Final Exam
5/10/2016
50 questions, 170 minutes, 20 pages
Version 389654

Name:_____

Student ID:_____

GSI name/Lab number:_____

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On my honor, I have neither given nor received any assistance in the taking of this exam.

Signed: _____

Instructions:

1. Write your full name on the front of the bubble sheet and write the name of your primary GSI and lab group number in the box labeled Version.

2. Bubble in your student ID number.

3. Record your answers on the bubble sheet (use pencil). There is one correct answer for each question. Multiple bubbles, incomplete bubbles, or stray marks will be marked incorrect.

4. You may use one 8.5" by 11" sheet of notes (both sides) for this exam provided you do not disturb those sitting nearby.

5. No electronic devices are permitted in your work area.

6. There will be no questions regarding the exam during the examination time except in cases where there is a missing page or printing problem with your exam.

7. At the end of the exam, hand in the completed bubble sheet AND the questions to your primary GSI.

1. You enter the following code into the Matlab command window. What is the value of `y`?

```
x1=1; x2=2; x3=3; x4=4;
fh = @(x1,x2) x1 + x2/2 + x3 + x4;

x1=x4; x2=x3;
x3=1; x4=5;
y = fh(x2, x1)
```

   (a) 9
   (b) 11
   (c) 11.5
   (d) 12
   (e) 12.5

2. Which of the following statements is true?

   (a) A `for` loop always has an `if` statement.
   (b) A `while` loop always follows a defined index sequence.
   (c) A `for` loop always has a larger tolerance.
   (d) A `while` loop always has a termination condition.
   (e) A `while` loop never enters an infinite loop.

3. Consider the following code:

```
a = 3;        % Line 1
b = (a == 4); % Line 2

c = class(a); % Line 3
d = class(b); % Line 4
```

   The variables `c` and `d` are character arrays that describe the class of the variables `a` and `b`. If displayed in the command window, what will `c` and `d` output, respectively?

   (a) `double` and `double`
   (b) `integer` and `logical`
   (c) `double` and `logical`
   (d) `uint8` and `double`
   (e) `numeric` and `logical`

4. Consider the following code:

```
a1 = [2, 1; 3, 2; 6, 13];
a2 = ['abc', 'def', 'ghi'];
a3 = {'Hello World', 94705, [2 4 6]};
```

What are the classes of `a1`, `a2`, and `a3`, respectively?

(a) `matrix`, `char`, and `cell`

(b) `double`, `struct`, and `cell`

(c) `double`, `char`, and `struct`

(d) `matrix`, `struct`, and `cell`

(e) `double`, `char`, and `cell`

5. We define the variables `x` and `y` in the workspace by executing the following statements in the command line:

```
x = 2;
y = 3;
```

Which of the following pieces of code would throw an error when executed in the command line?

(a) `z = x + y;`

(b) `x == y;`

(c) `x + y == 5`

(d) `x + y = 5`

(e) None of the above.

6. Consider the following function:

```
function [y] = my_square(x)
    y = x .^ 2;
end
```

Which of the following pieces of code would throw an error when executed in the command line?

(a) `y = my_square(3);`

(b) `x = my_square([3, 4, 5, 6]);`

(c) `y = my_square([2, 3])*my_square([1; 2]);`

(d) `y = my_square(4, 3);`

(e) None of the above.

7. You have a matrix A with NaNs in it and wish to replace them all with zeros. Which of the following lines will accomplish this? Note: the following line is returned when the command `help isnan` is executed.

"`isnan(A)` returns an array the same size as `A` containing logical 1 (true) where the elements of `A` are `NaN`s and logical 0 (false) where they are not."

