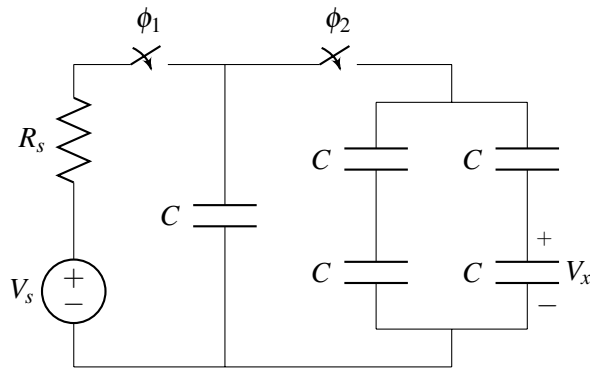
Solutions:

The rank is 2, because we can add a scalar to all possible resulting voltages in this circuit and still satisfy the potential difference constraints it imposes on the solutions. You could also mechanically compute the rank by row reduction.

A basis for the null space of F is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

PRINT your name and student ID: _____

8. Charge-Sharing (8 points)

Initially, all capacitors are uncharged, and have the same capacitance C. For $t < 0$, the switch ϕ_1 is *on* and ϕ_2 is *off* and the circuit has achieved steady state. Later, at $t = 0$, ϕ_1 is *off* and ϕ_2 is *on*; then the system is allowed to reach steady state.

What is the voltage V_x at $t \gg 0$?

Solutions:

$V_x = \frac{V_S}{4}$.

Note that C_{eq} of 2 capacitors of capacitance C in series is $C/2$ and in parallel is $2C$. Based on the symmetry of the setup, the voltage splits evenly across the capacitor on the left and the capacitor setup on the right. This implies half the initial voltage V_S exists on both sides. On the capacitor network on the right, symmetry indicates a secondary split across each capacitor in series.

Section 2 $\left(44 \text{ points}\right)$

9. Operation$^{\text{TM}}$ (10 points)

You're an intern at Hasbro working on improvements to the game Operation. Operation is a board game that consists of an "operating table" (the board), overlaid with a drawing of the "patient". In the surface of the board/patient are a number of metal-lined cavities filled with plastic bones and other body parts. The player tries to remove these objects with metal tweezers without touching the edge of the cavity opening. Touching the cavity opening turns on a light and causes a loud buzzer to go off.
You are given the following supplies:

- buzzer: This can be modeled as a resistor, and it buzzes when a voltage is applied across it.

- lamp: The lamp can be modeled as a resistor, and it lights up when current runs through it. The brightness of the lamp is directly proportional to the current passing through it.

- battery: An ideal voltage source.

- switch: When the player makes a mistake and touches the cavity opening with the tweezers, the tweezers act as a switch and close the circuit.
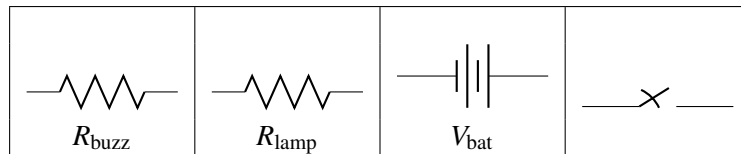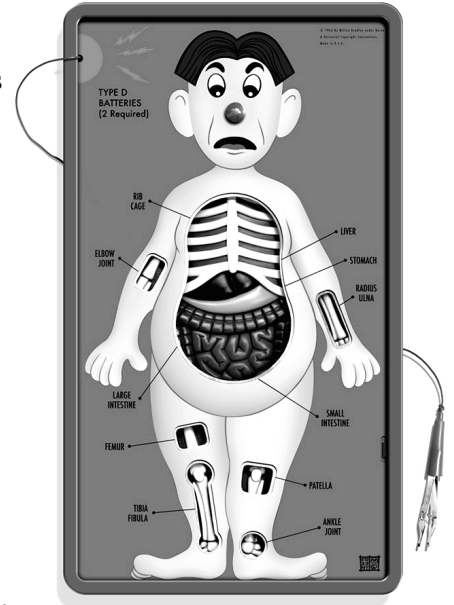
(a) **(4 points)** Design a circuit that makes the buzzer buzz and the lamp turn on when the player makes a mistake. **Solutions:** The objective is to have it so current flows through the buzzer and lamp when the user turns on the switch, meaning any answer with no closed circuit when the switch is open works. A couple of possible solutions:
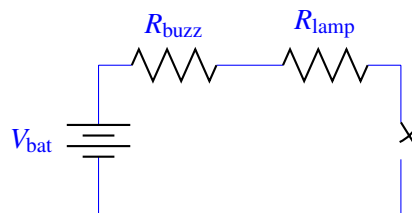

