Department of Physics, University of California, Berkeley

Final Examination Physics 7B, Section 2, Prof. Smoot

12:30 AM - 3:30 PM, 14 May 2004

Name:____

SID No:____

Discussion Section:____________________________

Name of TA:__________________________________

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Problem 8	

Score:____

Answer all eight problems. Write clearly and explain your work. Partial credit will be given for incomplete solutions provided your logic is reasonable and clear. Cross out any parts that you don't want to be graded. Enclose your answers with boxes. **Express all numerical answers in SI units**. Answers with no explanation or disconnected comments will not be credited. If you obtain an answer that is questionable, explain why you think it is wrong.

Constants and Conversion factors

Avogadro number, N_A	6.022×10^{23}
Permittivity of vacuum, ϵ_0	8.85×10^{-12} F·m^{-1}
Permeability of vacuum, μ_0	$4\pi \times 10^{-7}$ T·m·A^{-1}
Speed of light in vacuum, c	$1/\sqrt{\mu_0 \epsilon_0} = 3 \times 10^8$ m/s
Charge of electron, q_e	-1.602×10^{-19} C
Universal gas constant, R	8.315 J·mol^{-1}·K^{-1} = 1.99 cal·mol^{-1}·K^{-1}
Boltzmann constant, k	1.381×10^{-23} J·K^{-1}
Stefan-Boltzmann constant, σ	5.67×10^{-8} W·m^{-2}·K^{-4}
Acceleration due to gravity, g	9.8 m·s^{-2}
Specific heat of water	1 kcal·kg^{-1}·°C^{-1}
Heat of fusion of water	80 kcal·kg^{-1}
1 atm	1.013×10^5 N·m^{-2}
1 kcal	4.18×10^3 J $\quad x^\circ F = \frac{9}{5} y^\circ C + 32^\circ F$
1 hp	746 W $\quad$ 1 ft-lb = 1.356 N-m
1 liter	10^3 cm^3

Equations and formulae:

$$\frac{dN_v}{dv} = 4\pi N \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}}$$

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\rho}{P}}$$

$$\overline{v} = \sqrt{\frac{8kT}{\pi m}}$$

$$TV^{\gamma-1} = constant$$

$$W = \frac{P_1 V_1}{\gamma - 1}\left[\left(\frac{V_1}{V_2}\right)^{\gamma-1} - 1\right]$$

$$\gamma = \frac{C_P}{C_V}$$

$$\frac{dQ}{dt} = \sigma \epsilon A T^4$$

$$\frac{dQ}{dt} = -\kappa A \frac{dT}{dx}$$

$$
\begin{aligned}
\vec{F} &= q(\vec{E} + \vec{v} \times \vec{B}) \\
\mathbf{p} &= q\mathbf{d} & \vec{\mu} = NI\vec{A} \\
\vec{\tau} &= \mathbf{p} \times \mathbf{E} & \vec{\tau} = \vec{\mu} \times \vec{B} \\
U &= -\mathbf{p} \cdot \mathbf{E} & U = -\mu \cdot \vec{B} \\
V_{ab} &= -\int_a^b \mathbf{E} \cdot d\mathbf{l} \\
U_{ab} &= qV_{ab} \\
\mathbf{E} &= -\nabla V \\
u_E &= \frac{1}{2}\epsilon_0 E^2 & u_B = \frac{1}{2\mu_0} B^2 \\
V &= IZ \\
\mathbf{J} &= \sigma \mathbf{E} \\
R &= \rho \frac{l}{A} \\
E_H &= v_d B \\
d\mathbf{B} &= \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2} \\
\Phi_B &= \int \mathbf{B} \cdot d\mathbf{a} \\
\omega_{LRC} &= \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \\
\alpha &= \frac{R}{2L} \\
\phi &= tan^{-1}\left(\frac{R}{2L\omega_{LRC}}\right) \\
Z_{LRC} &= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \\
\overline{P} &= I_{rms}^2 Z \cos\phi \\
I_{rms} &= \frac{1}{\sqrt{2}} I_0 \\
I_d &= \epsilon_0 \frac{d\Phi_E}{dt} \\
\mathbf{S} &= \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}
\end{aligned}
$$

1. [25 points] Short Questions

(a) [5 points] Circle T or F for True or False

T F (i) The principle of equipartition of energy states that in equilibrium the thermal energy is shared among all active degrees of freedom and is randomly distributed with an average energy of kT/2.