(a) `A = A(A == isnan(A))`

(b) `A = A(isnan(A))`

(c) `A(isnan(A)) = 0`

(d) `A(A == isnan(A)) = 0`

(e) `A(A == 0) = NaN`

8. You enter the following code into the Matlab command window. What is the value of `z`?

```
x = 1:2:7;
y = 12:-3:3;
z = x.*y == 27
```

(a) 1

(b) 0

(c) [0, 1, 1, 0]

(d) [0, 1, 0, 0]

(e) [1, 0, 1, 0]

Consider this function for the next two questions:

```
function [result] = collision(sprite1, sprite2)

result = zeros(1,2);
laser = 0;
rocket = 0;

if strcmpi(sprite1, 'laser')
    laser = 1;
elseif strcmpi(sprite1, 'rocket')
    rocket = 1;
elseif strcmpi(sprite1, 'player')
    player = player + 1;
end

if strcmpi(sprite2, 'laser')
    laser = laser + 1;
elseif strcmpi(sprite2, 'rocket')
    rocket = rocket + 1;
elseif strcmpi(sprite2, 'player')
    player = player + 1;
end

if (laser == 1) && ((rocket == 1) || (player == 1))
    if strcmpi(sprite1, 'laser')
        result(2) = 1;
    else
        result(1) = 1;
    end
end

end
```

9. What is the value of `result` after the following is executed in the command window?

```
sprites = {'rocket','laser'};
result = collision(sprites{2}, sprites{1})
```

   (a) `[0 1]`
   (b) `[0 0]`
   (c) `[1 0]`
   (d) `[1 1]`
   (e) Matlab throws an error

10. What is the value of `result` after the following is executed in the command window?

```
result = collision('player', 'rocket')
```

   (a) `[0 1]`
   (b) `[0 0]`
   (c) `[1 0]`
   (d) `[1 1]`
   (e) Matlab throws an error

---

11. Consider the Matlab variable `M`, which represents the following matrix:

$$M = \begin{bmatrix} 1 & 3 & -2 & 4 \\ 0 & 1 & 5 & 12 \\ 2 & 3 & -1 & -3 \end{bmatrix}$$

Which of the following commands will return the matrix

$$\begin{bmatrix} 0 & 5 \\ 2 & -1 \end{bmatrix}$$

   (a) `M(:,1:3)`
   (b) `M(2:3,1:3)`
   (c) `M(2:3,[2,4])`
   (d) `M([1,2],[0,2])`
   (e) `M(2:3,[1,3])`

12. Consider the two following functions to compute Fibonacci numbers:

```
function x = Fib1(n)
var1 = 0;
var2 = 1;
for j=1:n
```

```
        temp = var1;
        var1 = var2;
        var2 = var2 + temp;
    end
    x = var1;
end

function x = Fib2(n)
if n==0
    x = 0;
elseif n==1
    x = 1;
else
    x = Fib2(n-1)+Fib2(n-2);
end
```

What are the time complexities of `Fib1` and `Fib2`, respectively? (C is a real constant)

(a) $O(n)$ and $O(n^2)$

(b) $O(n)$ and $O(\log_2(n))$

(c) $O(n)$ and $O(C^n)$

(d) $O(n^3)$ and $O(\log_2(n)$

(e) $O(n^2)$ and $O(C^n)$

13. Consider the following function:

```
function [result] = RecursiveFunction(n,p)
if n==p
    result=0;
elseif n>p
    result=RecursiveFunction(n-1,p)+1;
elseif p>n
    result=RecursiveFunction(n,p-1)-1;
end
```

What is the value of x after the following is executed?

```
x = RecursiveFunction(3,6)
```

(a) 3

(b) 9

(c) −3

(d) −9

(e) 0

14. Remember that in a 64-bit binary representation of numbers, each number is represented using a sequence of 64 zero(s) and/or one(s). Which of the following statements is true?

(a) All numbers that can possibly be represented by a 64-bit binary representation lie in the interval $[-2^{32}, 2^{32}]$.

(b) There exists a 64-bit binary representation which can represent all the real numbers in the interval $[-2^{32}, 2^{32}]$.

(c) At most 2^{64} different numbers can be represented by a given 64-bit binary representation.

(d) Only integers can be represented using 64-bit binary representations.

(e) All of the above.