For putting elements in series, the order of components does not matter—closing the switch creates a closed loop for charge to flow.
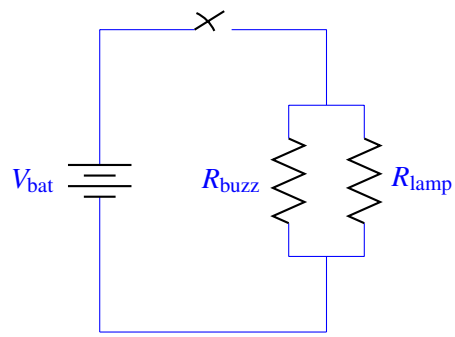
For elements in parallel, the switch must not be parallel to the buzzer or the lamp, otherwise there will be a closed circuit independent of the switch.

(b) **(6 points)** Market research has shown that buzzers scare small children, so you've been told to <u>remove the buzzer</u> and use <u>two identical lamps</u> instead of just one. **Assuming all power dissipated by the lamps is converted to light, design a circuit that maximizes the total light emitted by the lamps. Calculate the power P dissipated by the lamps.** *Note: You do not need to rigorously prove that your circuit maximizes the light emitted, but explain your design choices.* **Solutions:** First, we know that there should be no closed circuits when the switch is open. This can be written as:
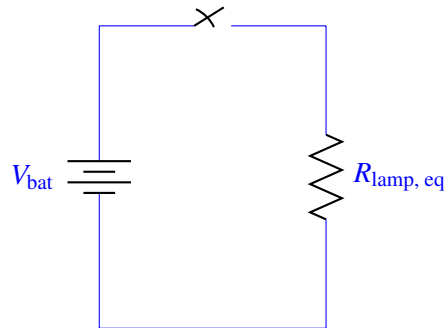

where R_{total} represents the equivalent resistance of your arrangement of lamps.

Next, we'll try to maximize the lamps' brightness. In other words, we're trying to maximize the power dissipated by $R_{lamp,\ eq}$.

$$P = IV$$
$$= \frac{V^2}{R}$$

From the diagram above, we know the voltage is fixed at V_{bat}, so we need to maximize the current drawn from the battery, i.e. minimize $R_{lamp,\ eq}$. This can be done by placing the lamps in parallel:
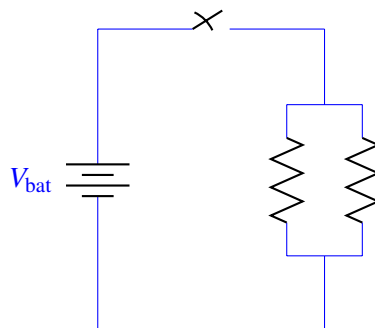
From here, we know $R_{lamp,\ eq} = \frac{R_{lamp}}{2}$ and the current seen by each resistor is the same, so we can calculate the power:

$$P = \frac{V_{bat}^2}{\frac{R_{lamp}}{2}}$$
$$= 2\left(\frac{V_{bat}^2}{R_{lamp}}\right)$$

10. **This Is Your Brain on EE (18 points)**

Medical instruments such as electroencephalograms (EEG) take very small electric signals from the body and scale them into a voltage that we can measure. EEG's take voltage readings from the scalp and are used to determine brain activity. There are some challenges in designing a circuit for measuring the brain:

- The human body is very noisy, so we have to take multiple sensor readings and average them.
- The brain's electrical signals are very small, usually in the microvolt range.

In this problem you will design an EEG circuit that will tackle these problems.