T F (ii) If the temperature difference across a conductor triples, the rate at which it transfers heat energy increases by a factor of nine.

T F (iii) When a system goes from equilibrium state 1 to state 2, the change in the internal energy is the same for all processes.

T F (iv) The internal energy of a given amount of an ideal gas at equilibrium depends only on its absolute temperature.

T F (v) For any material that expands when heated, C_P is greater than C_V.

(b) [5 points]

T F (vi) The linkage of flux from circuit A to circuit B is the same as the linkage of flux from circuit B to circuit A.

T F (vii) All materials are diamagnetic. It is sometimes over ridden by ferromagnetism or paramagnetism.

T F (viii) Lenz's law states that two parallel wires carrying current in the same direction will oppose each other and push each other apart.

T F (ix) The wave equation can be derived from Maxwell's equations.

T F (x) Electromagnetic waves are transverse waves.

(c) [5 points] A circuit contains a capacitor and resistor in series is connected to an AC source. The circuit schematic is shown in the accompanying figure.

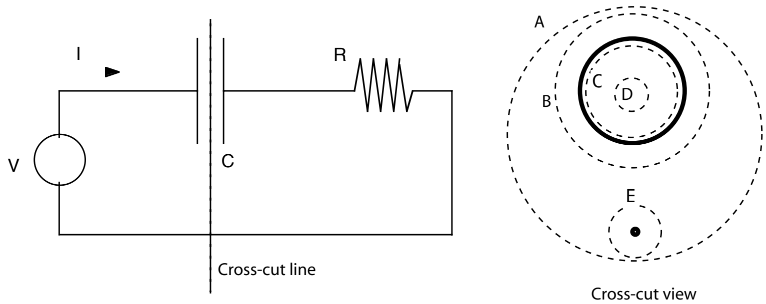


Figure 1: sketch of circuit for problem 1 (c)

The right portion of the figure shows a cross-section through the circuit at the mid-point of the parallel plate capacitor. There are 5 imaginary closed paths (A, B, C, D, E) drawn around portions of the capacitor and the return wire (the wire coming back to the AC source).

(i) Rank the absolute value of the $\int \vec{B} \cdot d\vec{s}$ in order (maximum = 1 to minimum = 5). (Break ties by which has the highest mean absolute or rms magnetic field.)

Path	(i) $\lvert\int \vec{B} \cdot d\vec{s}\rvert$	(ii) $\lvert\int \vec{E} \cdot d\vec{s}\rvert$
A		
B		
C		
D		
E		

(ii) If the capacitor is replaced by a solenoid - inductor that produces a uniform magnetic field inside the solid circle (between imaginary circles C and B and perpendicular to them), rank the absolute value of $\int E \cdot d\vec{s}$ around the imaginary paths in order (maximum = 1 to minimum =5).

(d) [5 points] Circle correct answer

(i) If the rms voltage in an AC circuit (made of resistors, capacitors, and inductors) is doubled, the peak current is

(A) increased by a factor of 2.
(B) decreased by a factor of 2.
(C) increased by a factor of $\sqrt{2}$.
(D) decreased by a factor of $\sqrt{2}$.
(E) not enough information to determine the change.

(ii) If the current in an inductor is doubled, its stored energy will

(A) increase by a factor of 2.
(B) decrease by a factor of 2.
(C) increase by a factor of 4.
(D) increase by a factor of 8.
(E) not changed.

(iii) A positively charge particle is moving northward in a static magnetic field The magnetic force on the particle is toward the northeast. What is the direction of the magnetic field?

(A) Upward
(B) West
(C) South
(D) Downward
(E) This situation cannot exist.
(F) Upward at an angle of 45° to East .

(iv) If the AC frequency driving an inductor is doubled, the inductive reactance of the inductor will

(A) increase by a factor of 2.
(B) not change.
(C) decrease by a factor of 2.
(D) increase by a factor of 4.
(E) decrease by a factor of 4.

