

**Midterm 1
EE16B
Fall 2016**

NAME: _____

Instructions

Read all of the instructions and all of the questions before beginning the exam.

There are 5 problems in this exam. The total score is 100 points. Points are given next to each problem to help you allocate time. Do not spend all your time on one problem.

IMPORTANT

- **If you do not put your answers within the boxes labeled 'Solution' THEY WILL NOT BE COUNTED (no matter how correct they may be in the bottom left back corner of the third to last page of the exam.)**

- **If you have more than one solution in the box, that box will be given zero points.**

Unless otherwise noted on a particular problem, you must show your work in the space provided, on the back of the exam pages or in the extra pages provided at the back of the exam.

Be sure to provide units where necessary.

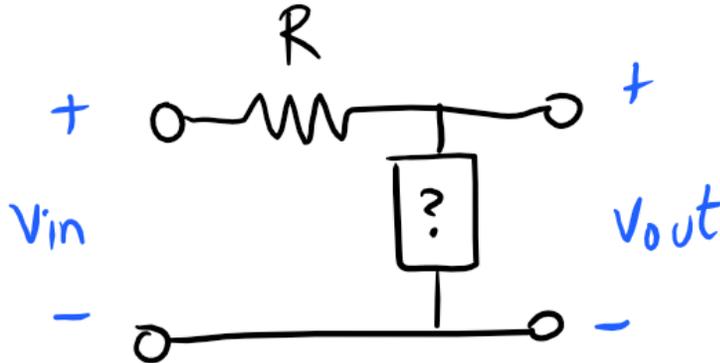
GOOD LUCK!

PROBLEM	POINTS	MAX
1		15
2		20
3		20
4		15
5		30

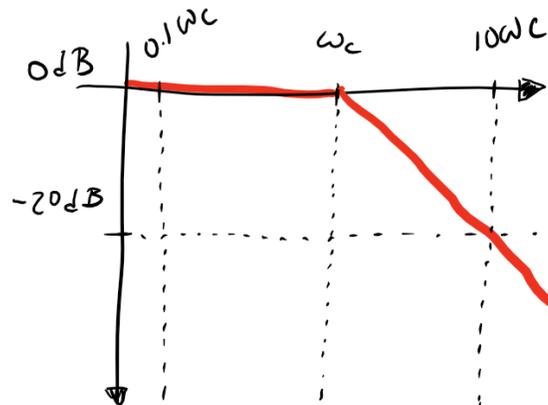
“It is the certainty that they possess the truth that makes men cruel.”
— Anatole France

Problem 1 Bode plots (15 points)

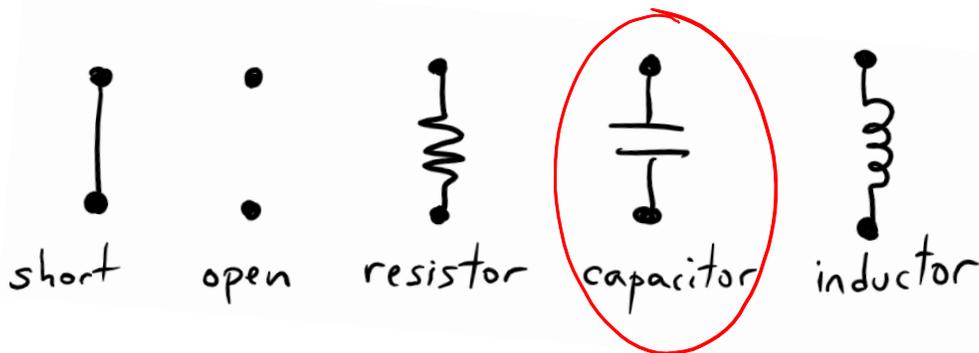
Consider the circuit below. The box represents a missing circuit component. $\mathbf{H}(\omega) = \mathbf{V}_{\text{out}} / \mathbf{V}_{\text{in}}$ where \mathbf{V}_{out} and \mathbf{V}_{in} are phasors.