(a) **(4 points)** In the circuit below, determine V_{out} in terms of v_1, v_2, v_3, R_1, R_2, and R_3. Use the parallel operator ‖ wherever applicable.
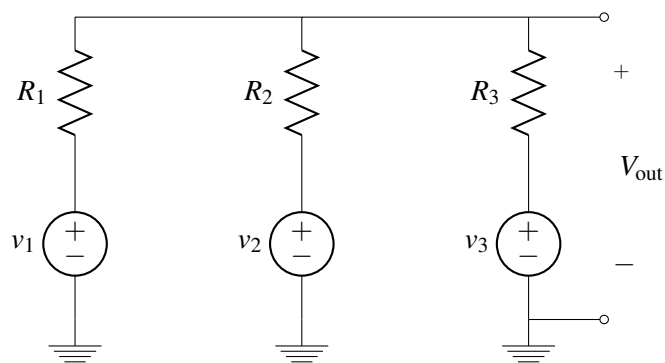
Solutions:

Using superposition:

$$V_{\text{out}} = \frac{R_2\|R_3}{R_1 + R_2\|R_3}v_1 + \frac{R_1\|R_3}{R_1\|R_3 + R_2}v_2 + \frac{R_1\|R_2}{R_1\|R_2 + R_3}v_3$$

PRINT your name and student ID: _____

(b) **(7 points)** Suppose we want to mitigate noise and interference from our circuit and body by averaging the reading across multiple areas of our scalp. Design a circuit that takes the average of three EEG readings. Model the kth EEG reading as a voltage source labelled v_k in series with a source resistance of $1k\Omega$. Also, provide an equation for V_{out} in terms of the v_k's. You may use resistors on any value for this design.

**Hint: the circuit from part (a) might be useful.**

Solutions:
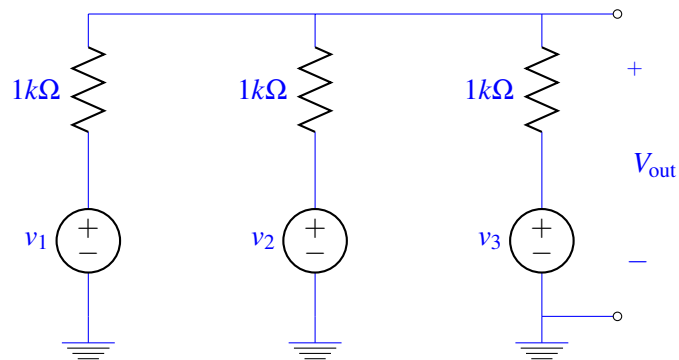
$$V_{out} = \frac{1}{3}(v_1 + v_2 + v_3)$$

(c) **(7 points)** Let the typical voltage range of values for EEG signals be from $-10mV$ to $10mV$. Design a circuit that takes the output of your circuit in part (b) and scales the EEG signal, while maintaining the sign, to a voltage range of $-2.5V$ to $2.5V$. Give an equation that relates the output to the input. You may use any combination of resistors or ideal op-amps for this question.
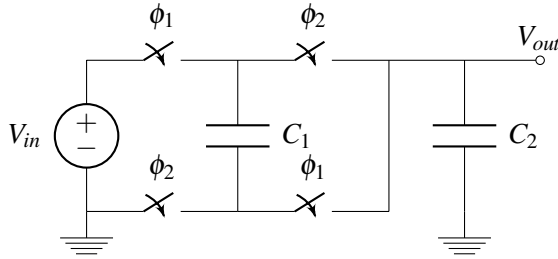
Solutions:
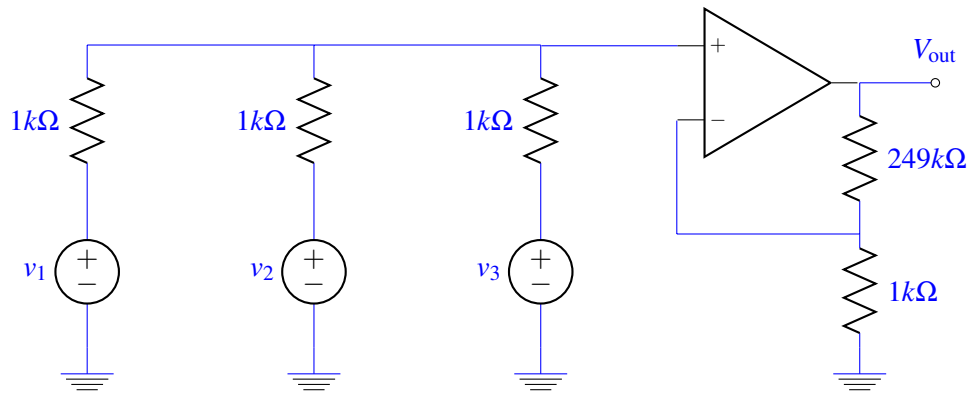
$$V_{out} = 250V_{in}$$