(v) An ideal transformer has N_p turns on the primary and N_s turns on its secondary. The power dissipated in a load resistance R connected across the secondary is P_s, when the primary voltage is V_p. The current in the primary is then

(A) P_s/V_p
(B) $\frac{N_p}{N_s}\frac{P_s}{V_p}$
(C) $\frac{N_s}{N_p}\frac{P_s}{V_p}$
(D) $\left(\frac{N_s}{N_p}\right)^2\frac{P_s}{V_p}$
(E) $\left(\frac{N_s}{N_p}\right)^2\frac{V_p^2}{R}$

(e) [5 points] Circle correct answer

(i) Compasses point north because

(A) the north star attracts them.
(B) the Earth has an electric charge.
(C) there are electric currents in the iron core of the Earth.
(D) there are magnetic monopoles near the North Pole.
(E) there is a very large bar magnet in the Earth.

(ii) The Earth's magnetic field flips are used for

(A) creating new permanent magnets.
(B) proving the Earth has a solid iron core..
(C) generate useful power.
(D) geologic dating.
(E) bird migration.

(iii) Which is **not** a property of EM waves?

(A) E and B waves have the same velocity $v_E = V_B = c = 1/\sqrt{\epsilon_0 \mu_0}$.
(B) E and B waves are in phase.
(C) EM waves are transverse with E and B perpendicular to the direction of wave motion
(D) E and B magnitudes are in the ratio $E/B = c$.
(E) EM waves can self-propagate. They require no medium for propagation.
(F) EM waves carry both energy and momentum.
(G) The electric field of a wave decreases as one over the square of the distance from the source.

(iv) Magnetism comes from

(A) magnetic monopoles.
(B) moving quanta of light.
(C) quantization of charge.
(D) moving electric charge.
(E) magnesia and lodestones transferred to soft iron.

(v) AC is used instead of DC power because

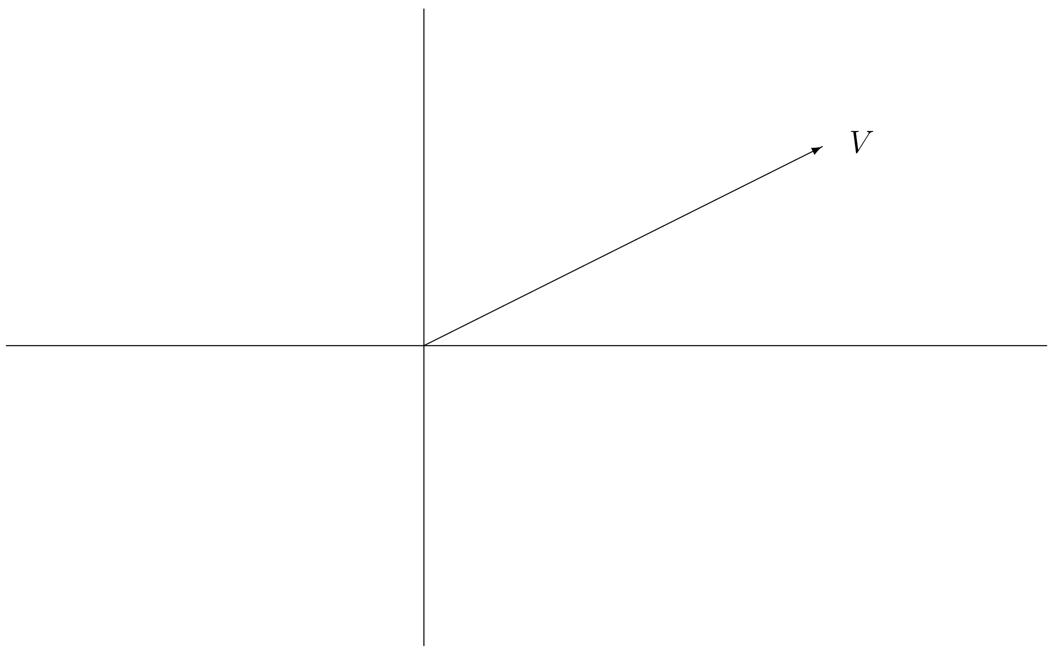
(A) it is safer.
(B) it carries more power.
(C) it makes the use of transformers straight forward.
(D) it is higher voltage
(E) Westinghouse had deeper-pocket backers than Edison.