We would like the Bode plot of the magnitude of the transfer function, $|\mathbf{H}(\omega)|$, to be as shown below.



a) What element would you place inside the box? Circle one below. (5 points)

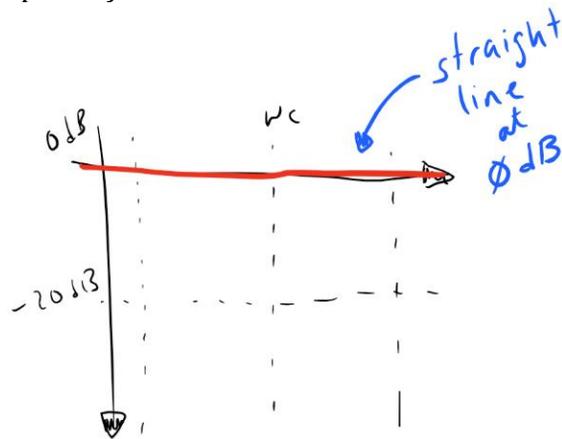


b) We would like $\omega_c = 100$ rad/s (see plot above). If $R = 1$ k Ω , what is the value of the component in a)? (5 points)

Solution:

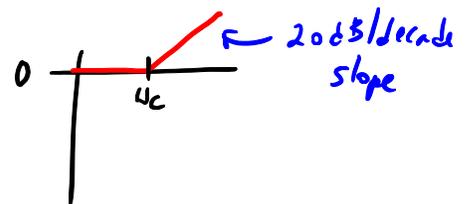
$$\omega_c = \frac{1}{RC} ; 100 = \frac{1}{10^3 C} ; \boxed{C = 10^{-5} \text{ F}}$$

c) What expression would you multiply $\mathbf{H}(\omega)$ by to make the Bode plot look like the one below? Please keep the expression symbolic. (5 points)



Solution: We need a simple zero at ω_c

$$\left(1 + j\frac{\omega}{\omega_c}\right) \text{ where } \omega_c = \frac{1}{RC}$$

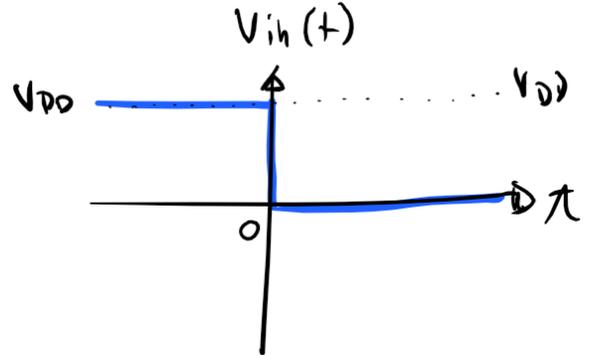
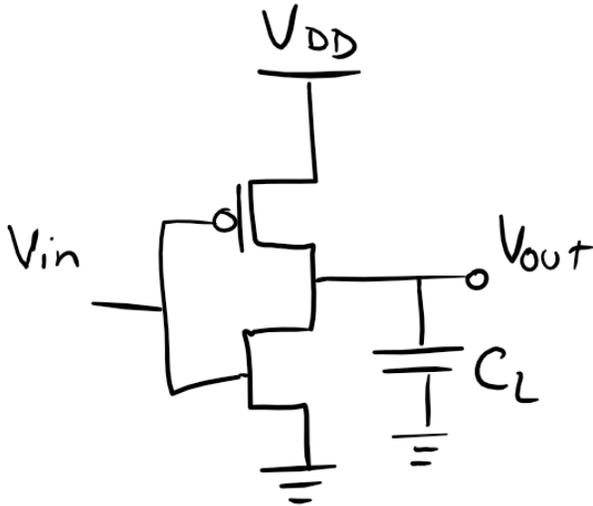


"I just try to live every day as if I've deliberately come back to this one day to enjoy it as if it was the full final day of my extraordinary life,"

-Tim Lake, **About Time**

Problem 2 (20 points)

Consider the circuit below. Please apply the "switch with resistor" model of a transistor when solving this problem. Assume R_{DS} is the 'on' resistance and $|V_{th,n}| = |V_{th,p}| \ll V_{DD}$.



