1.

(a) The initial total energy is just the kinetic energy. There is no electric potential energy because the particle m is infinitely far away.

$$E_i = K$$

The final total energy is just the electric potential energy. There is no kinetic energy because the particle is at rest. Using U=qV,

$$V = kQ/r$$

$$E_f = qV = kQq/r$$

Using energy conservation,

$$K = kQq/r$$

$$r = kQq/K$$

(b) Now since the final distance is 2r, the final state has some kinetic energy.

$$K = kQq/(2r) + \tfrac{1}{2} m v_f^2$$

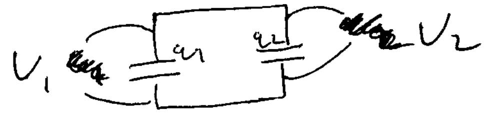
Using the result of (a),

$$K = K/2 + \tfrac{1}{2} m v_f^2$$

$$v_f = (K/m)^{1/2}$$

Problem #2

a) $V_1 = V_2 = V$



$q_0 = q_1 + q_2$ (conservation of charge)

$\Rightarrow q_0 = C_1 V + C_2 V \Rightarrow \boxed{V = \dfrac{q_0}{C_1 + C_2}}$

b) Before

$U_i = U_1 + U_2$

$= \frac{1}{2} C_1 V_0^2 + 0$

$= \boxed{\frac{1}{2} C_1 V_0^2}$

After

$U_f = U_1 + U_2$

$= \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2$

$= \frac{1}{2} (C_1 + C_2) V^2$

$= \frac{1}{2} (C_1 + C_2) \dfrac{q_0^2}{(C_1 + C_2)^2}$

$= \boxed{\frac{1}{2} \dfrac{q_0^2}{(C_1 + C_2)}}$

c) $U_i = \frac{1}{2} C_0 V_0^2$

for parallel plate

$C_0 = \dfrac{\varepsilon_0 A}{d}$

$\Rightarrow U_i = \boxed{\frac{1}{2} \left(\dfrac{\varepsilon_0 A}{d} \right) V_0^2}$

After dielectric is inserted

$C_0 \to K C_0$

but since the capacitor is no longer connected to a battery

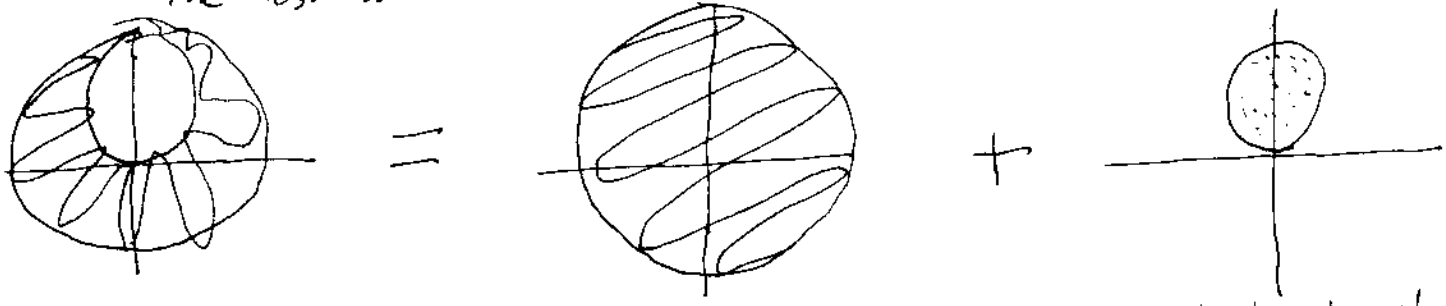
$Q \to Q \Rightarrow V_0 \to V_0 / K$

$\Rightarrow U_f = \frac{1}{2} \left(\dfrac{K \varepsilon_0 A}{d} \right) \left(\dfrac{V_0}{K} \right)^2 = \boxed{\dfrac{1}{2K} \left(\dfrac{\varepsilon_0 A}{d} \right) (V_0)^2}$

Since $K > 1$ the energy stored in the capicator goes down in the presence of the dielectric. This is becauce the capacitor expended work to bring the dielectric in, thus reducing its potential energy.