2. [25 points] A resistor R, induct or L, and a capacitor C are connected in parallel to an AC voltage source V at angular frequency ω.

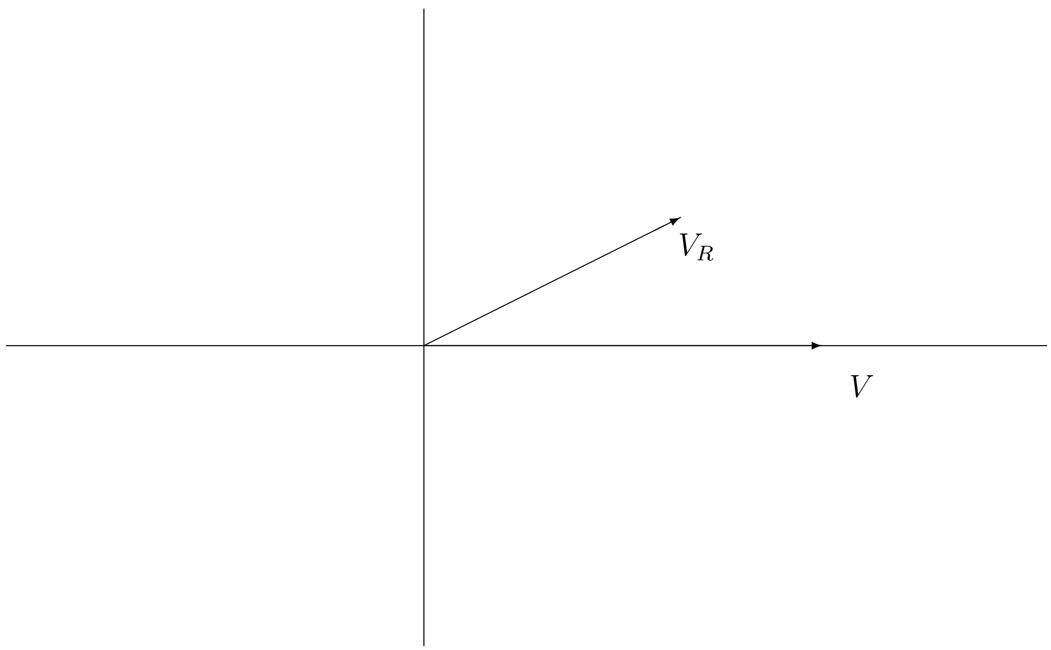
(a) [5 points] Find the current I and RMS current I_{RMS} from source V

(b) [5 points] Find the phase angle ϕ between V and I.

(c) [5 points] Make a diagram showing the phasors for each of the three components. Show both the voltages and currents on the following plot which shows the phasor for the voltage source. Label carefully each.



(d) [5 points] If the components, R, L, and C are connected in series with the AC voltage source, draw the phasor diagram showing currents and voltages. Given the voltage phasor of the source V and the voltage phasor of the resistor V_r



(e) [5 points] For the series configuration what value of capacitance C delivers the most power into the resistor R, when $R = 90\Omega$, $L = 100$ microHenries, and $\omega = 10^5$ Hz? What is that power?

3. [15 points] Consider a cylinder of radius R and uniform charge density ρ.

(a) [4 points] What is the electric field inside of cylinder?

(b) [4 points] What is the electric field E outside the cylinder?

(c) [5 points] What is the potential inside and outside the cylinder? What did you take as the location of zero potential?

(d) [2 points] Graph the field and potential as a function of r.

4. [20 points] Toyota and Honda both sell **Hybrid Car** models: the Prius and Insight respectively. An electric motor provides power to the wheels. An internal combustion heat engine operates at low speeds turning an alternator to charge a battery and at high-way speeds in parallel with the electric motor to provide power to the wheels.

