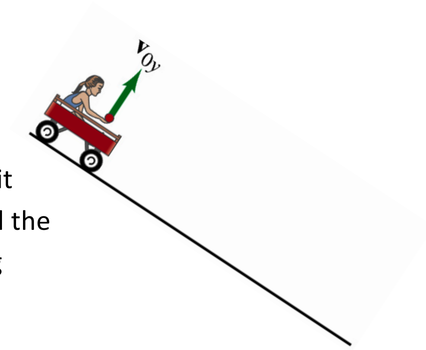
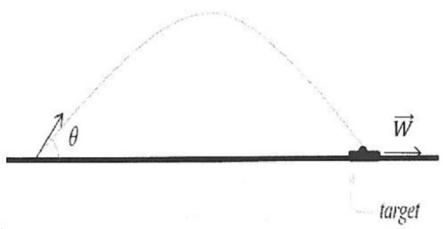
Spring 2016 Physics 7A Lec 001 (Yildiz) Midterm I

1. (15 points) Small cart is rolling down an inclined track and accelerating. It fires a ball straight out of the cannon as it moves. After it is fired, what happens to the ball? Does it fall back to the incline behind the cart, right into the cart or in front of the incline? Explain your reasoning conceptually.



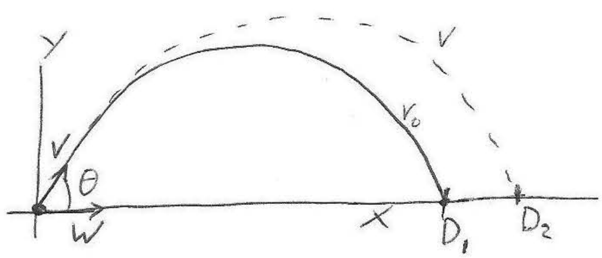
Answer: Because the track is inclined, the cart accelerates. However, the ball has the same component of acceleration along the track as the cart does! This is essentially the component of g acting parallel to the inclined track. So the ball is effectively accelerating down the incline, just as the cart is, and it falls back into the cart.

2. (20 points)



A tennis ball launcher is placed on a grass lawn, and a target is placed flat on the ground beside it. This target is attached to a car by a rope (see figure), and at the instant the launcher fires a ball at an angle, θ, and a speed, v_0, the target travels to the right at a constant speed, W. When the tennis ball lands, it hits the center of the target. At the instant this ball lands, a second ball is launched at the same angle, but at a speed, v. What must v/v_0 be so that the second tennis ball also hits the center of the target? (You should get a pure number.)

2.



First launch: $Wt = D_1 = v_0 \cos\theta\, t$

$\Rightarrow W = v_0 \cos\theta$.

Time of flight from y-equation

$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

$0 = v_0 \sin\theta\, t - \frac{1}{2} g t^2$

$\Rightarrow t = \frac{2 v_0 \sin\theta}{g}$

Range formula from x-equation

$D_1 = v_0 \cos\theta\, t = \frac{2 v_0^2 \cos\theta \sin\theta}{g}$

Second launch:

$D_2 - D_1 = W \cdot t'$, where t' is the time of flight of the second ball.

As above, $t' = \frac{2 v \sin\theta}{g}$.

Using the range formula again, $D_2 = \frac{2 v^2 \cos\theta \sin\theta}{g}$.

$\Rightarrow W \cdot \frac{2 v \sin\theta}{g} = D_2 - D_1 = (v^2 - v_0^2)\, \frac{2 \cos\theta \sin\theta}{g}$

So $v^2 - v_0^2 = \frac{W \cdot v}{\cos\theta} = v_0 v$ using the result for W above.

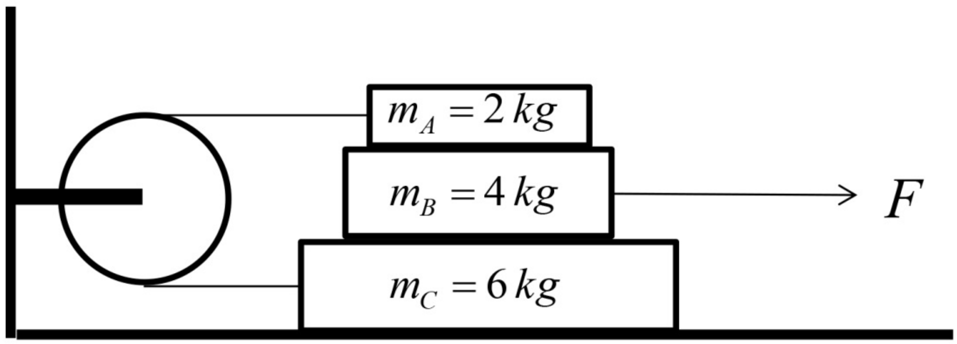
Define $\lambda = \frac{v}{v_0}$, then $\lambda^2 - \lambda - 1 = 0$.

