MIDTERM 1 - ME 104

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a) $\begin{cases} \underline{e}_r = \cos\theta\, \underline{i} + \sin\theta\, \underline{j} \\ \underline{e}_\theta = -\sin\theta\, \underline{i} + \cos\theta\, \underline{j} \end{cases}$

$\dot{\underline{e}}_r = \dot\theta \dfrac{d\underline{e}_r}{d\theta} \overset{①}{=} \dot\theta(-\sin\theta\, \underline{i} + \cos\theta\, \underline{j}) = \boxed{\dot\theta\, \underline{e}_\theta}$ ①

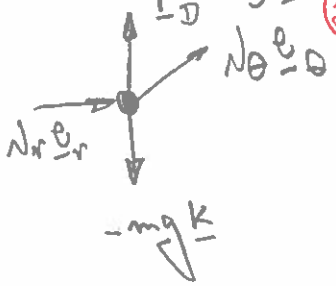
$\dot{\underline{e}}_\theta = \dot\theta \dfrac{d\underline{e}_\theta}{d\theta} \overset{①}{=} \dot\theta(-\cos\theta\, \underline{i} - \sin\theta\, \underline{j}) = \boxed{-\dot\theta\, \underline{e}_r}$ ①

$\underline{r}_B = R\underline{e}_r + z\underline{k}$

$\underline{v}_B = \dot{\underline{r}}_B = \boxed{\dot R \underline{e}_r + R\dot\theta\, \underline{e}_\theta + \dot z \underline{k}}$ ②

b) $R = \text{const.}$, $\dot\theta = \text{const.}$, $\dot z = \text{const.} > 0$ $\Rightarrow \underline{a}_B = -R\dot\theta^2 \underline{e}_r$ ①

FBD:



$\underline{F} = m\underline{a}$

$\underline{F}_D - mg\underline{k} + \underbrace{N_r \underline{e}_r + N_\theta \underline{e}_\theta}_{\underline{N}} = -R\dot\theta^2 \underline{e}_r$ ②

$\Rightarrow$ $\cdot\, \underline{e}_\theta$: $N_\theta = 0$

$\cdot\, \underline{e}_r$: $N_r = -R\dot\theta^2$ ③

$\cdot\, \underline{k}$: $F_D = mg$

$\Rightarrow$ $\boxed{\begin{aligned} \underline{N} &= -R\dot\theta^2 \underline{e}_r \\ \underline{F}_D &= mg\underline{k} \end{aligned}}$ ①

c)

- $\underline{r}_c = R_0 \underline{e}_r + h\underline{k}$
- $\underline{v}_c = \dot{\underline{r}}_c = R_0 \dot{\theta} \underline{e}_\theta \overset{(1)}{=} \underbrace{(\dot{\theta}\underline{k})}_{\underline{\omega}} \times (R_0 \underline{e}_r + h\underline{k}) = \underline{\omega} \times \underline{r}_c$ (1)

- $\underline{a}_c = \dot{\underline{v}}_c = \underbrace{\dot{\underline{\omega}}}_{\underline{0}} \times \underline{r}_c + \underline{\omega} \times \underbrace{\dot{\underline{r}}_c}_{\underline{\omega} \times \underline{r}_c} \overset{(1)}{=} \underline{\omega} \times (\underline{\omega} \times \underline{r}_c)$ (1)

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(2)

$$\underline{e}_t = \frac{d\underline{r}}{ds} \quad , \quad \underline{e}_t \cdot \underline{e}_t = 1$$

a)

$$\frac{d}{ds}(\underline{e}_t \cdot \underline{e}_t) = \frac{d}{ds}(1) \qquad 4$$

$$\Rightarrow \quad \underline{e}_t \cdot \frac{d\underline{e}_t}{ds} = 0 \quad , \quad \underline{e}_t \cdot K\underline{e}_n = 0$$

$$\Rightarrow \boxed{\underline{e}_t \cdot \underline{e}_n = 0}$$

b)

$$\underline{v} = \dot{\underline{r}} = \frac{d\underline{r}}{ds}\frac{ds}{dt} = \dot{s}\,\underline{e}_t$$

$$\underline{a} = \dot{\underline{v}} = \ddot{s}\,\underline{e}_t + \dot{s}\,\dot{\underline{e}}_t \qquad 6$$

$$\dot{\underline{e}}_t = \frac{d\underline{e}_t}{ds}\frac{ds}{dt} = \dot{s} K \underline{e}_n$$

$$\Rightarrow \quad \underline{a} = \ddot{s}\,\underline{e}_t + K\dot{s}^2\,\underline{e}_n$$

c)

$$\underline{v} \times \underline{a} = \dot{s}\,\underline{e}_t \times (\ddot{s}\,\underline{e}_t + K\dot{s}^2\,\underline{e}_n)$$

$$= K\dot{s}^3\,\underline{e}_b$$

$$\|\underline{v} \times \underline{a}\| = K\|\underline{v}\|^3 \quad (\dot{s} = \|\underline{v}\|) \qquad 4$$

$$\Rightarrow \quad K = \frac{\|\underline{v} \times \underline{a}\|}{\|\underline{v}\|^3}$$

d)

$$\underline{v}_B = R_0 \dot{\theta}\, \underline{e}_\theta + \dot{z}\, \underline{k}$$

$$\underline{a}_B = -R_0 \dot{\theta}^2\, \underline{e}_R$$

$$\underline{\omega} = \dot{\theta}\, \underline{k} = \frac{30(2\pi)}{60}\, \underline{k} = \pi\, \underline{k} \ \text{rad/s}$$

$$\dot{z} = 1 \ \text{m/s}$$

$$R_0 = 0.2 \ \text{m}$$

$$\Rightarrow \underline{v}_B = 0.6283\, \underline{e}_\theta + \underline{k} \qquad \|\underline{v}_B\| = 1.181 \ \text{m/s}$$

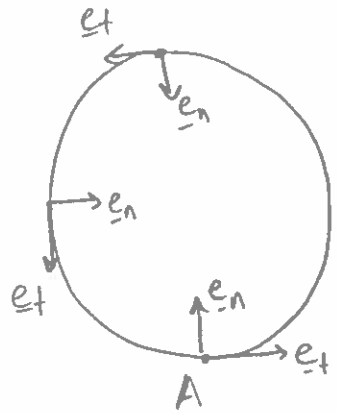
$$\underline{a}_B = -1.9739\, \underline{e}_R$$

$$\underline{v}_B \times \underline{a}_B = 1.2403\, \underline{k} - 1.9739\, \underline{e}_\theta \qquad 6$$

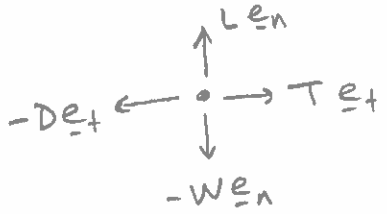
$$\|\underline{v}_B \times \underline{a}_B\| = 2.3312$$

$$\rho = \frac{\|\underline{v}_B\|^3}{\|\underline{v}_B \times \underline{a}_B\|} = \underline{\underline{0.7066 \ \text{m}}}$$

e)



FBD



$$\underline{F} = m\,\underline{a}$$

$$(T-0)\,\underline{e}_t + (L-mg)\,\underline{e}_n = m\,\underline{a}$$

① $T - D = m a_t = 0$ $\boxed{T = D}$

② $L - mg = m(24.1127)$

$$L = m(33.9227)$$

@ A $\boxed{\underline{a} = 24.1127\, \underline{e}_n}$