a) If $v_{in}(t)$ is as plotted above, please provide a differential equation in $v_{out}(t)$. (5 points)

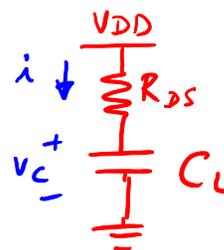
Solution:

$$\frac{dv_c}{dt} + \frac{1}{RC} v_c = \frac{V_{DD}}{RC}$$

$$v_c(t < 0) = 0V \quad \text{For } t \geq 0$$

$$-V_{DD} + iR_{DS} + v_c = 0$$

$$RC \frac{dv_c}{dt} + v_c = V_{DD}$$



Recall $i = C \frac{dv_c}{dt}$

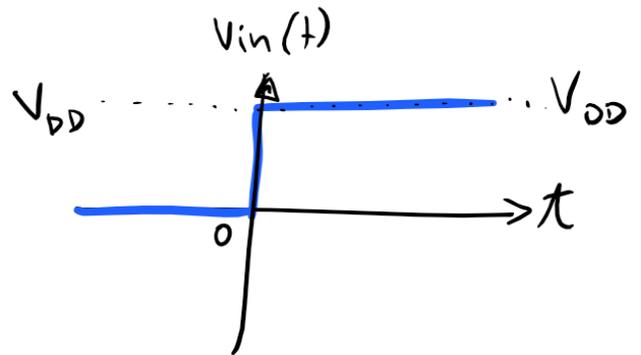
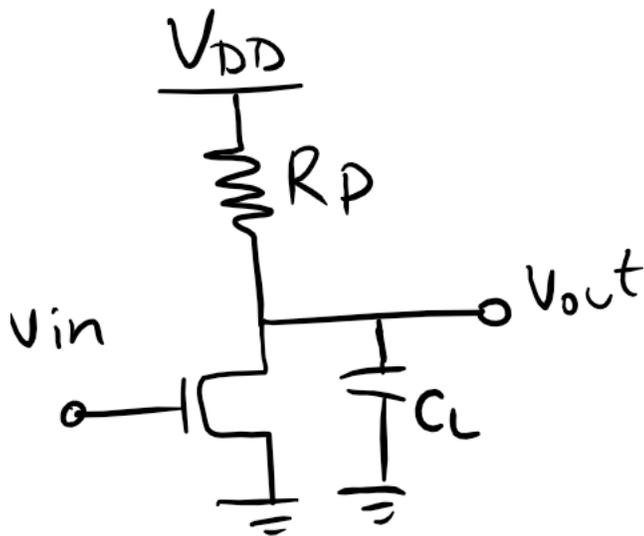
b) Please provide an expression for $v_{out}(t)$. (5 points) $t \geq 0$ Note $v_c(\infty) = V_{DD}$

Solution:

$$v_c(t) = V_{DD} + -V_{DD} e^{-t/RC} = V_{DD} (1 - e^{-t/RC})$$

$$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty)) e^{-t/RC}$$

c) Consider the circuit below. Please apply the "switch with resistor" model of a transistor when solving this problem. Assume $|V_{th,n}| = |V_{th,p}| \ll V_{DD}$.

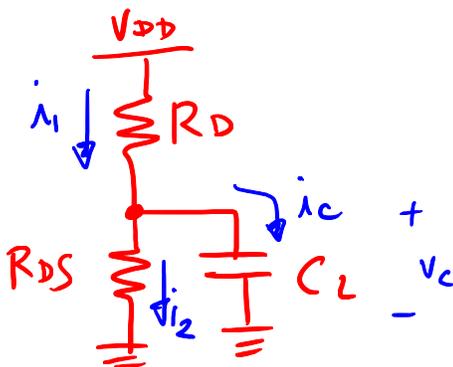


If $v_{in}(t)$ is as plotted above, please provide a differential equation in $v_{out}(t)$. (5 points) $t \geq 0$