We could have also used an inverting summer circuit and then invert the output of that with unity gain. There are several correct solutions to this problem.

11. Voltage Regulators (16 points)

In this problem we will explore how we can use charge sharing to make a better voltage regulator.

(a) **(4 points)** Switches ϕ_1 are both *on* in Phase 1. All other switches are *off*. Find the charge on and voltage across each capacitor as a function of V_{in}, C_1, and C_2. Assume the capacitors are uncharged before Phase 1.

Solutions: The two capacitors in series have a total capacitance of $\frac{C_1 C_2}{C_1 + C_2}$. We know there is V_s volts across this capacitor, and thus $V_s \frac{C_1 C_2}{C_1 + C_2}$ charge. The charge on C_1 must be equal to the charge on C_2. Knowing the charge on each capacitor, we know the voltage across both. Therefore the voltage across C_1 is $\frac{C_2}{C_1 + C_2} V_s$. The voltage across C_2 is similarly $\frac{C_1}{C_1 + C_2} V_s$.

(b) **(4 points)** In Phase 2, switches ϕ_2 are both *on*, all other switches are *off*. What are the charges on C_1 and C_2 in Phase 2 as a function of V_{out}, C_1, and C_2?

Solutions: The charge on C_1 is simply $C_1 V_{out}$. Similarly the charge on C_2 is $C_2 V_{out}$.

(c) **(4 points)** Find V_{out} as a function of V_{in}, C_1, and C_2.

Solutions: We know the total charge in the system is conserved between Phase 1 and Phase 2. There was charge $V_s \frac{C_1 C_2}{C_1 + C_2}$ on each capacitor, so the total charge in Phase 1 is $2V_s \frac{C_1 C_2}{C_1 + C_2}$. Since we know charge is conserved, thus must be equal to the total charge in Phase 2.

$$2V_s \frac{C_1 C_2}{C_1 + C_2} = (C_1 + C_2)V_{out}$$
$$V_{out} = 2\frac{C_1 C_2}{(C_1 + C_2)^2} V_s$$

(d) **(4 points)** Assume we have chosen $C_1 = C_2 = C$. One of the most important metrics in voltage regulators is efficiency. For a switching circuit such as this, we calculate efficiency as the total energy stored in the capacitors in Phase 2 divided by the total energy stored in the capacitors in Phase 1. Find the efficiency of this circuit.

Solutions: In both phases, the energy stored on the capacitors is the same. Thus there is no loss in switching. Intuitively we can see this because initially, both capacitors had $\frac{V_2}{2}$ volts on them. Then, when in parallel, they had the same V_s.

Mathematically we can see this by calculating the energy on the capacitors in each case.

$E = \frac{1}{2}CV^2$

$E_1 = \frac{1}{2}C_1(V_1)^2 + \frac{1}{2}C_2(V_2)^2$

$E_1 = \frac{1}{8}C(V_{in})^2 + \frac{1}{8}C(V_{in})^2$

$E_2 = \frac{1}{2}C_1(V_{out})^2 + \frac{1}{2}C_2(V_{out})^2$

$E_2 = \frac{1}{8}C(V_{in})^2 + \frac{1}{8}C(V_{in})^2$

We see the energy in both phases is equal. This system is thus 100% efficient.

You may use this page for scratch work but **it will not be graded.**

You may use this page for scratch work but **it will not be graded.**