15. Under IEEE 754 double-precision standards, each number is represented using 64 binary digits (bits). One bit is used to represent the sign (+/-), 52 bits are used to represent a fractional coefficient, and 11 bits are used to represent an exponent to which 2 is raised. The distance from one representable number to the next is called the gap. What is true about the gap for this standard?

(a) The gap is determined by the smallest representable floating point number.

(b) The gap is determined by the largest representable floating point number.

(c) The gap is constant for all numbers.

(d) The gap becomes larger as the magnitude of the number represented becomes larger.

(e) The gap becomes smaller as the magnitude of the number represented becomes larger.

16. What is the time complexity of the following function, in Big-O notation?

```
function [out] = myFunction(n)
out = 0;
while n > 1
    n = n/10;
    out = out + 1;
end
end
```

(a) $\mathcal{O}(n)$

(b) $\mathcal{O}(n^2)$

(c) $\mathcal{O}(n^3)$

(d) $\mathcal{O}(\log n)$

(e) $\mathcal{O}(C^n)$

17. Consider the following lines of code:

```
t = linspace(0, 10, 100);           % Line 1
y1 = 2*t;                           % Line 2
y2 = 5*(t.^2);                      % Line 3

figure(1);                          % Line 4
plot(t, y1, 'ro', 'LineWidth', 2)   % Line 5
grid on;                            % Line 6
plot(t, y2, 'bo', 'LineWidth', 2)   % Line 7
```

After executing the code above, how many lines will be present on Figure 1?

(a) 1 line (`y1` vs `t`)

(b) 1 line (`y2` vs `t`)

(c) 2 lines (`y1` vs `t` and `y2` vs `t`)

(d) 0 lines

(e) 2 lines (`t` vs `y1` and `y2` vs `y1`)

18. You load and manipulate your favorite self-portrait (named `awesome_selfie.png`) into Matlab using the following commands. After executing the following code, an image similar to the original image is showing. What is different about the new image?

```
img = double(imread('awesome_selfie.png'))/255;
[r c l] = size(img);
img(1:round(r/2), 1:round(c/2), :) = 0;
image(img)
```

(a) The top half is now black

(b) The right half is now red

(c) The top-left quadrant is now blue

(d) The left half is now green

(e) The top-left quadrant is now black

19. Consider the following piece of code, where N and M are integers strictly greater than 2.

```
A = zeros(N,M,3);
for i=1:N
    for j=1:M
        for k=1:3
            A(i,j,k)=1/(1+i*j);
        end
    end
end
```

Executing the command `image(A)` would display:

(a) an image where black and white are the only colors

(b) an image where all pixels are some shade of green

(c) an image where all pixels are some shade of red

(d) an image where all pixels are some shade of gray

(e) an image where all pixels are some shade of blue

20. We would like to analyze data which varies over 2D (x, y) space. Before making a plot, we execute the following code.

```
x = linspace(-50, 50, 200);
y = linspace(150, 300, 200);
[xx, yy] = meshgrid(x, y)
```

After executing the code, which of the following statements is true?

(a) Every element of `x` is equal to at least one element of `xx`.

(b) `x` has 200 elements and the size of `xx` is 200×200.

(c) `x` contains values ranging from -50 to 50, while `xx` contains values ranging from -50 to 300.

(d) Both (a) and (b) are true

(e) All of the above are true

The following two questions are related.

21. The following code is an incomplete recursive implementation of the root-finding Bisection Method. Input `f` is a function handle, and `a`, `b` and `tol` are scalars of class `double`.

```
function [ R ] = myBisection(f, a, b, tol )
% Approximates a root, R, of f bounded by a and b
% such that |f(R)| < tol
% f is a function handle. All other inputs are scalar doubles

if sign(f(a)) == sign(f(b))
    error('The scalars a and b do not necessarily bound a root')
end

m=(a+b)/2; % get midpoint

if abs(f(m)) < tol % stopping condition
    R = m;
elseif sign(f(a)) == sign(f(m))
    R = ;% to be completed
elseif sign(f(b)) == sign(f(m))
    R = ;% to be completed
end

end
```

How would you complete the first and second missing parts, respectively?

