EECS 16A   Designing Information Devices and Systems I
Fall 2016   Babak Ayazifar, Vladimir Stojanovic   Midterm 1

Exam location: 2050 VLSB, Last Name: Tong-Zzz

PRINT your student ID: _____

PRINT AND SIGN your name: _____, _____ _____
                                                (last)   (first)   (signature)

PRINT your Unix account login: ee16a-_____

PRINT your discussion section and GSI (the one you attend): _____

Name and SID of the person to your left: _____

Name and SID of the person to your right: _____

Name and SID of the person in front of you: _____

Name and SID of the person behind you: _____

Section 0: Pre-exam questions (3 points)

1. What other courses are you taking this term? (1 pt)

2. What activity do you really enjoy? Describe how it makes you feel. (2 pts)

Do not turn this page until the proctor tells you to do so. You can work on Section 0 above before time starts.

Section 1 $\left(18 \text{ points}\right)$

Unless told otherwise, you must show work to get credit. There will be very little partial credit given in this section.

3. **True/False (6 points, 1 point for each question)**

   Answer each of the following questions by circling True or False. No work is necessary for credit on this part.

   (a) **(True) (False)** The pivot columns of matrix A forms a basis for the column space of A.

   (b) **(True) (False)** Let A be a 2×2 matrix, where $A^2 = 0$. Then A is the zero matrix.

   (c) **(True) (False)** Let A, B, C be some arbitrary matrices. Then, $(AB)C = A(BC)$.

   (d) **(True) (False)** An $M \times N$ matrix has at most N pivots.

   (e) **(True) (False)** $AB = BA$ where A and B are $N \times N$ matrices.

   (f) **(True) (False)** Applying any pair of 2×2 rotation matrices to an input vector is a commutative operation.

4. Proof (7 points)

(a) Prove that if $A\vec{x} = 0$ for some nonzero $\vec{x}$, then the columns of A are linearly dependent.

(b) Prove that if $A^2 = 0$ where A is an arbitrary square matrix, then the columns of A are linearly dependent.

5. Inverse of a Matrix (5 points)

Find the inverse of A, if it exists. If not, explain why. $A = \begin{bmatrix} 5 & 4 & 2 \\ 1 & 2 & 1 \\ 9 & 6 & 3 \end{bmatrix}$.

Section 2 (55 points)

6. Directional Shovels (10 points)

Kody and Nara were found exceptional at taking measurements to figure out light intensities, and they were both granted admission to a graduate school. Unfortunately, they both supported their new school's football team while they were playing against Berkeley and angry Berkeley fans found them and left them in a room at an unknown location under the ground. As compassionate people, Berkeley fans left some tools in the room that can help them escape.

(a) Kody found a shovel in the room and figured that it can operate in the following directions:
$\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \}$. Is it possible for them to escape to Berkeley by digging in the given directions to a

point which is located at $\begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$ given that they are at point $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$? If so, find the scalars that multiply

the vectors such that they reach Berkeley.

(b) While Kody was busy planning his escape to Berkeley, Nara found a pick-axe in the room that can operate in the following directions: $\{\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}\}$. Nara is convinced that the axe she found is better, but Kody disagrees. Show that Kody's shovel can reach anywhere that Nara's pick-axe can.

