PHYSICS 7B, Lecture 3 – Spring 2015
Final exam, C. Bordel
Tuesday, May 12, 2015
8-11 am

Make sure you show all your work and justify your answers in order to get full credit.

Problem 1: Thermodynamic process (20 points)

n moles of a diatomic ideal gas undergoes a reversible thermodynamic process from temperature and volume (T_1,V_1) to temperature and volume (T_2,V_2), with $V_2>V_1$, following the curve $T/V^2=const$. You may assume that the 2 temperatures are in the range [100-1000 K].

a) Sketch the corresponding path on a P-V diagram. Is this process one of the 4 thermodynamic processes you know? Justify.
b) Calculate the work done by the gas and represent it graphically on the P-V diagram.
c) Calculate the change in internal energy and the heat gained by the gas.
d) Calculate the change in entropy of the gas. *Hint:* the first law of thermodynamics might be useful!

Problem 2: Electric potential (20 points)

We consider two infinite and hollow coaxial cylinders of radii R_1 and R_2 ($R_1 < R_2$) carrying uniform electric charges per unit length, $-\lambda$ and $+\lambda$, respectively (see Fig.1).

a) Determine the difference in electric potential between the 2 cylindrical shells.
b) Draw some electric field lines and equipotential surfaces resulting from this charge distribution. Justify.
c) Describe the trajectory of an electron leaving the inner shell with zero velocity and moving towards the outer shell. Determine, in terms of the electric potentials V_1 and V_2 of the two shells, its final speed when it strikes the outer shell.

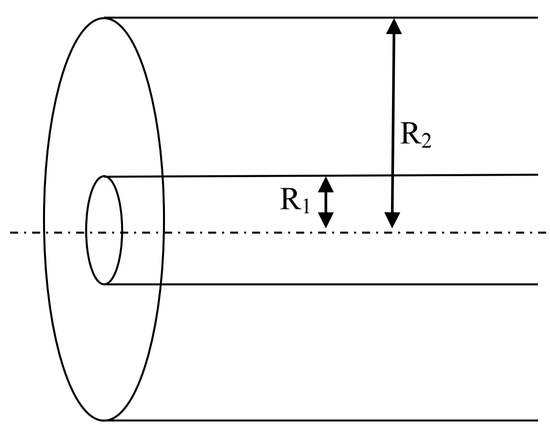

Figure 1

<u>Problem 3</u>: DC circuit (20 points)

A Wheatstone bridge is a type of "bridge circuit" used to make measurements of resistance. The unknown resistance to be measured, R_x, is placed in the circuit with accurately known resistances R_1, R_2, and R_3, as shown in Fig. 2. One of these, R_3, is a variable resistor which is adjusted so that when the switch is closed momentarily, the ammeter *A* shows zero current flow.

Determine R_x in terms of R_1, R_2, and R_3.

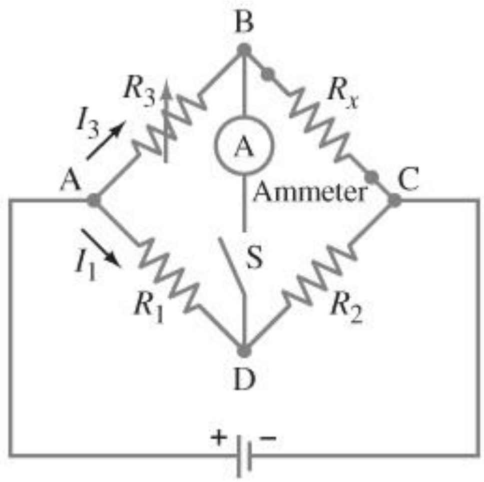

Figure 2

<u>Problem 4</u>: Magnetic field (20 points)

A single piece of wire carrying current *I* is bent so it includes a circular loop of radius a, and a long linear section of length $L >> a$, as shown in Fig. 3.
Determine the magnitude and direction of the magnetic field created at the loop center.

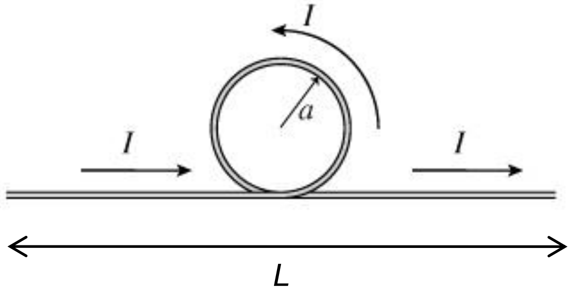

Figure 3

Problem 5: Hall effect (20 points)

A Hall probe used to measure magnetic field strengths consists of a rectangular slab of material with free-electron density n, width w and thickness t, carrying a current I along its length b. The slab is immersed in a magnetic field of magnitude B oriented perpendicular to its large rectangular face, as shown in Fig.4.