(a) `myBisection(f,a,m,tol)` and `myBisection(f,m,b,tol)`

(b) `myBisection(f,a,b,tol-f(a))` and `myBisection(f,a,b,tol-f(b))`

(c) `myBisection(f-f(a),a,b,tol)` and `myBisection(f-f(b),a,b,tol)`

(d) `myBisection(f,m,b,tol)` and `myBisection(f,a,m,tol)`

(e) `myBisection(f,a,b,tol)` and `myBisection(f,a,b,tol)`

22. Assuming the above code for the Bisection Method is correctly completed, what will be the value of `R` after executing the following commands?

```
f = @(x) x/2;
[ R ] = myBisection(f, -2, 1, 0.2 )
```

   (a) -0.5

   (b) 0.25

   (c) 0.125

   (d) -0.125

   (e) None of the above

---

23. When generating a fractal, we used the following root-finding function.

```
function [ConvergeOrNot, ConvergeToRoot] = NewtonConv(d, z, n, tol)
```

   where `d` is the order of a polynomial $z^d - 1$, `z` is a complex number as the initial guess, `n` is the number of iterations for which we apply Newton's method, and `tol` is the tolerance which determines if the output has converged after `n` iterations of Newton's method. If the iteration result is within tolerance of a root, `ConvergeOrNot` equals 1; if the iteration result is not within tolerance of any roots, `ConvergeOrNot` is 0. The function `NewtonConv` was then applied to all numbers in a specified 2D grid in the complex plane.
   If `ConvergeOrNot=0` is represented by white, and `ConvergeOrNot=1` is represented by other colors, consider the following cases with different inputs `n` and `tol`: (`d` and `z` are the same for all cases)
   1. `n=10; tol=0.0001;`
   2. `n=100, tol=0.0001;`
   3. `n=100, tol=0.01;`
   Which of these following statements is true?

   (a) Case 3 will produce the most white space, and Case 1 will produce the least white space.

   (b) Case 1 will produce the most white space, and Case 3 will produce the least white space.

   (c) Case 3 will have longer computation time than Case 1, no matter what the size of the domain is.

   (d) Case 1 will have the longest computation time, no matter what the size of the domain is.

   (e) Both (b) and (c) are true.

24. The Secant Method is a root-finding method similar to Newton's Method but uses a finite difference instead of the function's actual derivative.
    Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{1}$$

where x_n is the approximation of the root at nth iteration, f is the function, and f' is the derivative of f.

Secant Method:

$$x_{n+1} = x_n - f(x_n)\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \tag{2}$$

Which of the following is true about the Secant Method?

(a) To use the Secant Method, we must know the analytical form of the derivative $f'(x)$

(b) We only need one initial guess, x_0, to begin iteration of the Secant Method.

(c) The Secant Method requires use of `struct` arrays.

(d) The number of iterations required to find the root using the Secant Method depends on the initial guess(es).

(e) The Secant Method cannot be implemented using a *while* loop.

25. You are given a function handle, `f`, where the input `x` is a scalar double greater than zero. Assume that `f` executes in linear time with the magnitude of its input (that is, the time complexity of $f(x)$ is $\mathcal{O}(x)$). What is the time complexity of the following operation?

```
i = 1;
while i < k
    i = i + 1;
    for j = 1:N
        f(j);
    end
end
```

(a) $\mathcal{O}(kN^2)$

(b) $\mathcal{O}(kjN)$

(c) $\mathcal{O}(k^3)$

(d) $\mathcal{O}(N^2)$

(e) $\mathcal{O}(N^2 \log j)$

26. After the following code is executed, what are the values of `s1` and `s2`, respectively?

```
X = [1 2 3; 4 5 6];
Y = [6 5; 4 3; 2 1];
Z1 = X*Y;
Z2 = Y*X;
s1 = size(Z1)
s2 = size(Z2)
```

(a) `[2 2]` and `[2 2]`

(b) `[2 2]` and `[3 3]`

(c) `[3 3]` and `[2 2]`

(d) `[2 3]` and `[3 2]`

(e) `[3 3]` and `[3 3]`

The following two questions are related.