7. Graph Majors (30 points)

We'd like to understand how engineering undergrads change their majors. For simplicity, there are three majors we'll look at: EECS, CS, and MechE. Let's assume that students can only be studying one major at a time, and must be studying one of these three majors. Let's also assume that once a week, students can choose to switch to another major, or stick with what they're studying. So, a discrete time step represents one week.
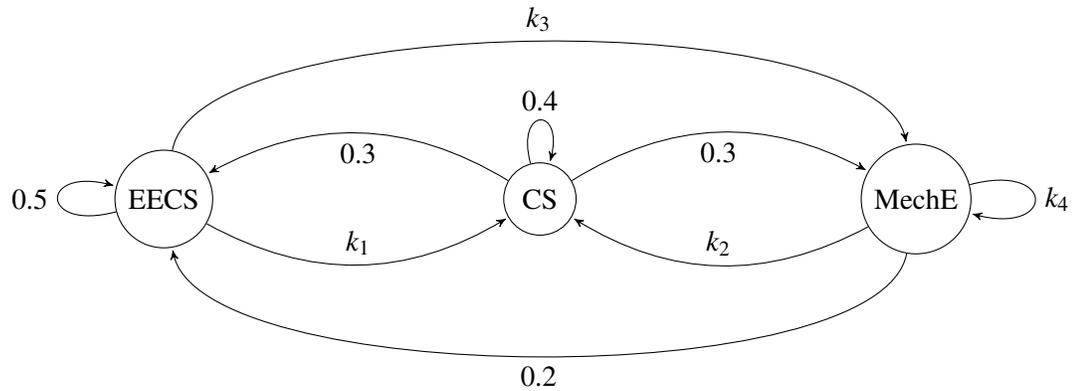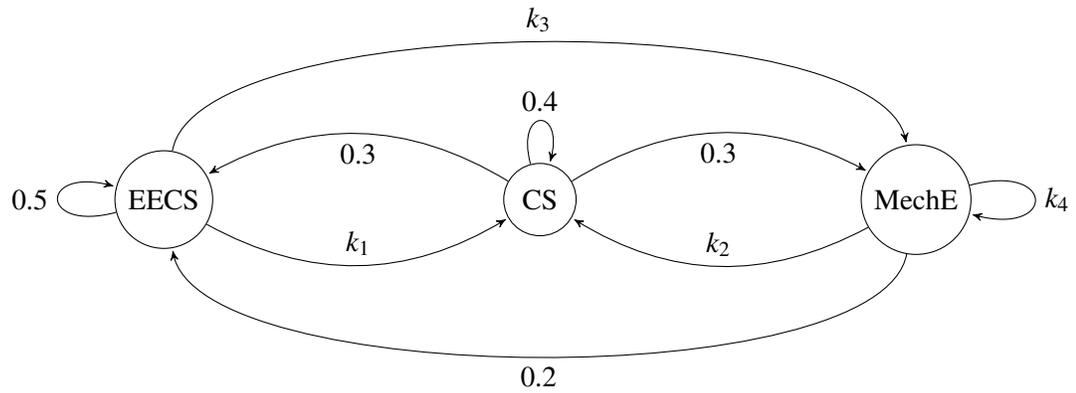

At the start of week n, the number of EECS, CS, MechE students are $x_e[n]$, $x_c[n]$, and $x_m[n]$, respectively.

Let $\vec{x}[n] = \begin{bmatrix} x_e[n] \\ x_c[n] \\ x_m[n] \end{bmatrix}$. Also let $\vec{k} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix}$.
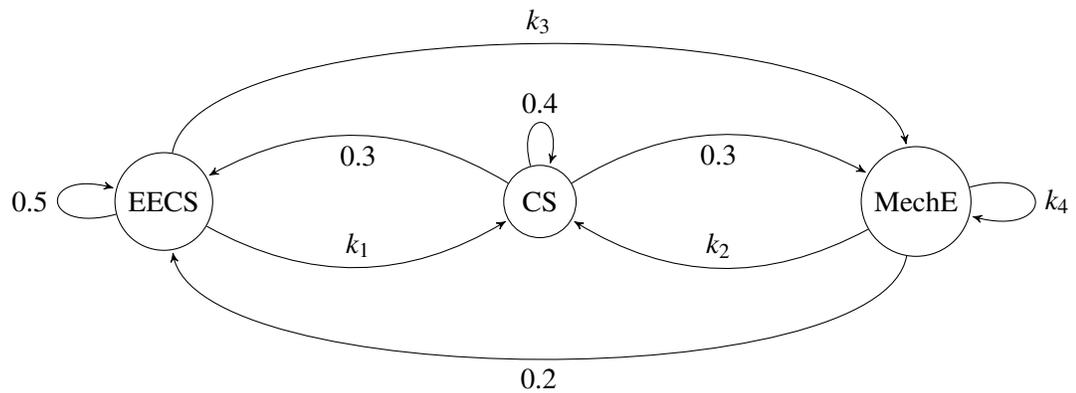
(a) Write the transition matrix, A, such that $\vec{x}[n+1] = A\vec{x}[n]$.

(b) Assume that from one week to the next, no students drop out or are enrolled to the system – in other words, the total number of students is conserved. Write a system of four linear equations that relate k_1, k_2, k_3, k_4. *Hint: you should use $x_e[n]$, $x_c[n]$, $x_m[n]$, $x_e[n+1]$, $x_c[n+1]$, $x_m[n+1]$ in your answer.*

(c) Let $\vec{x}[10] = \begin{bmatrix} 100 \\ 200 \\ 200 \end{bmatrix}$ and $\vec{x}[11] = \begin{bmatrix} 150 \\ 100 \\ 250 \end{bmatrix}$. Rewrite your four linear equations from part (b) in the form

$T\vec{k} = \vec{b}$, where $\vec{k}$ is the vector defined above and $\vec{b}$ is a vector of constants. Do **not** solve for $\vec{k}$.

(d) Now let us redefine our graph transition matrix A such that $A = \begin{bmatrix} 0.6 & 0.4 & 0.2 \\ 0.3 & 0.2 & 0.3 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$. Given $\vec{x}[923]$, is it possible to find $\vec{x}[2]$? Give a mathematical justification and a brief explanation of how. Do not make any assumptions derived from previous parts of this problem.