Hybrid Car Infomation

Make Model	Toyota Prius	Honda Civic Hybrid	Honda Insight
City/Highway	51/60 mpg	47/48 mpg	60/65 mpg
Electric Motor	perm mag. AC synchronous	10 kW perm mag. DC	same
Battery Pack	201.6 V, 21 kW	144 V (120× 1.2 V 6.5 A-hr)	same
Heat Engine	4-cylinder 1.5 liters	3 cylinder 1 liter	same
Heat Engine Power	76 hp @ 5000 rpm	67 hp @ 5700 rpm	65 hp @ 5700 rpm
Hybrid Sys. net Power	110 hp (82 kW)	73 hp	71 hp

(a) [5 points] Show a calculation for the peak torque for the electric motor of the Honda Insight. Assume that the magnetic field is 1 Tesla, the battery voltage is 200 V (rounded number) the net resistance of the motor is 0.3 ohms, and the number of turns times the motor coil area is 3000 cm^2 (0.3 m^2).

(b) [5 points] Compute the torque of the electric motor versus its RPM (revolutions per minute of the motor) and plot is on this graph of torque vs. rpm for an internal combustion engine. Assume that the resistance of the motor winding is independent of the RPM.

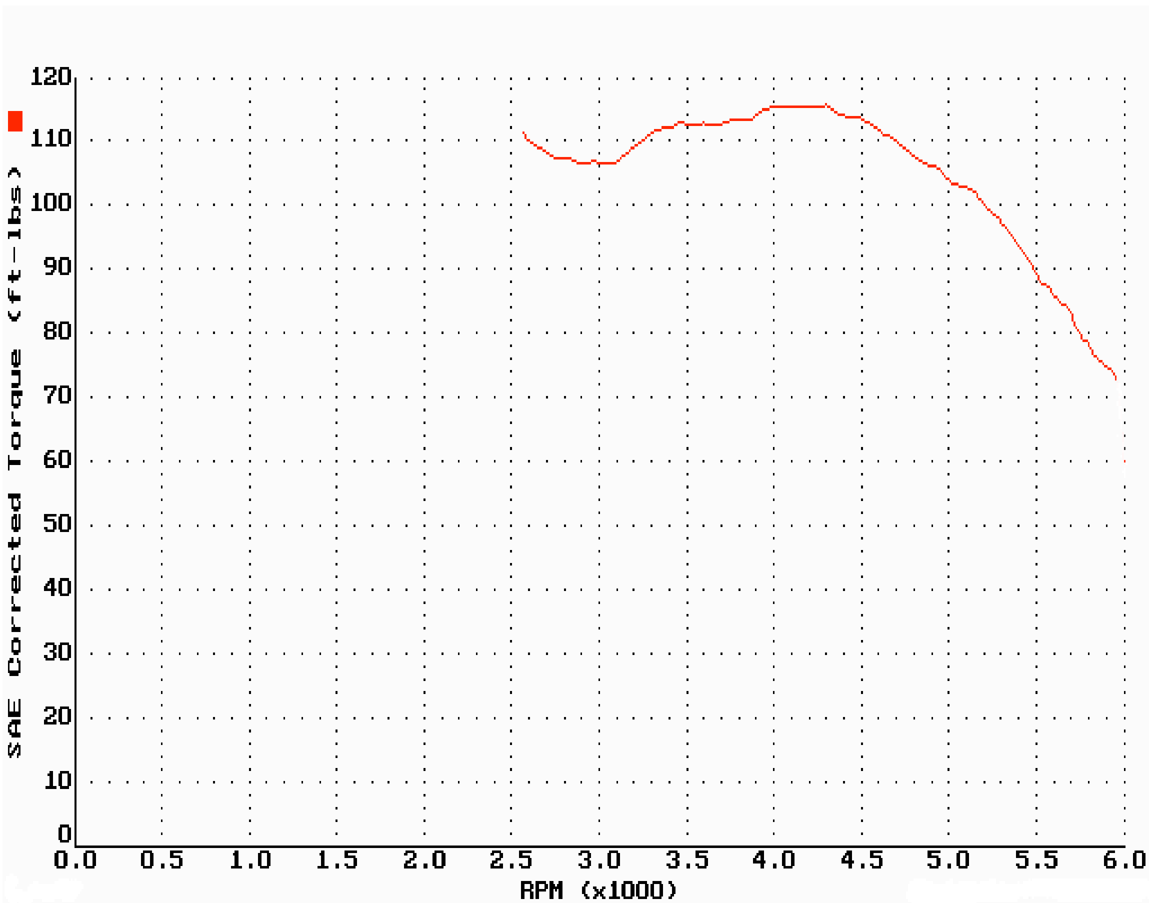


Figure 2: Torque versus RPM

(c) [5 points] From the numbers in the table compute the power output of the electric motor and plot is on this graph of horsepower vs. rpm for an internal combustion engine. *Hint:* From 7A remember Power is torque times angular frequency ($P = \tau\omega$) or that the power output of a motor is given by its Back-EMF times its current.

