1. Write in terms of π the following values :

$$(a)\,\Gamma\,(3.5)\,;\ (b)\,B\left(\frac{5}{6},\frac{7}{6}\right).$$

2. (a) Find the period T of a pendulum (with a string of length l) for swings from the angle $\frac{\pi}{3}$ to $-\frac{\pi}{3}$ and back in terms of complete elliptic integral.

(b) Let T_0 denote the period of a very small oscillation of the same pendulum. Use the approximation

$$K(k) \approx \frac{\pi}{2}\left(1+\frac{1}{4}k^2\right)$$

to estimate $\frac{T}{T_0}$.

3. Solve the differential equation

$$y'' = x^2 y$$

using power series.

4. Prove the following identities for the Legendre's polynomials

$$P_l'(x) = (2l-1)\,P_{l-1}(x) + (2l-5)\,P_{l-3}(x) + \cdots + P_0$$

if l is odd,

$$P_l'(x) = (2l-1)\,P_{l-1}(x) + (2l-5)\,P_{l-3}(x) + \cdots + 3P_1$$

if l is even.

Hint: write

$$P_l'(x) = \sum_{k=0}^{l-1} c_k P_k(x)$$

and find c_k using the orthogonality property.