$\lambda = \frac{1}{2} \pm \sqrt{\frac{5}{4}}$. λ has to be greater than 0, so we find

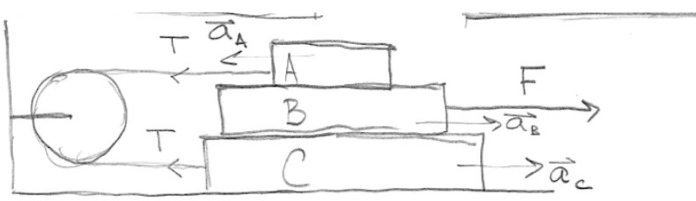
$\frac{v}{v_0} = \lambda = \frac{1 + \sqrt{5}}{2}$.

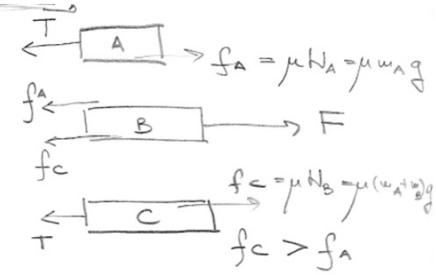
3. (25 points)

The figure below shows three blocks stacked on top of a *frictionless* table. The top block and the bottom block are connected to each other by a string through a pulley with negligible mass. The coefficient of kinetic friction between the three blocks is the same $\mu_k = 0.30$. When a force, F=40 N, is applied to the middle block, all three blocks accelerates, and thus slide with respect to one another. What is the acceleration of each of each block?



Use g = 10 m/s^2 for mathematical calculations.





$f_C > f_A \Rightarrow$ C goes to RIGHT, A goes to LEFT

$|\vec{a}_C| = |\vec{a}_A| \equiv a$

$m_C a = f_C - T = \mu(m_A+m_B)g - T$

$m_A a = T - f_A = T - \mu m_A g$

+

$(m_A+m_C)a = \mu m_B g$

$a = \boxed{\frac{\mu m_B g}{m_A+m_C} = |\vec{a}_C| = |\vec{a}_A|}$

$m_B a_B = F - f_A - f_C$

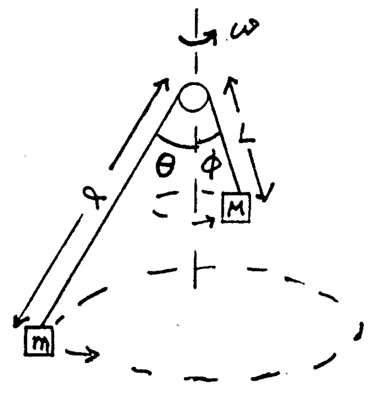
$m_B a_B = F - \mu(2m_A+m_B)g$

$\boxed{a_B = \frac{F-\mu(2m_A+m_B)g}{m_B}}$

$a_A = a_C = 1.47\ m/s^2$

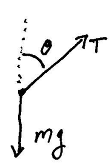
$a_B = 4.12\ m/s^2$

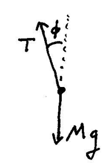
4. A double conical pendulum consists of two masses, m and M, connected by a massless string over a frictionless, massless pulley. The entire apparatus rotates freely at constant angular speed ω (rad/s) about the vertical axis (dashed line) with l and L constant. Find L, θ, ϕ, and the tension T in the string, in terms of m, M, l, ω, and g.



(i) Mass m   Mass M

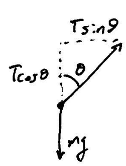
4 pts

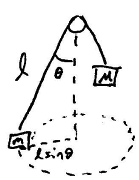




(ii) To find tension in the string, we will look at the radial (centripetal) force on mass m

8 pts





$$T \sin\theta = m a_r = \frac{mv^2}{r} = m r \omega^2$$

$$r = l \sin\theta \Rightarrow T \cancel{\sin\theta} = m l \cancel{\sin\theta}\, \omega^2$$

$$\Rightarrow \boxed{T = m l \omega^2}$$

If you had done the same thing with mass M, you would get:

$$T = M L \omega^2$$

(iii) Using our two expressions for the tension, we see:

8 pts

$$T = m l \omega^2 = M L \omega^2$$

Hence: $\boxed{L = \frac{m}{M} l}$

Now, using the vertical forces on mass m, we get:

$$T\cos\theta - mg = 0 \text{, Hence } \cos\theta = \frac{mg}{T} = \frac{\cancel{m}g}{\cancel{m}l\omega^2} = \frac{g}{l\omega^2}$$

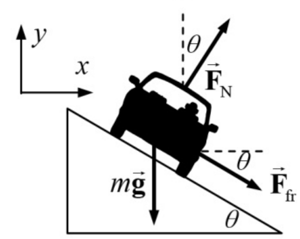
Similarly for Mass M: $\cos\phi = \frac{Mg}{T} = \frac{Mg}{ML\omega^2} = \frac{g}{L\omega^2} = \frac{g}{l\omega^2}\frac{M}{m}$

Hence: $\boxed{\theta = \cos^{-1}\left[\frac{g}{l\omega^2}\right]}$ and $\boxed{\phi = \cos^{-1}\left[\frac{g}{l\omega^2}\frac{M}{m}\right]}$

5. (20 points)

A banked curve of radius R in a new highway is designed so that a car traveling at speed v_0 can negotiate the turn safely on glare ice (zero friction). If a car travels too slowly, then it will slip toward the center of the circle. If it travels too fast, it will slip away from the center of the circle. If the coefficient of static friction increases, it becomes possible for a car to stay on the road while traveling at a speed within a range from $v_{\min}$ to $v_{\max}$. Derive formulas for $v_{\min}$ and $v_{\max}$ as functions of μ_s, v_0, and R.