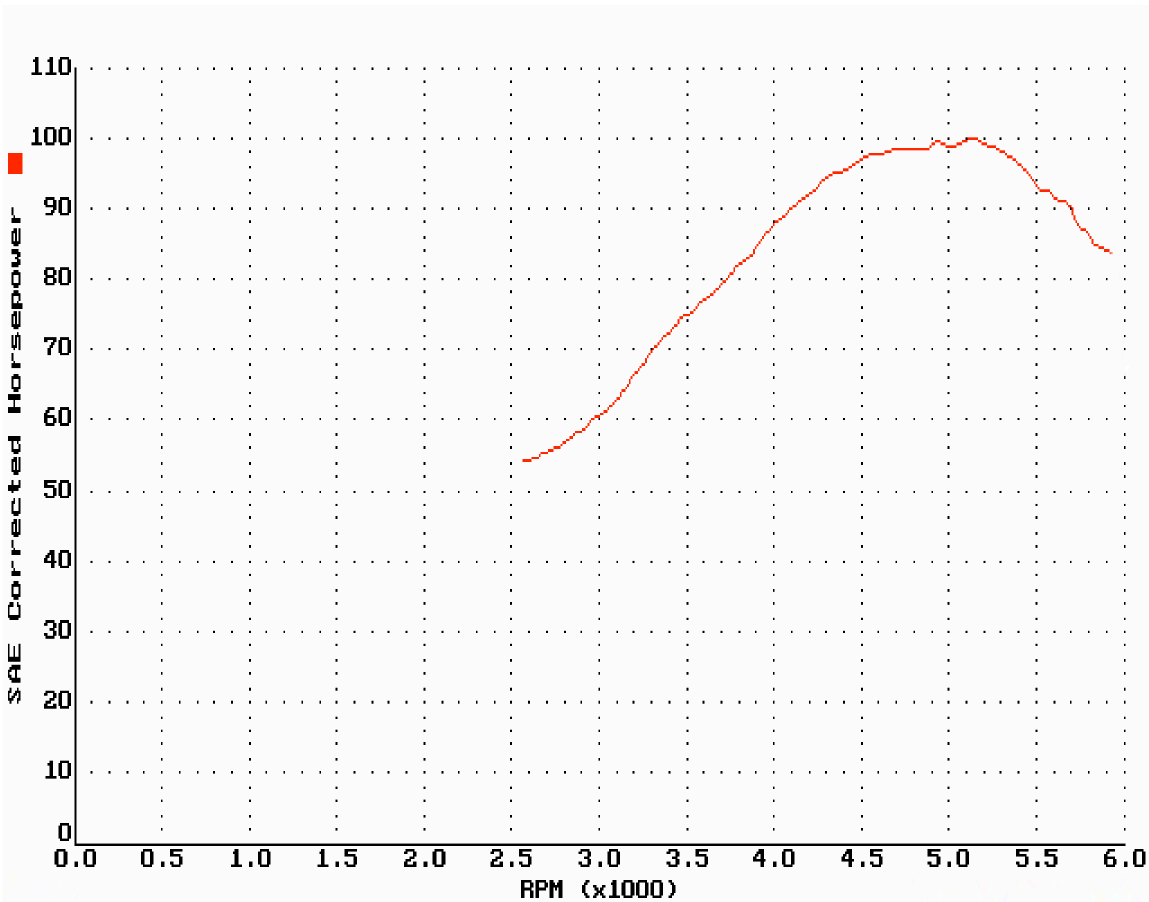


Figure 3: Horse Power versus RPM

(d) [5 points] Heat Engine efficiency and pollution question. Honda's Insight can go 700 miles on a single tank of gas. The Honda Civic hybrid can go 650 miles on a tank of gas, and the Toyota Prius can go about 500 miles. This is because the system has a higher efficiency than a standard internal combustion drive car. Hybrid cars get about 50 to 70 miles per gallon which is about twice a typical car.