3. Our charge distribution exhibits no convenient symmetry $\Rightarrow$ can't use Gauss' Law directly, nor integrate $\int dE$ easily.

But, we notice that our charge distribution is identical to the <u>superposition</u> of a positively charged ball of radius $2a$, plus a negatively charged ball of radius a. We can use Gauss' Law on those configurations independently then add the results.



positively charged ball A

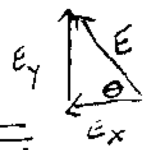
negatively charged ball B

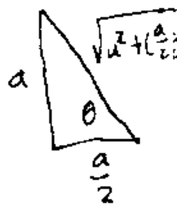
E-field inside a charged sphere:

$$\int E \cdot dA = \frac{Q_{enc.}}{\epsilon_0}$$

$$\underbrace{E \cdot 4\pi r^2}_{\text{surface of Gaussian sphere}} = \underbrace{\tfrac{4}{3}\pi r^3}_{\text{volume of Gaussian sphere}} \rho/\epsilon_0 \quad \Rightarrow \quad E = \frac{\rho r}{3\epsilon_0}\hat{r}$$

So $\vec{E}_A = \dfrac{\rho r}{3\epsilon_0}\hat{r} = \dfrac{\rho}{3\epsilon_0}\sqrt{a^2+(\frac{a}{2})^2}\ \hat{r}$.





Resolve into x, y components: $E_x = -E\cos\theta = E \cdot \dfrac{a}{2\sqrt{a^2+(\frac{a}{2})^2}}$

$$E_y = E\sin\theta = E \cdot \frac{a}{\sqrt{a^2+(\frac{a}{2})^2}}$$

$$\vec{E}_A = \frac{\rho}{3\epsilon_0}\sqrt{\tfrac{5}{4}a^2} \cdot \frac{a}{2\sqrt{\frac{5}{4}a^2}}(-\hat{x}) + \frac{\rho}{3\epsilon_0}\sqrt{\tfrac{5}{4}a^2} \cdot \frac{a}{\sqrt{\frac{5}{4}a^2}}(\hat{y})$$

$$\vec{E}_A = \frac{\rho}{3\epsilon_0}\,\frac{a}{2}\,(-\hat{x}) + \frac{\rho}{3\epsilon_0}\,a\,(\hat{y})$$

negatively charged

$$\vec{E}_B = \frac{-\rho r}{3\epsilon_0}\,\hat{r}$$

relative to the center of this negative sphere, which is located at $(0, a)$

point: $(-\frac{a}{2}, a)$

sphere center: $(0, a)$

$\hat{r} = (-\frac{a}{2}, 0) = -\hat{x}$

$$\vec{E}_B = \frac{\rho(\frac{a}{2})}{3\epsilon_0}\,\hat{x}$$

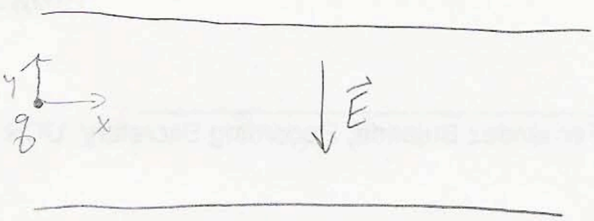
[no $\hat{y}$ component]

$$\vec{E}_{total} = \vec{E}_A + \vec{E}_B$$

$$= \boxed{\frac{\rho}{3\epsilon_0}\,a\,\hat{y}}$$

Ramesh exam:

4



a) $\vec{F} = q\vec{E} = qE(-\hat{y}) \longrightarrow m\vec{a} = qE(-\hat{y})$

$\vec{F} = m\vec{a} \longrightarrow$

acceleration in $-\hat{y}$ direction only

$a_y = -\frac{qE}{m}$

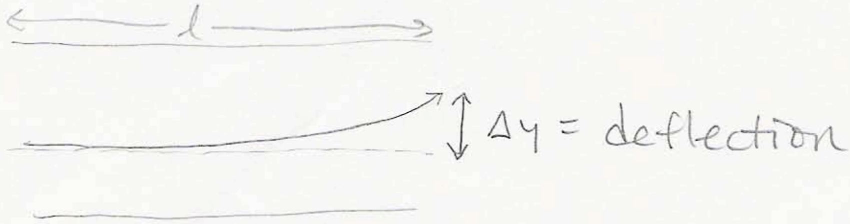
$y = \frac{1}{2}a_y t^2 = -\frac{qE}{2m}t^2 = y$

$x = 0$

(starts at rest at origin)

$\longrightarrow \boxed{\begin{aligned} x &= 0 \\ y &= -\frac{qE}{2m}t^2 \end{aligned}}$

b) electron $\longrightarrow$ $q = -e$ $\rightarrow$ Force is in $+y$ direction



Initially, velocity is in x-direction.

$q = -1.6 \times 10^{-19}$ C

$m_e = 9.1 \times 10^{-31}$ kg

$KE = \frac{1}{2}mV_x^2$

$V_x = \sqrt{\frac{2KE}{m}}$

Time to reach end of tube

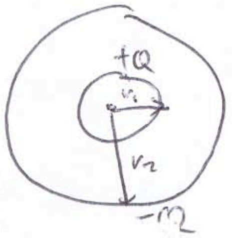
$l = V_x t$

$t = \frac{l}{V_x} = l\sqrt{\frac{m}{2KE}}$

y at time t: $y = \frac{1}{2}a_y t^2 = -\frac{qE}{2m}t^2 = \frac{eE}{2m}\left(\frac{l^2 m}{2KE}\right) = \frac{eEl^2}{4KE} = 3.375 \times 10^{-4}$ m

$\Delta y = y(t) - y(0) = y(t) = \boxed{3.375 \times 10^{-4}\text{ m} = \Delta y}$

5 10 points: Capacitance of concentric spherical shells



- $C = \dfrac{Q}{V(r_1) - V(r_2)}$

- $V(r_1) - V(r_2) = -\displaystyle\int_{r_1}^{r_2} \vec{E} \cdot dr$

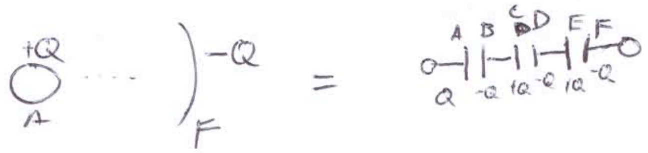
$$= -\int_{r_1}^{r_2} \frac{Q}{4\pi\epsilon_0 r^2}\, dr$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Rightarrow \boxed{C = 4\pi\epsilon_0 \frac{r_1 r_2}{r_2 - r_1}}$$

10 points: Combining multiple capacitors

$$C = \frac{Q}{V_A - V_F}$$



Put +Q on A and −Q on F, and compute the voltage drop $V_A - V_F$.

(1) ~~No charge lives inside conductors.~~ Due to screening all conducting surfaces in problem have either +Q or −Q.

(2) No voltage drop occurs between B and C, nor between D and E. They are shorted, and therefore at same potential.

(statement (1) is equivalent to the statement "capacitors connected in series"
" (2) is " " " " "neglect capacitance of B−C, D−E")

$$\left.\begin{aligned} V_A - V_B &= C_{AB}^{-1} \cdot Q \\ V_C - V_D &= C_{CD}^{-1} \cdot Q \\ V_E - V_F &= C_{EF}^{-1} \cdot Q \end{aligned}\right\} \Rightarrow C_{eff}^{-1} = C_{AB}^{-1} + C_{CD}^{-1} + C_{EF}^{-1}$$

$$= \frac{1}{4\pi\epsilon_0 R}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right)$$

$$\Rightarrow \boxed{C_{eff} = \frac{240}{37}\epsilon_0 R}$$