Consider the linear system $Ax = b$. The size of A is $p \times m$, and the size of x is $m \times 1$. You can assume that all rows in A and all equations in the system are linearly independent.

27. You want to use Matlab's backslash operator to solve the system (with `x = A\b`). What will the backslash operator do when executing `x = A\b` if $p > m$?

    (a) Backslash will return a single solution, which uniquely and directly solves the system.

    (b) The system is over-determined. Backslash will return a single approximation which minimizes the squared error.

    (c) The system is under-determined. Backslash will return an error.

    (d) Backslash will use Simpson's Rule.

    (e) Backslash will solve the system using polynomial interpolation.

28. You want to use Matlab's backslash operator to solve the system (with `x = A\b`). What will the backslash operator do when executing `x = A\b` if $p = m$?

    (a) Backslash will return a single solution, which uniquely and directly solves the system.

    (b) The system is over-determined. Backslash will return a single approximation which minimizes the squared error.

    (c) The system is under-determined. Backslash will return an error.

    (d) Backslash will use Simpson's Rule.

    (e) Backslash will solve the system using polynomial interpolation.

29. Consider the following lines of code:

```
A = [2 1; 0 1];
I = [1 0; 0 1];
B = (A+I)^2-(A+I).^2;
```

    What is the value of `B`?

    (a) `[0 4;0 0]`

    (b) `[0 0;0 0]`

    (c) `[0 -4;0 0]`

    (d) `[2 6;4 2]`

    (e) Error: Matrix dimensions don't agree.

30. The root mean square error (RMSE) is a common metric used to evaluate a regression model based on how well it fits the observed data. It is defined by

$$RMSE = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (\hat{y}_i - y_i)^2}$$

where $\hat{y}_i, i = 1, 2, ..., m$ represent values predicted by the regression model, and $y_i, i = 1, 2, ..., m$ represent the observed data points. In MATLAB, $\hat{y}$ is represented by the $1 \times m$ array `y_hat` and y is represented by the $1 \times m$ array `y`. To compute RMSE using MATLAB, you could use which of the following commands?

(a) `RMSE = sqrt(mean((y_hat-y).^2))`

(b) `RMSE = sqrt((1/length(y))*sum((y_hat-y).^2))`

(c) `RMSE = (mean((y_hat-y).^2))^(0.5)`

(d) All of the above

(e) None of the above

31. Consider the following piece of code, given data points stored in the column vectors `x` and `y`.

```
A = [ones(size(x)), x];
alpha = A\y
alpha =
    -4.33
    3.56
```

According to MATLAB, what is the equation of the least squares best fit?

(a) $y = -4.33 + 3.56x$

(b) $x = -4.33 + 3.56y$

(c) $y = -4.33x + 3.56$

(d) $y = -4.33x + 3.56x^2$

(e) $y = -4.33x$

The following two questions are related.

32. Consider a data set that consists of two arrays, `t1` and `y1`. The array `t1` goes from 0 to 100 and has ten equally spaced elements (i.e. `t1 = linspace(0,100,10)`). The array `y1` has ten elements, each of which corresponds to one of the time values listed in the array `t1`. You wish to perform some interpolation to get a better understanding of the data between the times $t = 0$ and $t = 100$. Assuming `t1` and `y1` are already in your workspace, consider the following code:

```
t2 = linspace(0, 100, 1000);         % Line 1
y2 = interp1(t1, y1, t2, 'spline');  % Line 2

a = numel(y2);                       % Line 3 - "numel" returns the "number of elements"
```

Based on the code above, what is the value of `a`?