A gallon of gas contains about 60 kilowatt-hours of chemical energy and is made up of chains of C_6H_{14} to $C_{10}H_{22}$. Its energy density is 45.7 MJ/kg. How much heat energy does a hybrid car getting 60 mpg produce in one hour at 60 mph on the level? How many kg of CO_2 and how many kg of H_2O in the same time? Assume that the ratio of carbon C to hydrogen H is 8 to 18. What is the volume of the gases at 0° C?

5. [15 points] Cell Phones and Microwaves

(a) [4 points] Older cellular phones operate in a frequency band of 824 to 894 MHz. What is their wavelength range? Newer phones (PCS) operate near 1900 MHz. What is the typical wavelength? Microwave ovens work at 2.45 GHz. What is the wavelength?

(b) [4 points] A cell phone communication tower 5 km away transmits 500 watts of power. What is the electric field at the cell phone in volts per meter? (Assume the power is beamed out in roughly a 120° by 10° fan covering 1/30th of a spherical surface.)

(c) [7 points] Which will give a bigger EMF an antenna rod 15 cm long or a circular loop antenna with a 15 cm diameter and 50 turns. Calculate the EMF from both antennas and compare.

5. [15 points] Two stiff parallel wires a distance l apart in a horizontal plane act as rails to support a light metal rod of mass m (perpendicular to each rail). A magnetic field $\vec{B}$, directed vertically upward, acts throughout. At time $t = 0$, the rails are connected to a constant current source and a current I begins to flow through the rails and the rod.

(a) [5 points] Determine the speed of the rod, which starts from rest at $t = 0$, as a function of time.

(b) [5 points] What is the voltage required to maintain the current I as a function of the rod velocity (and thus of time), if the voltage at the initial zero velocity is V_0 and the resistance of the rails is negligible compared to that of the rod plus the power source?

(c) [5 points] If the rod is uniform rectangular cross-section with width w parallel to the rails, and height h perpendicular to the rails, and develops a voltage V_w parallel to the rails what is the drift velocity of the charge carriers in the rod and what is their number density? What is the direction of the voltage, if the charge carriers have negative charge ? I.e. which side of the bar is positive?

6. [20 points] *CSI: Heart Attack or Electrocution?* One morning a man walks barefooted away from a picnic onto moist ground towards a tower that supports electric transmission lines. Suddenly, he collapses. His relatives at the picnic table see him fall, and upon reaching him a few seconds later, find that he is in ventricular fibrillation. The man dies before the emergency team arrives. Later the family files a lawsuit against the power company, claiming that the victim was electrocuted because of accidental current leakage from the tower. You are part of the forensic team investigating the death - was it a heart attack or electrocution?

Investigation of the power company records/data reveals that there was indeed an electrical fault at the tower that morning - for about 1 second a current $I = 100$ amperes leaked from the tower grounding rod into the ground. Assume that the current spreads uniformly (hemispherically) into the ground. Let ρ be the resistivity of the ground and r the distance from the rod (rounded with 1 cm radius).

Remember to express all answers first in symbolic form and then evaluate for $\rho = 100\ \Omega\ m$ and $r = 10$ m which is the man's distance from the rod when he falls.

(a) [4 points] Find the expression for the current density J as a function of r..

(b) [4 points] Find the electric field as a function of r and evaluate it at the man's position.

(c) [4 points] Find the electric potential as a function of r. What is the potential at the man's position?

(d) [3 points] If the man had one foot 0.5 m closer to the rod, find the potential difference between the man's feet. (Use one foot at 9.75 m and the other at 10.25 m.)

(e) [3 points] Assume that the resistance of a bare foot on wet soil has the typical value of

300 Ω and the resistance of the torso interior is the commonly accepted value of 1000 Ω. What was the current across the victim's torso?

(f) [2 points] The human heart can be put into fibrillation by a current of 0.1 to 1.0 A through the torso. Was the victim's fibrillation due to current leakage from the rod?

