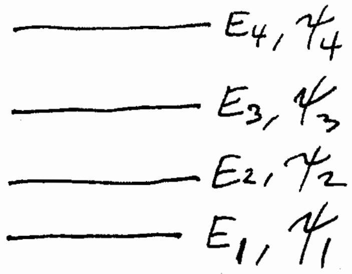
Physics 137B, Fall 2004
Quantum Mechanics II
Midterm I 10/8/2004, 9:10-10:00 a.m.

No books, notes, or calculators are allowed. Please start each of the 4 problems on a fresh side. You should make an effort to answer every problem.

I. Consider a one-dimensional potential well with single-particle energy eigenvalues $E_1, E_2, E_3, E_4, \ldots$, with corresponding orbital eigenstates $\psi_1, \psi_2, \psi_3, \psi_4, \ldots$. Here is an energy level diagram:



(a) (12 pts) Suppose 5 identical, noninteracting particles are placed in the well. Find the ground state energy when

(i) the particles all have spin 1/2.
(ii) the particles all have spin 1.
(iii) the particles all have spin 3/2.

(b) (13 pts) Suppose two identical noninteracting spin 1/2 particles are placed in the well. What is the degeneracy (i.e., number of degenerate states) of the ground state, including spin? What is the degeneracy of the first excited state, including spin? Write a physically allowed wavefunction (it does not need to be normalized), including spin, for one of the first excited states, and give the total spin angular momentum quantum number, that is, the number j such that $(bfS_1 + \mathbf{S}_2)^2\Psi = j(j+1)\hbar^2\Psi$, of your state.

II. (25 pts) A system is composed of two stationary particles having spin-1/2 and spin-1 respectively. An interaction between the particles leads to the following Hamiltonian:

$$H = A\mathbf{S}_1 \cdot \mathbf{S}_2. \quad (1)$$

What are the allowed energy levels of this system? What are the degeneracies?

III. Consider a particle in a box: the particle is confined to move in one dimension between $x = 0$ and $x = L$. The energy eigenfunctions are

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L}) \tag{2}$$

and the energy levels are

$$E_n = \frac{\pi^2 n^2 \hbar^2}{2mL^2}. \tag{3}$$

For this question, you may leave your answers in the form of a well-defined definite integral: **you do not need to do the integral**, but it should be well-defined and have the correct units.

(a) (15 pts) How much does the energy of the ground state ($n = 1$) change to first order if a **weak** spring connects the particle to $x = L/2$? That is, there is now an additional potential energy

$$V'(x) = k(x - L/2)^2/2. \tag{4}$$

(b) (5 pts) Will the first-order energy gain of the ground state $E_1^{(1)}$ be larger or smaller than the first-order energy gain of the first excited state $E_2^{(1)}$? You do not need to do a computation as long as you explain convincingly the reason for your answer (say, by drawing a picture of these states and explaining).

IV. For this problem you do not need to do any calculations to support your answers.

(a) (14 pts) Suppose that the electron in a hydrogen atom is subjected to a weak perturbation

$$H' = aL^2 + bS_z \tag{5}$$

How does this split the 8 initially degenerate $n = 2$ levels? That is, give the new energy shifts to first order in the perturbation. You do not need to give the corresponding eigenstates.

(b) (6 pts) Are the energy levels different if instead $H' = aL^2 + bS_x$? This is the same perturbation as before except that now the direction of the S term is along x. A straight yes or no answer is fine. Are the energy eigenstates different? (again a straight yes or no is fine)

(c) (10 pts) The perturbation Hamiltonian that is generated by a magnetic field B along the $\hat{z}$ direction is

$$H' = B\mu_B(L_z + gS_z)/\hbar. \tag{6}$$

How does this split the eight initially degenerate $n = 2$ levels? This time give both the energy shifts and, for each level, the quantum numbers ℓ, m_ℓ, and m_s that correspond to that energy.