The probe's magnetic sensitivity is defined as $K_H=\mathcal{E}/IB$, where $\mathcal{E}$ is the magnitude of the Hall voltage.

a) Describe the origin of the Hall effect and explain, based on the geometry of the set-up, between which 2 sides of the slab the Hall voltage can be measured.
b) Calculate K_H in terms of the material's characteristics.
c) As possible candidates for the material used in a Hall probe, consider a typical metal ($n \approx 10^{29}/m^3$) and a semiconductor ($n \approx 10^{22}/m^3$). Which one would be the best choice to maximize the probe's sensitivity and why?
d) Assuming that the free charges responsible for the electric conduction are electrons, of electric charge -e, which side of the slab has the higher potential? Explain.

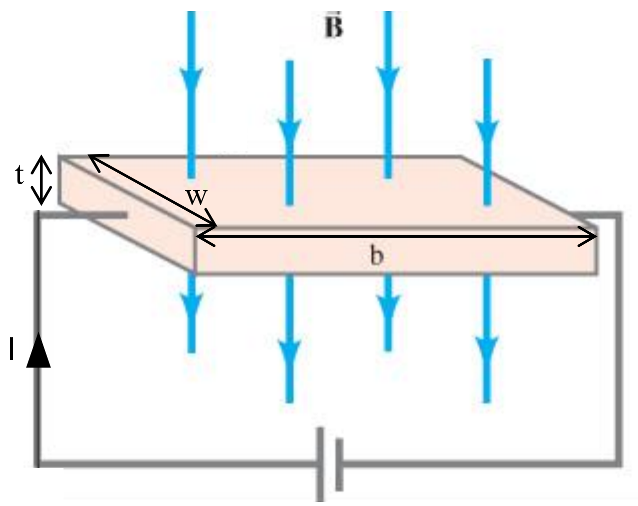

Figure 4

Problem 6: Electromagnetic induction (20 points)

A uniform horizontal magnetic field of magnitude B exists above a level defined to be $y = 0$. Below $y = 0$, the field abruptly becomes zero, as shown in Fig.5.
A square loop of side length a is made of a metallic wire of mass m, resistivity ρ, and diameter $d \ll a$. The loop is held in a vertical plane with its lower horizontal side at $y = 0$. Initially at rest, it is then allowed to fall under gravity, with its plane perpendicular to the direction of the magnetic field.

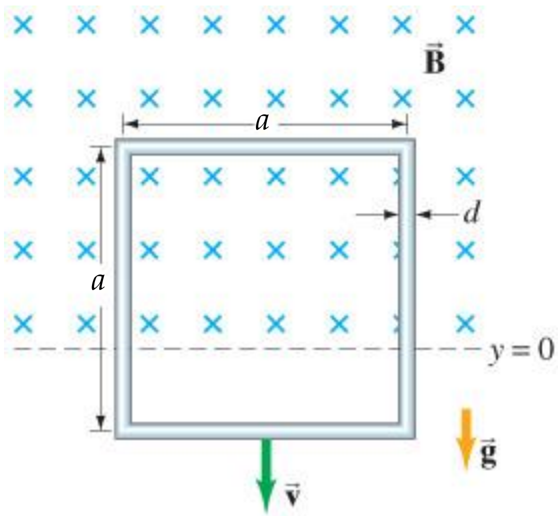

Figure 5

a) Without any calculation, predict the direction of the induced current in the loop.
b) Calculate the induced *emf* and induced current as a function of the instantaneous speed v.
c) Determine the terminal speed v_T achieved by the loop before its upper horizontal side exits the field.

Problem 7: Inductance, LR circuit (20 points)

At time t=0, the switch of the circuit shown in Figure 6 is closed in order to connect the battery to the rest of circuit.
a) Calculate the equivalent inductance L_{eq} of the three inductors (L_1, L_2, L_3). Ignore any mutual inductance.
b) Establish the differential equation satisfied by the current $I(t)$.
c) How many time constants does it take for the potential difference across the resistor to reach 90 % of its maximum value?