Solution:

$$\frac{dv_c}{dt} + \left(\frac{1}{R_D C} + \frac{1}{R_{DS} C} \right) v_c = \frac{V_{DD}}{R_D C}$$

For $t \geq 0$



$$i_1 = i_2 + i_c$$

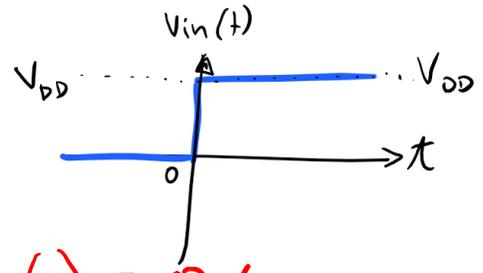
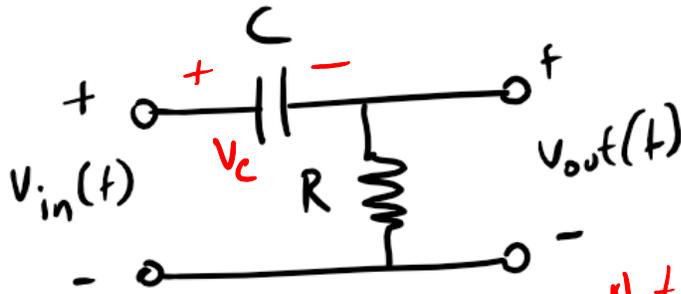
$$\frac{V_{DD} - v_c}{R_D} = \frac{v_c}{R_{DS}} + C \frac{dv_c}{dt}$$

