MATH 110, Fall 2015, midterm test #1.

Name :
Student ID # :
GSI name :
Discussion meeting time :

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit. Partial credit **may** be given but only for significant progress towards a solution. Show all relevant work in logical sequence and indicate all answers clearly. Cross out all work you do not wish considered. Books and notes are allowed. Electronic devices are not allowed during the test.

1. (10pp.) Consider the linear map $T : \mathcal{P}_2(\mathbb{R}) \to \mathbb{R}^3 : f \mapsto (f(-1), f(0), f(1))$. Determine the dimensions of $\operatorname{null} T$ and $\operatorname{range} T$. Is T invertible?

2. (10pp.) Let $U := \{f \in \mathcal{P}_3(\mathbb{R}) : f(1) = 0\}$ and $W := \{f \in \mathcal{P}_3(\mathbb{R}) : f'(1) = 0\}$. Choose a basis for $U \cap W$ and extend it in two ways: to a basis of U and to a basis of W. Is the resulting list of all vectors (the basis for $U \cap W$ and all the vectors you added for the two extensions) a basis for $\mathcal{P}_3(\mathbb{R})$? Explain your answer. (Note: you are *not* required to prove that U and W are subspaces of $\mathcal{P}_3(\mathbb{R})$; they are.)

3. (10pp.) Prove or disprove: the three linear maps $\mathbb{I}$, D, $D^2 (= D \circ D)$ are linearly independent in $\mathcal{L}(\mathcal{P}(\mathbb{R}), \mathcal{P}(\mathbb{R}))$. ($\mathbb{I}$ is the identity map; D is the differentiation map $f \mapsto f'$.)

4. (10pp.) Let

$$T : \mathbb{R}^{2,2} \to \mathbb{R}^{2,2} : \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mapsto \begin{bmatrix} b & c \\ d & a+c \end{bmatrix}.$$

Prove that T is a linear map and find its matrix representation with respect to the standard basis of $\mathbb{R}^{2,2}$

$$(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix})$$

(used as a basis both for the domain and co-domain).