From Example 5-15 in the textbook, the no-friction banking angle is given by $\theta = \tan^{-1}\dfrac{v_0^2}{Rg}$. The centripetal force in this case is provided by a component of the normal force. Driving at a higher speed with the same radius requires more centripetal force than that provided by the normal force alone. The additional centripetal force is supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. The center of the circle of the car's motion is to the right of the car in the diagram. Write Newton's second law in both the x and y directions. The car will have no acceleration in the y direction, and centripetal acceleration in the x direction. Assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\text{fr}} = \mu_s F_{\text{N}}$.



$$\sum F_y = F_{\text{N}}\cos\theta - mg - F_{\text{fr}}\sin\theta = 0 \;\rightarrow\; F_{\text{N}}\cos\theta - \mu_s F_{\text{N}}\sin\theta = mg \;\rightarrow$$

$$F_{\text{N}} = \frac{mg}{(\cos\theta - \mu_s\sin\theta)}$$

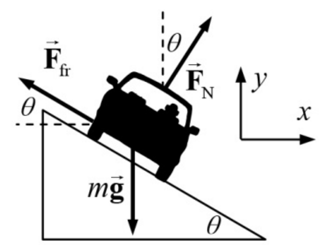
$$\sum F_x = F_{\text{R}} = F_{\text{N}}\sin\theta + F_{\text{fr}}\cos\theta = mv^2/R \;\rightarrow\; F_{\text{N}}\sin\theta + \mu_s F_{\text{N}}\cos\theta = mv^2/R \;\rightarrow$$

$$F_{\text{N}} = \frac{mv^2/R}{(\sin\theta + \mu_s\cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed, which is the maximum speed that the car can have.

$$\frac{mv^2/R}{(\sin\theta + \mu_s \cos\theta)} = \frac{mg}{(\cos\theta - \mu_s \sin\theta)} \rightarrow$$

$$v_{\max} = \sqrt{Rg \frac{\sin\theta}{\cos\theta} \frac{(1+\mu_s/\tan\theta)}{(1-\mu_s \tan\theta)}} = v_0 \sqrt{\frac{(1+Rg\mu_s/v_0^2)}{(1-\mu_s v_0^2/Rg)}}$$



Driving at a slower speed with the same radius requires less centripetal force than that provided by the normal force alone. The decrease in centripetal force is supplied by a force of static friction, upward along the incline. See the free-body diagram for the car on the incline. Write Newton's second law in both the x and y directions. The car will have no acceleration in the y direction, and centripetal acceleration in the x direction. Assume that the car is on the verge of skidding, so that the static frictional force is given by $F_{\text{fr}} = \mu_s F_{\text{N}}$.

$$\sum F_y = F_{\text{N}} \cos\theta - mg + F_{\text{fr}} \sin\theta = 0 \rightarrow$$

$$F_{\text{N}} \cos\theta + \mu_s F_{\text{N}} \sin\theta = mg \rightarrow F_{\text{N}} = \frac{mg}{(\cos\theta + \mu_s \sin\theta)}$$

$$\sum F_x = F_{\text{R}} = F_{\text{N}} \sin\theta - F_{\text{fr}} \cos\theta = mv^2/R \rightarrow F_{\text{N}} \sin\theta - \mu_s F_{\text{N}} \cos\theta = mv^2/R \rightarrow$$

$$F_{\text{N}} = \frac{mv^2/R}{(\sin\theta - \mu_s \cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/R}{(\sin\theta - \mu_s \cos\theta)} = \frac{mg}{(\cos\theta + \mu_s \sin\theta)} \rightarrow$$

$$v_{\min} = \sqrt{Rg \frac{\sin\theta}{\cos\theta} \frac{(1-\mu_s/\tan\theta)}{(1+\mu_s \tan\theta)}} = v_0 \sqrt{\frac{(1-\mu_s Rg/v_0^2)}{(1+\mu_s v_0^2/Rg)}}$$

Thus $\boxed{v_{\min} = v_0 \sqrt{\frac{(1-\mu_s Rg/v_0^2)}{(1+\mu_s v_0^2/Rg)}}}$ and $\boxed{v_{\max} = v_0 \sqrt{\frac{(1+Rg\mu_s/v_0^2)}{(1-\mu_s v_0^2/Rg)}}}$.