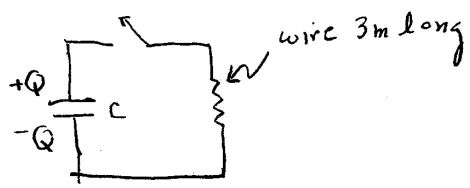
Physics 7B Spring 2000 Final Exam R. Packard (7 problems)

1. (35pts) A capacitor C=2 μfd is charged with $Q=3x10^{-4}C$ and is connected in series with a wire of length l=3m. The wire has a circular cross-section with radius $r=1x10^{-5}$ m, a resistivity $\rho=9.7x10^{-8}\Omega m$, a mass $m=10^{-4}kg$ and specific heat c_p of 450J/kgC°. When the switch (shown below) is closed at t=0 the capacitor discharges through the resistor.

a. How long after the switch closes does the voltage on the capacitor reach 75 volts? Call this time t_1,
b. At t_1 what is the voltage across the resistance?
c. How much thermal energy is deposited in the wire by the discharge process?
d. What is the temperature rise of the wire due to the complete discharge process? Ignore heat lost to the environment during the discharge process.



Problem 1 (Packard Exam)

(a) $R = \frac{\rho L}{A} = \frac{(9.7\times10^{-8})(3)}{\pi(1\times10^{-5})^2} = 926.28\,\Omega$

$Q(t) = Q_o e^{-t/RC}$; $V_c(t) = \frac{Q_o}{C} e^{-t/RC}$

$V(t_1) = 75\text{ V} \Rightarrow \ln\left(\frac{75C}{Q_o}\right) = -\frac{t_1}{RC} \Rightarrow t_1 = RC\ln\left(\frac{Q_o}{CV_o}\right)$

or $t_1 = (926.28)(2\times10^{-6})\ln\left(\frac{3\times10^{-4}}{2\times10^{-6}(75)}\right)$

$= \underline{1.28\times10^{-3}\text{ s}}$.

(b) $V_R = V_c = 75\text{ V}$

(c) $U = Q_o^2/2C = (3\times10^{-4})^2/2(2\times10^{-6}) = \underline{0.0225\text{ J}}$.

(d) $Q = mc_p\Delta T \Rightarrow \Delta T = \frac{Q}{mc_p} = \frac{0.0225}{(10^{-4})(450)} = \underline{0.5\,^\circ\text{C}}$.

Grading Scheme

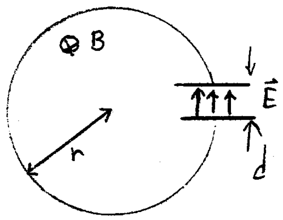
(a) 20 pts. i) correct evaluation of resistance = 10 pts
ii) correct expression for $V(t)$ or $Q(t)$ or $I(t)$ + correct solution for t_1 = 10 pts.

(b) 5 pts. For full credit, you need to state that $V_R(t = t_1)$ is precisely 75 V, as demanded by Kirchoff's Rule. If you got an approximate numerical answer by plugging in the result of part (a), a point was deducted.

(c) 5 pts. Two points off for incorrect expression for initial energy stored in capacitor or for not finding total power (i.e., from $t=0$ to $t=\infty$)

(d) 5 pts. Some points taken off for algebraic errors or miss-application of $Q = mc_p\Delta T$.

2. (20pts) A flat circular loop (radius r=10cm) of wire connects the two sides of a parallel plate capacitor with gap d=0.1mm. The breakdown electric field strength of the air in the gap is $3\text{x}10^6$Nt/C. A uniform magnetic field perpendicular to the plane of the loop is increasing exponentially in time: $B = 2e^{3t}\,(T)$. What is the value of the magnetic field when the gap breaks down?



Changing magnetic flux induces emf:

$$\Phi_B = \int B \cdot dA = BA = B\pi r^2$$

$$\frac{d\Phi_B}{dt} = \frac{dB}{dt}\pi r^2 = 3B\pi r^2$$

(since $B = 2e^{3t}$, $\frac{dB}{dt} = 3B$)

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} \quad \text{(don't care about direction)}$$

But since the wire is a conductor, there is no electric field in it. The electric field exists only in the gap: $\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = Ed$.