(a) 10

(b) 100

(c) 1000

(d) It depends on the values in the data `y1`

(e) It is impossible to know

33. Now, consider a modification to line 1 from the code presented in the previous problem:

```
t2 = linspace(0, 100, 10);        % Line 1
y2 = interp1(t1, y1, t2, 'spline');  % Line 2
```

Is this a good example of interpolation?

(a) No. No new information is gained.

(b) Yes, although now the interpolation is only occurring from t = 0 to t = 5.

(c) Yes, but only because we chose a spline interpolation method.

(d) No. 'spline' is never a good choice when using the command `interp1`.

(e) Yes, because we used `linspace` to create the array `t2`.

---

34. Consider the time-dependent heat diffusion equation in one dimension. Using finite differences to express the spatial derivatives, it can be shown that the temperature value at each grid point can be estimated as a weighted average of the neighbors (note that T_j^n represents $T(x_j, t_n)$). Given temperature values of $T = (0, 0, 1, 0, 0)$ at $x = (0, 1, 2, 3, 4)$ at time zero, what does this formula predict the temperature will be at $x = 3$ at the next time step?

$$T_j^{n+1} = T_j^n + 0.4(T_{j-1}^n - 2T_j^n + T_{j+1}^n) \tag{3}$$

(a) 0

(b) 0.2

(c) 0.4

(d) 0.8

(e) 1

(f) None of the above

35. Curves are part of many modern building designs. To create a smooth surface which is pleasing to the eye, cubic splines are used to fit a curve to a set of points. Fitting a cubic spline involves

(a) integration of the arclength of the curve

(b) finding a unique solution of a system of equations

(c) least-squares regression to find the best fit curve

(d) determining the order of the truncation error

(e) linear interpolation between points

36. Assume that at a specific time, BART train cars have a position $x(t)$, a velocity $v(t)$, and an acceleration given by $a(t) = 3.0 + 0.1t^2$ ft/s^2, where t is time in seconds. For a train initially at rest ($v(0) = 0$ and $x(0) = 0$), you can estimate the train's position x after 6 seconds using the first three terms of a Taylor series expansion: $x(6) = x(0) + x'(0)\Delta t + x''(0)\Delta t^2/2$ where $\Delta t = 6$ s. How would you estimate the error associated with this 3-term series?

    (a) $a(0)\Delta t^2/2$

    (b) $a'(0)\Delta t^3/6$

    (c) $v'(0)\Delta t^2/2$

    (d) $x''(0)\Delta t^2/2$

    (e) $x'''(0)\Delta t^2/2$

The following two questions are related.

37. A vector `x` contains values $x_1, x_2, ..., x_n$, and a vector `f` contains values $f(x_1), f(x_2), ..., f(x_n)$ for a function f. We wish to approximate the first derivative $f'(x)$ and store it in the vector `df`. We use the forward difference approximation:

$$\left.\frac{df}{dx}\right|_{x_k} \approx \frac{f(x_{k+1}) - f(x_k)}{x_{k+1} - x_k}$$

We start by writing the following code. What is wrong with this code?

```
n = length(x);
df = zeros(size(x));
for k = 1:length(x)
    df(k) = (f(k+1) - f(k)) / (x(k+1) - x(k));
end
```

    (a) It will compute the central difference approximation instead of the forward difference approximation.

    (b) It will estimate the second derivative instead of the first derivative.

    (c) It will throw an error when $k = n$ because `f(n+1)` does not exist.

    (d) It will throw an error when $k = 1$ because `df(1)` is not defined.

    (e) It will throw an error because `/` should be `./`

38. For the same arrays `f` and `x` from the previous problem, we wish to compute the central difference approximation for $f'(x_2), f'(x_3), ..., f'(x_{n-1})$. The central difference approximation is given by

$$\left. \frac{df}{dx} \right|_{x_k} \approx \frac{f(x_{k+1}) - f(x_{k-1})}{x_{k+1} - x_{k-1}}$$

To do this, we write the following partial code. What code should be added to complete the line?

```
df2 = (f(3:end) - f(1:end-2)) ./ % missing code
```

   (a) `(x(2) - x(1))`
   (b) `(x(end) - x(1))`
   (c) `(x(1) - x(3))`
   (d) `(x(3:end) - x(1:end-2))`
   (e) `(x(2:end-1) - x(1:end-2))`


The following two questions are related.