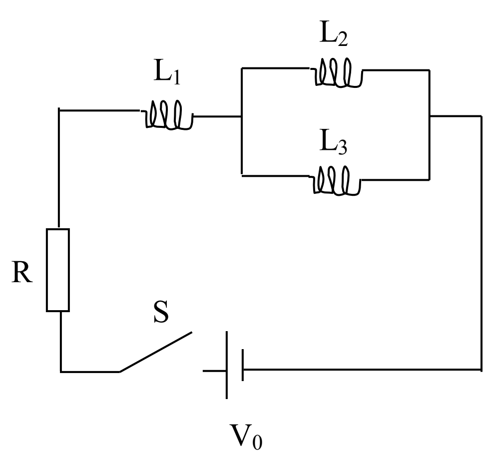

Figure 6

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$PV = NkT = nRT$$

$$\frac{1}{2} m\overline{v^2} = \frac{3}{2} kT$$

$$f_{Maxwell}(v) = 4\pi N \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

$$E = \frac{d}{2} nRT$$

$$Q = mc\Delta T = nC\Delta T$$

$$Q = mL \text{ (For a phase transition)}$$

$$dE = -PdV + dQ$$

$$W = \int PdV$$

$$C_P - C_V = R = N_A k$$

$$PV^\gamma = \text{const. (For an adiabatic process)}$$

$$\gamma = \frac{C_P}{C_V} = \frac{d+2}{d}$$

$$C_V = \frac{d}{2} R$$

$$\frac{dQ}{dt} = -kA \frac{dT}{dx}$$

$$e = \frac{W_{net}}{Q_{in}}$$

$$e_{ideal} = 1 - \frac{T_L}{T_H}$$

$$dQ = TdS$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} \hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$\vec{E} = \int \frac{dQ}{4\pi\epsilon_0 r^2} \hat{r}$$

$$\rho = \frac{dQ}{dV}$$

$$\sigma = \frac{dQ}{dA}$$

$$\lambda = \frac{dQ}{dl}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Delta U = Q\Delta V$$

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$V = \int \frac{dQ}{4\pi\epsilon_0 r}$$

$$\vec{E} = -\vec{\nabla} V$$

$$Q = CV$$

$$C_{eq} = C_1 + C_2 \text{ (In parallel)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (In series)}$$

$$\epsilon = \kappa\epsilon_0$$

$$U = \frac{Q^2}{2C}$$

$$U = \int \frac{\epsilon_0}{2} |\vec{E}|^2 dV$$

$$I = \frac{dQ}{dt}$$

$$\Delta V = IR$$

$$R = \rho \frac{l}{A}$$

$$\rho(T) = \rho(T_0)(1 + \alpha(T - T_0))$$

$$P = IV$$

$$I = \int \vec{j} \cdot d\vec{A}$$

$$\vec{j} = nQ\vec{v_d} = \frac{\vec{E}}{\rho}$$

$$R_{eq} = R_1 + R_2 \text{ (In series)}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ (In parallel)}$$

$$\sum_{\text{junc.}} I = 0 \text{ (junction rule)}$$

$$\sum_{\text{loop}} V = 0 \text{ (loop rule)}$$

$$d\vec{F}_m = I d\vec{l} \times \vec{B}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{\mu} = NI\vec{A}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \cdot \vec{B}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \hat{r}}{r^2}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$M = N_1 \frac{\Phi_1}{I_2} = N_2 \frac{\Phi_2}{I_1}$$

$$L = N\frac{\Phi_B}{I}$$

$$U = \frac{1}{2}LI^2$$

$$U = \int \frac{1}{2\mu_0} |\vec{B}|^2 dV$$

$$\overline{g(v)} = \int_0^\infty g(v) \frac{f(v)}{N} dv$$

($f(v)$ a speed distribution)

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z}$$

$$d\vec{l} = dr\hat{r} + r d\theta\hat{\theta} + dz\hat{z}$$

(Cylindrical Coordinates)

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$d\vec{l} = dr\hat{r} + r d\theta\hat{\theta} + r\sin(\theta) d\phi\hat{\phi}$$

(Spherical Coordinates)

$$y(t) = \frac{B}{A}(1 - e^{-At}) + y(0)e^{-At}$$

solves $\frac{dy}{dt} = -Ay + B$

$$y(t) = y_{max}\cos(\sqrt{A}t + \delta)$$

solves $\frac{d^2y}{dt^2} = -Ay$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int_0^\pi \sin^3(x) dx = \frac{4}{3}$$

$$\int (1 + x^2)^{-1/2} dx = \ln(x + \sqrt{1 + x^2})$$

$$\int (1 + x^2)^{-1} dx = \arctan(x)$$

$$\int (1 + x^2)^{-3/2} dx = \frac{x}{\sqrt{1 + x^2}}$$

$$\int \frac{x}{1 + x^2} dx = \frac{1}{2}\ln(1 + x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\int \sin(x) dx = -\cos(x)$$

$$\int \cos(x) dx = \sin(x)$$

$$\int \frac{dx}{x} = \ln(x)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1 + x)^\alpha \approx 1 + \alpha x + \frac{(\alpha - 1)\alpha}{2}x^2$$

$$\ln(1 + x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$