EE 16B Midterm 2, October 25, 2016

Name: ________________

SID #: ________________

Discussion Section and TA: ________________

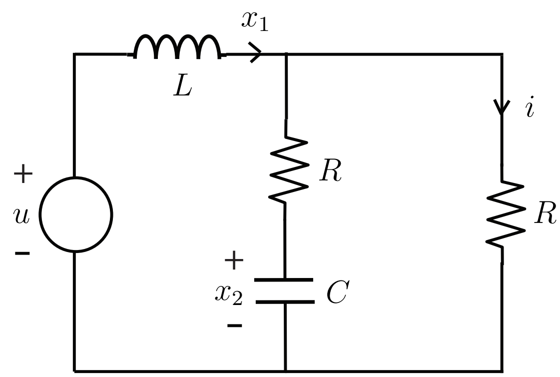
Lab Section and TA: ________________

Important Instructions:

- **Show your work.** An answer without explanation is not acceptable and does not guarantee any credit.
- **Only the front pages will be scanned and graded.** If you need more space, please ask for extra paper instead of using the back pages.
- **Do not remove pages**, as this disrupts the scanning. Instead, cross the parts that you don't want us to grade.

Problem	**Points**	**Score**
1	15	
2	10	
3	15	
4	25	
5	20	
6	15	
Total	100	

1. (15 points) Consider the circuit below that consists of two *identical* resistors, an inductor, a capacitor, and a voltage source whose voltage at time t is $u(t)$.



a) (5 points) Write an expression for the current i indicated on the circuit diagram in terms of x_1 (the inductor current) and x_2 (the capacitor voltage). Your answer should **not** involve any derivatives.

b) (10 points) Write a state space model using the states $x_1(t)$, $x_2(t)$, and input $u(t)$. Your final answer should specify the A and B matrices, with entries that depend on R, L, C.

Additional workspace for Problem 1b.

2. (10 points) Consider the scalar discrete-time system

$$x(t+1) = f(x(t))$$

where

$$f(x) = 2x - 2x^2.$$

a) (2 points) What is the solution $x(t)$ for $t > 0$ if $x(0) = 0.5$?

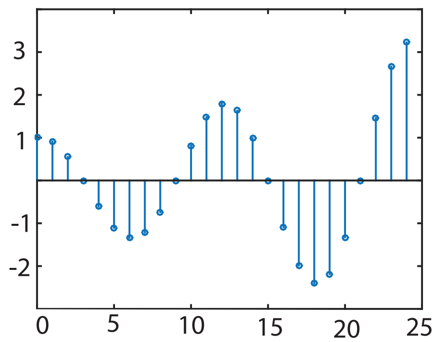
b) (3 points) Find **all** equilibrium points of the system.

c) (5 points) Linearize the system around each equilibrium and determine stability for the resulting linear models.

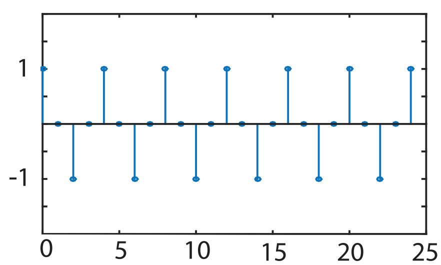
3. (15 points) Each plot below shows $x_1(t)$ obtained from the solution of

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} = A \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

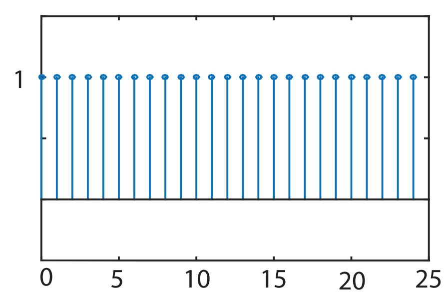
where A is one of the matrices below. Match each matrix to a plot and write the corresponding letter (a, b, c, or d) in the box next to each plot.