39. Taylor series can be used to derive finite difference formulas. Consider the Taylor series for $f(x_{j-1})$ and $f(x_{j+1})$ expanded about x_j. Combining these two expansions gives you which of the following formulas for the first derivative?

   (a) Central Difference
   (b) Backward Difference
   (c) Forward Difference
   (d) B or C
   (e) A or C

40. Which of the following formulas is the result of combining the series described in the previous question? The interval length $h = x_{j+1} - x_j$ is uniform for all j.

   (a) $f'(x_j) = \frac{x_{j+1} - x_{j-1}}{h} + O(h^2)$
   (b) $f'(x_j) = \frac{f(x_{j+1}) - f(x_{j-1})}{2h} + O(h)$
   (c) $f'(x_j) = \frac{f(x_{j+1}) - f(x_{j-1})}{2h} + O(h^2)$
   (d) $f'(x_j) = \frac{f(x_{j+1}) - f(x_{j-1})}{h} + O(h^2)$
   (e) $f'(x_j) = \frac{f(x_{j+1}) - 2f(x_j) + f(x_{j-1})}{h^2} + O(h^2)$

Refer to the following figure for the next two questions
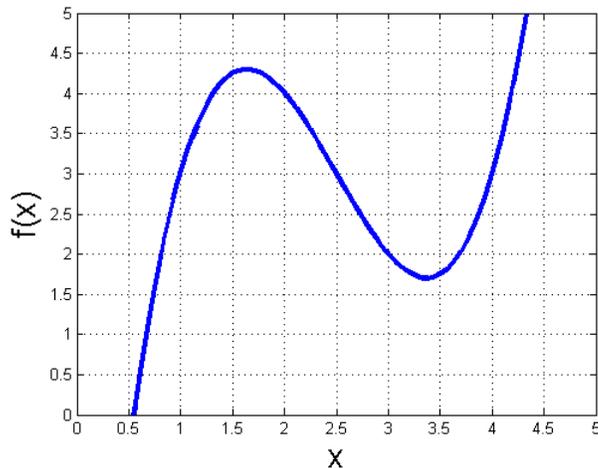
41. What is the **forward difference** approximation of $\frac{df}{dx}$ at $x = 3$? Use an interval width of $\Delta x = 1$.

    (a) -2

    (b) -1.5

    (c) -0.5

    (d) 1

    (e) 2

42. What is the left end-point Riemann Sum approximation of $\int_1^3 f(x)dx$? Assume the interval width is constant at $\Delta x = 1$.

    (a) 6

    (b) 6.5

    (c) 7

    (d) 9

    (e) 12

43. The first N terms of the Taylor series expansion about the origin for the exponential function are given by:

$$e^x \approx \sum_{n=0}^{N-1} \frac{x^n}{n!}$$

If an approximation for e^x were made by using the first three terms of the series, what would the order of the error be?

(a) $O(x)$

(b) $O(x^2)$

(c) $O(x^3)$

(d) $O(x^4)$

(e) The approximation becomes exact with three terms

44. We wish to approximate the integral $\int_a^b f(x)dx$. We subdivide $[a, b]$ into n intervals delimited by the points x_0, x_1, ..., x_n, where $x_0 = a$ and $x_n = b$. A $1 \times (n+1)$ array x is already defined in Matlab, along with a function handle f, which is able to compute f(x) without throwing an error. Which of the following formulas integrates the function f between a and b according to the previous subdivision **using Riemann's Rule and using left values** ($[x_0, x_1, ..., x_{n-1}]$).

