MATH 110, Fall 2015, midterm test #2.

Name :
Student ID # :
GSI name :
Discussion meeting time :

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit. Partial credit **may** be given but only for significant progress towards a solution. Show all relevant work in logical sequence and indicate all answers clearly. Cross out all work you do not wish considered. Books and notes are allowed. Electronic devices are not allowed during the test.

1. (10pp.) Let V be a (not necessarily finite-dimensional) vector space and let U be a subspace of V. Show that the space $(V/U)'$ is isomorphic to U^0. (A correct proof assuming that V is finite-dimensional will receive 5pp.)

2. (10pp.) Let V be a 4-dimensional vector space and let $T \in \mathcal{L}(V)$ be diagonalizable. Assuming the distinct eigenvalues of T are λ_1, λ_2, and λ_3, show that

$$p(T) = 0 \quad \text{where} \quad p(\lambda) := (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3).$$

3. (10pp.) Let V be a finite-dimensional vector space. Show that λ is an eigenvalue of $T \in \mathcal{L}(V)$ if and only if it is an eigenvalue of T'.

4. (10pp.) Consider the complex vector space $V := \operatorname{span}(1, \cos x, \sin x, \cos 2x, \sin 2x)$ with the inner product

$$\langle f, g \rangle := \frac{1}{2\pi} \int_0^{2\pi} f(t)\overline{g(t)}dt$$

and its subspace $U := \operatorname{span}(1, \cos x, \sin x)$. Find

(a) $g \in V$ which represents $\phi \in V' : f \mapsto f(0)$ in the sense $\langle f, g \rangle = \phi(f)$ for all $f \in V$;

(b) its orthogonal projection $P_U g$.