Math 1B Final Exam
Lecture 3, Spring 2010

The exam is closed book, apart from a sheet of notes 8"x11". Calculators are not allowed. It is your responsibility to write your answers clearly.

1. Using a trigonometric substitution, evaluate

$$\int_0^{\sqrt{3}/2} \frac{1}{(1-x^2)^{1/2}} dx$$

   Recall that $\sqrt{3}/2 = \sin \pi/3 = \cos \pi/6$.

2. Solve the equation

$$t\frac{du}{dt} = u + t^2 \cos t \quad (t > 0)$$

   and find a solution that satisfies $u(\pi/2) = 0$.

3. Use substitution and integration by parts to find:

$$\int (\cos x)^3 \, e^{\sin x} dx$$

4. Determine if the series absolutely converges, conditionally converges, or diverges.

$$\sum_{n=1}^{\infty} (-1)^n \ln\left(\frac{n}{2n+1}\right)$$

5. Show that integral

$$\int_1^{\infty} \frac{e^x}{x + e^{2x}} dx$$

converges or diverges using the comparison test.

6. Consider the differential equation

$$\frac{dy}{dx} = \frac{1 - x}{2y}$$

i) Sketch a direction field for the region $-1 \leq x \leq 3$, $0 < y \leq 3/2$, including at least 15 points. Also include labeled axes.

ii) Solve the differential equation. Express y explicitly in terms of x.

iii) Find a solution through $x = 2$, $y = 1$, and sketch it on the direction field graph.

7. Find the integral

$$\int_0^2 \frac{x}{(x^2 - 1)^2} dx,$$

if it converges. If it does not converge, show why that happens.

8. Find the Taylor series for

$$\frac{1}{2} x^2 (e^x - e^{-x})$$

around $x = 0$. What is the coefficient of x^n? What is its radius of convergence?

9. Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$.

i) Use the limit comparison test to show that the series is convergent.

ii) Determine whether

$$0 \leq \sum_{n=1}^{\infty} \frac{1}{n^2+1} \leq \frac{\pi}{2}$$

is true or false by comparing the series to an integral from 0 to infinity.

10. Find the Maclaurin series for

$$x \cos x - \sin x$$

and use that to find the limit

$$\lim_{x \to 0} \frac{x \cos x - \sin x}{x^3}$$

11. Find the radius and the interval of convergence of the power series

$$\sum_{n=2}^{\infty} (-1)^n \frac{x^n}{2^n \ln n}$$

12. Use the power series method to find the general solution to

$$y'' = x^2 y$$

Then, find the solution to the initial value problem $y(0) = 1, y'(0) = 5$.

13. Solve the differential equation

$$y'' + y' - 6y = 4x^2 e^x + e^{-x}$$