(a) `sum(0.5*(f(x(1:end-1))+f(x(2:end))).*(x(2:end)-x(1:end-1)))`

(b) `sum(f(x(1:end-1)).*(x(2:end)-x(1:end-1)))`

(c) `sum(f(x).*x)`

(d) `sum(f(x(1:end))*(x(end)-x(1)))`

(e) `sum(f(x(2:end))*(x(1:end)))`

45. Euler's method solves an ordinary differential equation, $\frac{dy}{dt} = f(y, t)$, by iterating using the following formula:

$$y_{i+1} = y_i + \Delta t f(y_i, t_i).$$

What is the form for using Euler's method to solve the following differential equation?

$$2\frac{dy}{dt} + 3y = e^{-5t}?$$

(a) $y_{i+1} = y_i + \Delta t \frac{-2e^{-5t_i}}{3y_i}$

(b) $y_{i+1} = y_i + \Delta t \left(-\frac{3}{2}y_i + \frac{e^{-5t_i}}{2} \right)$

(c) $y_{i+1} = \frac{5}{2}y_i + \Delta t e^{-5t_i}$

(d) $y_{i+1} = e^{\frac{5}{2}y_i} + \Delta t 5 t_i$

(e) None of the above

The following two questions are related.

Consider the following ordinary differential equation and its initial condition:

$$\frac{dg}{dt} = -g + 3 \qquad \text{(Ordinary differential equation)}$$
$$g(0) = 5 \qquad \text{(Initial condition)}$$

where g is the unknown function of t.

46. Which of the following statements is true when using Euler's method with a step $\Delta t = 1$ to estimate the value of g(1)?

(a) We assume that dg/dt is constant over the interval $[0, 1]$.

(b) The value we obtain for $g(1)$ may be different from the true value of $g(1)$.

(c) We first have to estimate the value of $g(0.5)$ before we can estimate the value of $g(1)$.

(d) (a) and (b), but not (c)

(e) (a) and (c), but not (b).

47. What is the value of $g(1)$ estimated using Euler's method with a step $\Delta t = 1$?

(a) 4

(b) 3

(c) -2

(d) 7

(e) None of the above

48. Matlab's `ode45` sometimes fails to obtain a result which closely matches the correct solution of an ODE. Which of the following is NOT a possible reason for this behavior?

(a) The ode45 algorithm cannot find a good solution if there is no analytical solution to the ODE.

(b) The ode45 algorithm may not be able to refine the step size properly for a rapidly varying function.

(c) If the integration period is long, the global error from the `ode45` prediction can eventually become large.

(d) If the step size is too large, the numerical solution can become unstable.

(e) If the step size is too large, the truncation error can become large.

49. Euler's method can be written as follows:

$$T(t_{i+1}) = T(t_i) + \left.\frac{dT}{dt}\right|_{t_i} \Delta t \tag{4}$$

Use this to solve for temperature T given the differential equation for temperature inside a building and a given ambient temperature T_A:

$$\frac{dT(t)}{dt} = \frac{1}{RC}(T_A(t) - T(t)) \tag{5}$$

Given that $R = 1$ and $C = 2$ and the following table, use Euler's method to solve for $T(t = 1)$.

time	T_A	T
0	60	70
1	65	?
2	68	?

What is $T(t = 1)$ according to Euler's method with a step size $\Delta t = 1$?

(a) 60

(b) 62.5

(c) 65

(d) 70

(e) 72

50. The predictor-corrector method can be used to solve an ODE of the form $\frac{dy}{dx} = f(x, y)$ as follows:

$$
\begin{aligned}
y^*_{i+1} &= y_i + f(x_i, y_i)\Delta x \\
y_{i+1} &= y_i + 0.5\left(f(x_i, y_i) + f(x_{i+1}, y^*_{i+1})\right)\Delta x
\end{aligned}
$$

Consider this ODE:

$$\frac{dy}{dx} = x - y^2$$

Given $y(0) = 2$, and $\Delta x = 1$, what is $y(1)$ from the predictor-corrector method?

(a) -2

(b) -1.5

(c) -1

(d) 0

(e) 2