$$\frac{V_{DD}}{R_D} - \frac{v_c}{R_D} = \frac{v_c}{R_{DS}} + C \frac{dv_c}{dt}$$

$$C \frac{dv_c}{dt} + \left(\frac{1}{R_D} + \frac{1}{R_{DS}} \right) v_c = \frac{V_{DD}}{R_D}$$

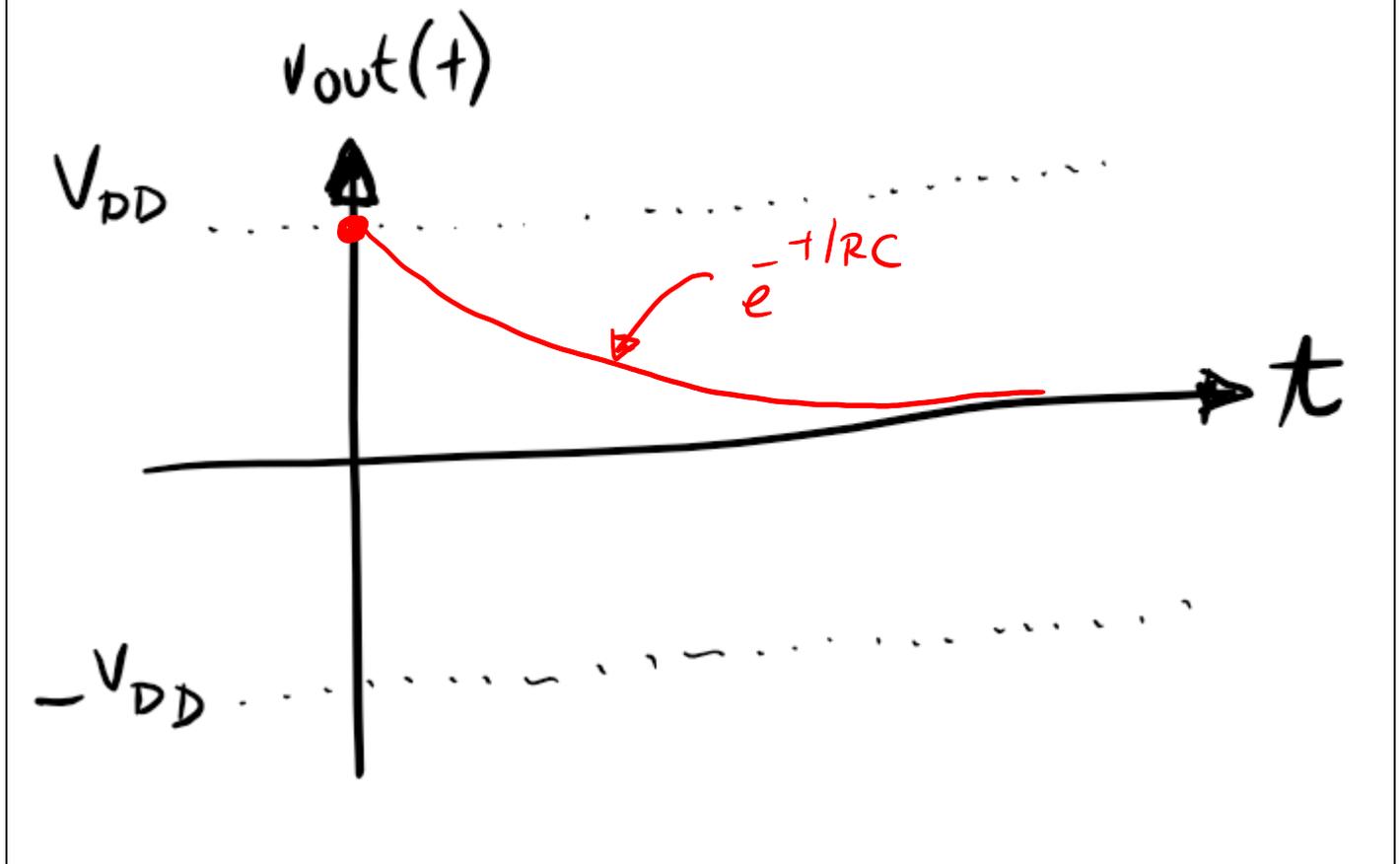
$$\frac{dv_c}{dt} + \left(\frac{1}{R_D C} + \frac{1}{R_{DS} C} \right) v_c = \frac{V_{DD}}{R_D C}$$