7. [25 points] **The Earth's Magnetic Field** The Earth's magnetic field can be well approximated by a dipole field. The Earth's radius is $R_e = 6378$ km and the magnetic field is about a Gauss (10^{-4} T) at its surface.

(a) [5 points] What electric current circling the equator at the outer radius of the fluid core $R_c = 3480$ km (about 2900 km deep) would be necessary to produce this magnetic field.

(b) [5 points] What is the magnetic moment of the Earth's field?

(c) [5 points] What is the energy stored in the Earth's field? *Hint:* Estimate the effective inductance of the current loop making the Earth's magnetic field as roughly $L = \Phi_B/I \sim BA/I$. Give a formula estimating the inductance of the Earth using only geometrical quantities and μ_0 and evaluate numerically.

(d) [5 points] If the conductivity of the Earth's core is roughly $\sigma = 4 \times 10^5$ / (Ohm - m) or the resistivity of $\rho = 2.5 \times 10^{-6}\ Ohm - m$ what is the decay time constant of the Earth's magnetic field. (Approximate the current density of the Earth as uniform over a cross-sectional area equal to the cross-sectional area of the molten core; i.e. that the cross-sectional area for computing the resistance is the same as the cross-sectional area of the Earth's molten core and the length of the resistor is the circumference of the core.)

(e) [5 points] Dynamo Theory of Magnetic field: The basic premise of dynamo theory is that all astrophysical bodies which contain anomalously long-lived magnetic fields also contain highly conducting fluids (e.g., the Earth's molten core, the ionized gas which makes up the Sun), and it is the electric currents associated with the motions of these fluids which maintain the observed magnetic fields. These currents, which are produced my the motion of a conductor through a magnetic field, in turn produce additional magnetic field. At first sight, this proposal, first made by Larmor in 1919 sounds suspiciously like pulling yourself up by your own shoelaces. However, there is really no conflict with the demands of energy conservation. The energy to make up ohmic losses is provided by thermal convection kinetic energy.

Give an argument why Ohm's law $\vec{E} = \rho\vec{J}$ ($V = IR$) must be modified to $\vec{E} + \vec{v} \times \vec{B} = \rho\vec{J}$ when the conducting material is moving with velocity $\vec{v}$.

Find the convection velocity necessary to keep the Earth's surface magnetic field essentially constant. *extra hint:* If $\vec{E}$ is zero, then the current in the effective inductance of the Earth is not changing.

8. [25 points] A **Far Infrared Sauna** designed to provide radiant (far infrared) for four people uses 2200 watts at 240 V AC.

(a) [5 points] There are eight identical heaters connected in parallel. What is the resistance (impedance) of a single heater and how much current flows through it at full power?

(b) [5 point] A conventional sauna directly heats rocks which then warms air which then rises and circulates finally contacting and warming a person's skin. In a far infrared sauna, the heating elements (resistor working at about 200°C imbedded in ceramic) emit radiation from 5 to 20 microns peaked at about 8 microns wavelength of which less than 20% goes to heating the air and and more than 80% is available to warm people in the sauna. What are the three methods of heat transfer illustrated in the two kinds of sauna and how do they work? What is the effective total area of the eight heaters?

(c) [5 points] The far-Infrared radiation

(i) What is the frequency range of the radiation?

(ii) What is the average and rms electric field in the sauna from the radiation right in front of a heater? (*Hint:* What is the E-field just in front of a 200°C blackbody?)

(iii) What is the radiation pressure in the sauna directly in front of a heater?

(iv) If a single person is in the sauna and absorbs 80% of the total heat, what is the force of the radiation on that person?

(d) [3 points] How much is the entropy change due to operating the Sauna for a 30 minute session (heater resistors at 200°C)? First calculate the entropy generated by the heaters and then that due to the degradation of the heat to the people and the sauna at about 50°C.

(e) [3 points] Now add in the power production. Consider the efficiency of electric power plant to be Carnot using superheated steam at 550°F and cold heat bath of 50°F? What is the efficiency of the power plant? What is the rate change of entropy for the electric power produced?

(f) [4 points] If, due to irreversible processes, the power plant efficiency is 80% Carnot efficiency, then what is rate of entropy production at the power plant?

End of Examination