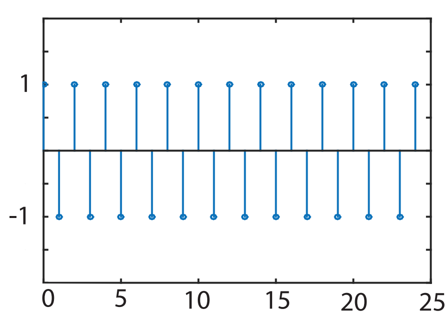
(a) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$



(b) $A = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$



(c) $A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$



(d) $A = 1.05 \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$

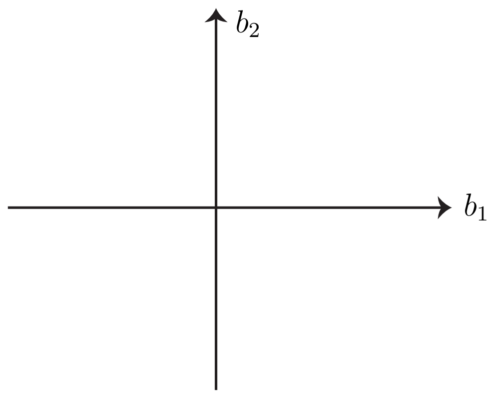
Additional workspace for Problem 3.

4. (25 points) Consider the system

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} = \begin{bmatrix} 1.5 & 1 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(t).$$

a) (5 points) Determine if the system is stable.

b) (5 points) Determine the set of all (b_1, b_2) values for which the system is **not** controllable and sketch this set of points in the b_1-b_2 plane below.



c) (6 points) Suppose $b_1 = 1$ and $b_2 = 0$. Design a state feedback controller such that the closed-loop system is stable.
Note: The problem doesn't prescribe specific eigenvalues, other than that they be in the stable region. Therefore controllability is not a necessary condition.

d) (9 points) Suppose $b_1 = 0$ and $b_2 = 1$. Design a state feedback controller such that the closed-loop system eigenvalues are $\lambda_1 = \lambda_2 = 0.5$.

Additional workspace for Problem 4d.

5. (20 points) Consider the system

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$
$$y(t) = x_1(t).$$

a) (4 points) Show that the system is observable.

b) (6 points) Suppose we measure $y(t) = x_1(t)$ at $t = 0$ and $t = 1$, and find

$$y(0) = 1, \qquad y(1) = 0.$$

Determine the unmeasured state $x_2(t)$ at $t = 0$ and $t = 1$:

$\boxed{x_2(0) = \qquad\qquad}$ $\boxed{x_2(1) = \qquad\qquad}$

Additional workspace for Problem 5b.

c) (10 points) Select values for l_1 and l_2 in the observer below such that $\hat{x}_1(t)$ and $\hat{x}_2(t)$ are guaranteed to converge to $x_1(t)$ and $x_2(t)$. (You are free to choose appropriate eigenvalues that guarantee convergence.)

$$\begin{bmatrix} \hat{x}_1(t+1) \\ \hat{x}_2(t+1) \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \end{bmatrix} + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} (\hat{x}_1(t) - y(t))$$

Additional workspace for Problem 5c.

6. (15 points) Suppose we have two systems,

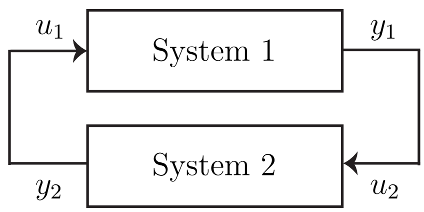
$$\text{System 1:} \quad \vec{x}_1(t+1) = A_1\vec{x}_1(t) + B_1u_1(t), \quad y_1(t) = C_1\vec{x}_1(t),$$

$$\text{System 2:} \quad \vec{x}_2(t+1) = A_2\vec{x}_2(t) + B_2u_2(t), \quad y_2(t) = C_2\vec{x}_2(t),$$

and connect the output of the first to the input of the second, and vice versa:

$$u_1(t) = y_2(t) \quad \text{and} \quad u_2(t) = y_1(t).$$

The dimensions of the states, inputs, and outputs above are arbitrary, except that the output of one system must have the same dimension as the input of the other. The resulting interconnection is shown in the block diagram below.



a) (5 points) Fill in the four blocks of the matrix below which describes the combined state model.

$$\begin{bmatrix} \vec{x}_1(t+1) \\ \vec{x}_2(t+1) \end{bmatrix} = \left[\begin{array}{c|c} \quad\quad & \quad\quad \\ \hline \quad\quad & \quad\quad \end{array}\right] \begin{bmatrix} \vec{x}_1(t) \\ \vec{x}_2(t) \end{bmatrix}.$$

b) (10 points) Show that stability of both System 1 and System 2 does **not** guarantee stability for the interconnection.

Hint: Construct an example where A_1 and A_2 each satisfy the discrete-time stability condition, but B_1, B_2, C_1, C_2 are such that the matrix you found in part (a) fails the stability condition.

Additional workspace for Problem 6b.