d) Given $v_{in}(t)$ plotted below, plot $v_{out}(t)$ for $t > 0$ for the circuit below. Assume $v_c(t \leq 0) = 0$ V. (5 points)



Note $v_c(0) = 0$ V

Solution:

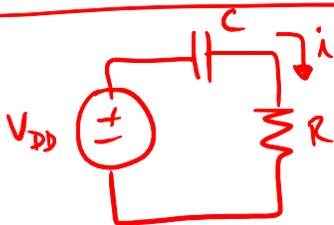


The voltage v_c cannot change instantly. This means that $v_{out}(0)$ has to be equal to V_{DD} because

$$v_{in}(0) = v_c(0) + v_{out}(0) \quad \text{or} \quad v_c(0) = v_{in}(0) - v_{out}(0)$$

$$0 = v_{in}(0) - v_{out}(0)$$

For $t > 0$,



$$i(0) = \frac{v_{out}(0)}{R} = \frac{V_{DD}}{R}$$

$$\text{KVL: } -V_{DD} + v_c + iR = 0$$

$$v_c + RC \frac{dv_c}{dt} = V_{DD}$$

Extra Space

$$V_C(t) = V_{DD} (1 - e^{-t/RC})$$

$$V_R(t) = V_{DD} - V_C(t)$$

$$= V_{DD} - V_{DD} + V_{DD} e^{-t/RC}$$

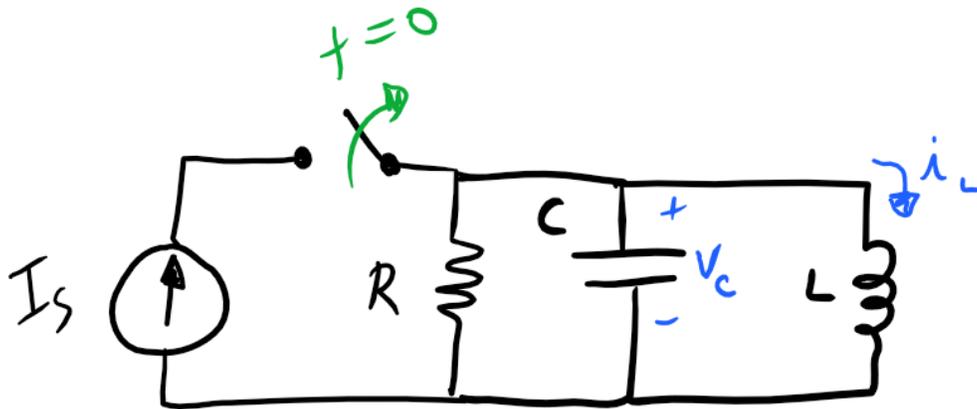
$$= \boxed{V_{DD} e^{-t/RC}}$$

"Look, it's been swell, but the swelling's gone down,"

-Tank Girl

Problem 3 (20 points)

Please consider the circuit below.



a) What is $v_c(t=0)$? (2.5 points)

Solution:

$$v_c(0) = 0$$

b) What is $i_L(t=0)$? (2.5 points)

Solution:

$$i_L(0) = I_s$$

c) Please provide a differential equation in $i_L(t)$ (and only in $i_L(t)$) for $t \geq 0$. (5 points)

Solution:

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC} \frac{d i_L}{dt} + \frac{1}{LC} i_L = 0$$

KCL: $0 = i_R + i_C + i_L$ $i_C = C \frac{d v_C}{dt}$

$v_C = v_R = v_L$ $0 = \frac{v_L}{R} + C \frac{d v_L}{dt} + i_L$ $v_L = L \frac{d i_L}{dt}$

$$= \frac{L}{R} \frac{d i_L}{dt} + LC \frac{d^2 i_L}{dt^2} + i_L$$

d) Please provide an expression for $i_L(t)$ for $t \geq 0$. Assume $L = 1 \mu\text{H}$, $R = 100 \Omega$, and $C = 1 \mu\text{F}$. (5 points)

Solution:

$$i(t) = e^{-\alpha t} (A \cos \omega_0 t + B \sin \omega_0 t)$$

$A = I_s$
 $B = \frac{\alpha I_s}{\omega_0}$

$$\frac{1}{RC} = \frac{1}{100 \cdot 10^{-6}} = 10^4 \quad \frac{1}{LC} = \frac{1}{10^{-6} \cdot 10^{-6}} = 10^{12}$$

$$\alpha = \frac{1}{2RC} = \frac{10^4}{2} = 5000 \quad \omega_0 = \sqrt{10^{12}} = 10^6$$

$\omega_0 \gg \alpha$ so this system is underdamped.

e) Please provide an expression for $v_C(t)$ for $t \geq 0$. (5 points)

Solution:

$$v_C(t) = L \frac{d i_L}{dt} = L e^{-\alpha t} (A \omega_0 \sin \omega_0 t - B \omega_0 \cos \omega_0 t)$$

Extra Space

$$d) i(t) = e^{-\alpha t} (A \cos \omega_D t + B \sin \omega_D t)$$

$$i_L(0) = \boxed{I_S = A}$$

$$v_L = v_C = L \frac{di}{dt} = L (A \cos \omega_D t + B \sin \omega_D t) (-\alpha e^{-\alpha t}) \\ + L e^{-\alpha t} (A \omega_D \sin \omega_D t - B \omega_D \cos \omega_D t)$$