(e) Let us redefine A as $A = \begin{bmatrix} 0.2 & 0.1 & 0.3 \\ 0.4 & 0.7 & 0.1 \\ 0.4 & 0.2 & 0.6 \end{bmatrix}$. Is $\vec{x}[5] = \begin{bmatrix} 120 \\ 120 \\ 260 \end{bmatrix}$ a valid state for this system? Explain. Assume the states begin with some $\vec{x}[0]$, where $\vec{x}[0]$ is not the zero vector $\vec{0}$. Do not make any assumptions derived from previous parts of the problem.

8. Transformation Basketball (15 points)

Kevin Bancroft just joined the Column Space Warriors. In order to better learn how to cooperate with the team before the season starts, he and his teammates are practicing some basketball drills.

(a) Kevin Bancroft and Draymond Blue-Gold are running a drill where they each have to run from a starting coordinate to an end coordinate. Kevin starts at point $k_s = [3\ 7]^T$ and wants to go to point $k_e = [-4\ 10]^T$. Draymond starts at point $d_s = [-6\ 1]^T$ and wants to go to point $d_e = [-7\ -5]^T$.
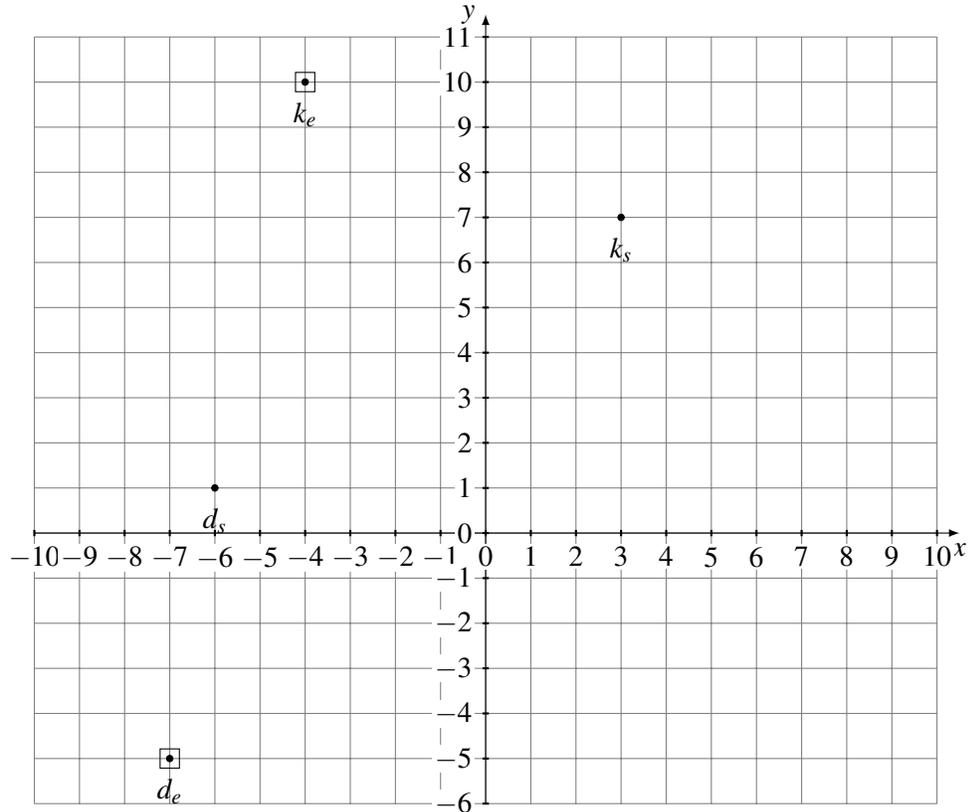
Each player must apply the same matrix transformation A on his starting point to reach reach his end point, such that $Ak_s = k_e$ and $Ad_s = d_e$. Derive the transformation matrix A, if possible. We also know that the transformation matrix A is of the form $A = \begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix}$, where a, b are real numbers.

(b) Steph Current noticed Kevin and Draymond running this drill, and decided to join them. For their next move, they will be using transformation matrix B.

$$B = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}$$

Describe what transformation matrix B performs to an input position in terms of rotations, scaling, and reflections.

(c) After a couple of drills, Kevin Bancroft came up with a new idea – he decided to race his teammates across the court to see who is faster. Kevin starts at point $k_s = [-2\ 1]^T$ and ends at $k_e = [3\ 2]^T$. Steph starts at $s_s = [0\ 0]^T$ and ends at $s_e = [-6\ -3]^T$. Can this be represented by a transformation matrix? Briefly justify why or why not.