$$v_L(0) = 0 = L (I_S) (-\alpha) + L (-B \omega_D)$$

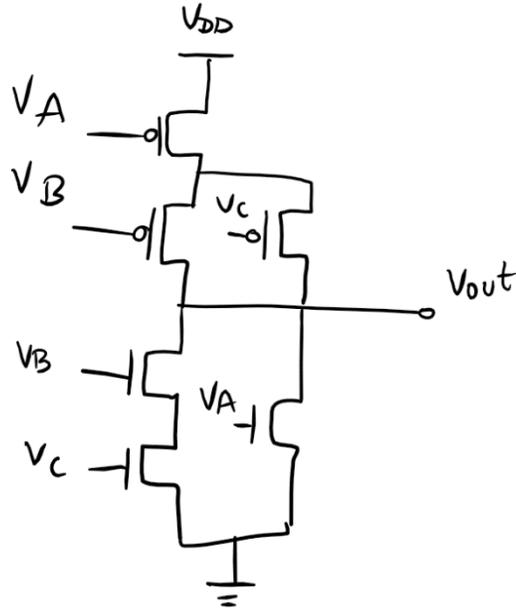
$$\alpha I_S = B \omega_D$$

$$\boxed{B = \frac{\alpha I_S}{\omega_D}}$$

“Ignoramuses are numerous in the palace,”
-ancient Sumerian proverb

Problem 4 (15 points)

a) Consider the circuit below. All inputs and outputs are digital.



Provide a logical expression that describes V_{OUT} as function of the inputs. (7.5 points)

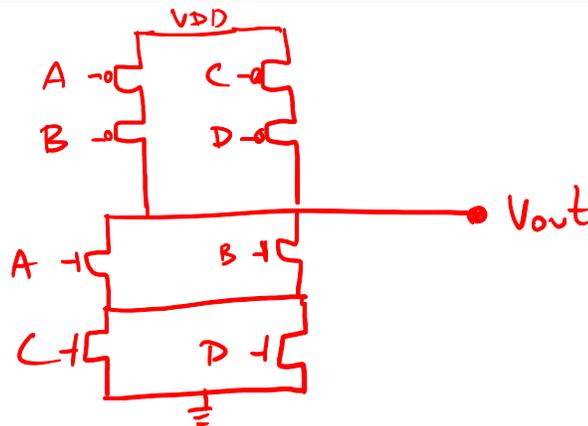
Solution:

$$OUT = (B \text{ AND } C) \text{ OR } A$$

b) Draw a *single* CMOS digital logic gate that performs the following function. (7.5 points)

$$OUT = \overline{(A \text{ or } B) \text{ and } (C \text{ or } D)}$$

Solution:



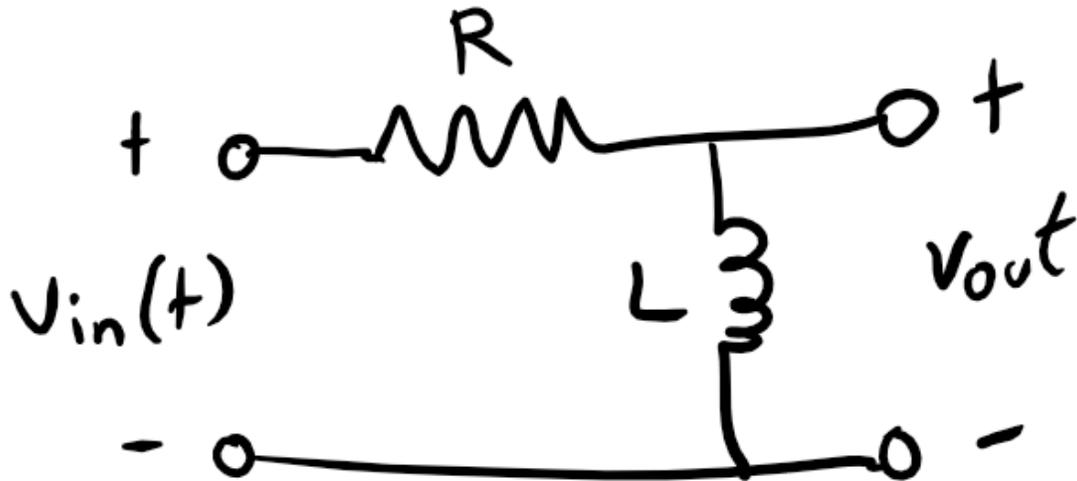
"I have always imagined that Paradise will be a kind of library."

– Jorge Luis Borges

Problem 5 (30 points)

For the following three circuits, provide an expression for the transfer function $H(\omega) = \frac{V_{OUT}}{V_{IN}}$ where V_{OUT} and V_{IN} are phasors. (10 points each)

a)



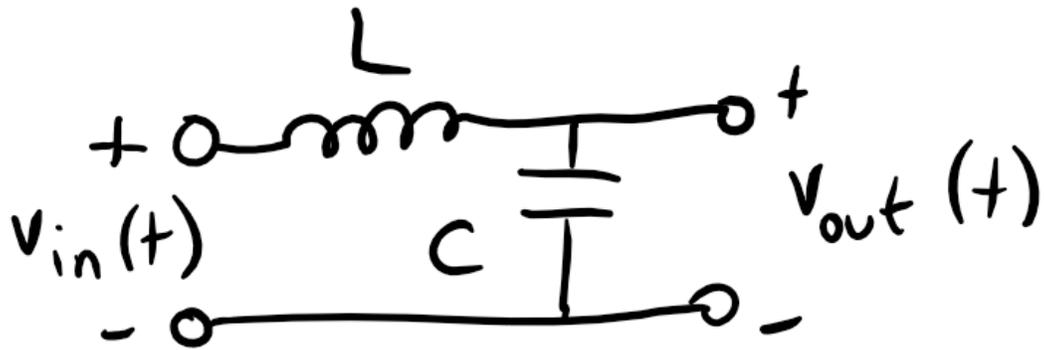
Any of these forms is fine.

Solution:

$$H(\omega) = \frac{j\omega L}{R + j\omega L} = \frac{j\omega(L/R)}{1 + j\omega(L/R)} = \frac{j\omega/\omega_c}{1 + j\omega/\omega_c}$$

$\omega_c = R/L$

b)

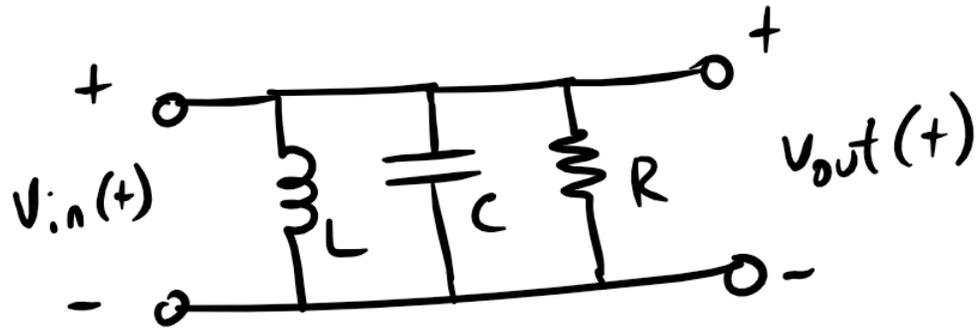


Solution:

$$\mathbf{H}(\omega) = \frac{\frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{1}{1 + (j\omega L)(j\omega C)} = \frac{1}{1 - \frac{\omega^2}{\omega_0^2}}$$

$\omega_0 = \sqrt{1/LC}$

c)



Solution:

$$\mathbf{H}(\omega) = \underline{1} \quad \left(v_{out} = v_{in} \right)$$

Extra Space

Factor	Bode Magnitude	Bode Phase
Constant K	$20 \log K$ 0 dB	$\pm 180^\circ$ if $K < 0$ 0° if $K > 0$
Zero @ Origin $(j\omega)^N$	0 dB slope = $20N$ dB/decade	$(90N)^\circ$ 0°
Pole @ Origin $(j\omega)^{-N}$	0 dB slope = $-20N$ dB/decade	0° $(-90N)^\circ$
Simple Zero $(1 + j\omega/\omega_c)^N$	0 dB slope = $20N$ dB/decade	0° $(90N)^\circ$
Simple Pole $\left(\frac{1}{1 + j\omega/\omega_c}\right)^N$	0 dB slope = $-20N$ dB/decade	0° $(-90N)^\circ$
Quadratic Zero $[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N$	0 dB slope = $40N$ dB/decade	0° $(180N)^\circ$
Quadratic Pole $\frac{1}{[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N}$	0 dB slope = $-40N$ dB/decade	0° $(-180N)^\circ$