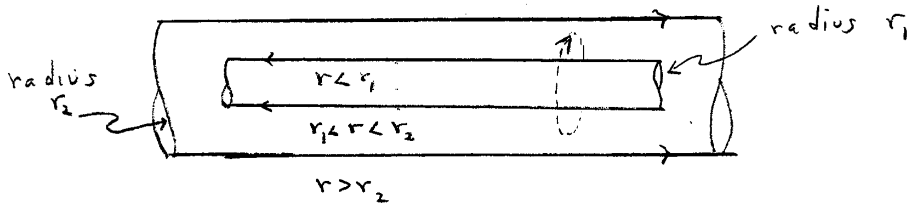
Breakdown occurs when $Ed = 3B\pi r^2 \Rightarrow B = \frac{Ed}{3\pi r^2} = \frac{(3\times10^6 V)(10^{-4} m)}{(3 s^{-1})\pi (0.1 m)^2} = \boxed{3.2\times10^3 T}$

Grading:

20: perfect score
19: mistake plugging in numbers
18: algebra mistake that gives result with wrong units
16: OK except Φ was used instead of $d\Phi/dt$, or E/d instead of Ed
14: used $\oint \vec{E} \cdot d\vec{l} = 2\pi r E$ (very common mistake)
10: found $\mathcal{E}$ correctly but didn't connect it to E, or confused it with E
4-6: stated Faraday's law but could not do area integral or time derivative
0: if no mention of Faraday's law

= hollow shells

3. (20pts) Consider a long cable made of two coaxial thin cylinders. The inner cylinder has radius r_1 and the outer cylinder has ~~radius r_2~~. The inner cylinder carries a current +I and –I flows in the outer conductor.
   a. Compute the magnetic field B for the three regions: $r<r_1$; $r_1<r<r_2$; and $r>r_2$ Neglect the thickness of the conductors. Do not just write down the answer from your notes. Show the calculation
   b. Use the result in part **a.** to compute the magnetic energy stored (per unit length) in the cable.



a) By symmetry, the B field lines are circles, and B depends only on radius. Use a circle of radius r, perpendicular to the cable, as the Amperian loop.

$\oint \vec{B}\cdot d\vec{l} = 2\pi r B$ around this loop.

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$ by Ampère's law; current enclosed is 0 for $r<r_1$, I for $r_1<r<r_2$, 0 $(I-I)$ for $r>r_2$.

Therefore $\boxed{\begin{array}{l} B(r<r_1)=0 \\ B(r_1<r<r_2) = \frac{\mu_0 I}{2\pi r}, \text{ direction as shown in diagram} \\ B(r>r_2)=0. \end{array}}$

b) Magnetic energy density is $u = \frac{B^2}{2\mu_0} = \frac{\mu_0 I^2}{8\pi^2 r^2}$ for $r_1<r<r_2$, 0 elsewhere

Energy in a thin cylindrical shell of length l, radius r, thickness dr is

$$dU = u\, 2\pi r l\, dr = \frac{\mu_0 I^2 l}{4\pi r}\, dr$$

Total energy in cable is

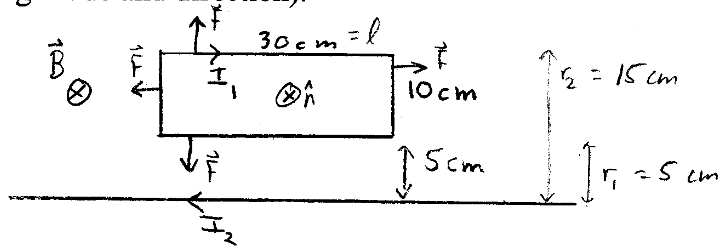
$$U = \int dU = \int_{r_1}^{r_2} \frac{\mu_0 I^2 l}{4\pi r}\, dr = \frac{\mu_0 I^2 l}{4\pi} \ln\frac{r_2}{r_1}$$

Energy per unit length is $\boxed{\frac{U}{l} = \frac{\mu_0 I^2}{4\pi} \ln\frac{r_2}{r_1}.}$

4. (20pts) A rectangular loop of wire with the dimensions shown below carries a current I_1=3 amps. The loop lies in a plane of a very long straight wire carrying a current I_2=22A.

a. Is there a torque on the loop? If your answer is yes compute the torque (magnitude and direction)

b. Is there a net force on the loop? If your answer is yes, compute the force (magnitude and direction).



10 pts. a) $\vec{B}$ produced by the straight wire points into the page at the loop, by the right hand rule. The magnetic dipole moment of the loop is also directed into the page by the right hand rule.

Therefore the torque is $\boxed{\vec{\tau} = \vec{\mu} \times \vec{B} = 0.}$

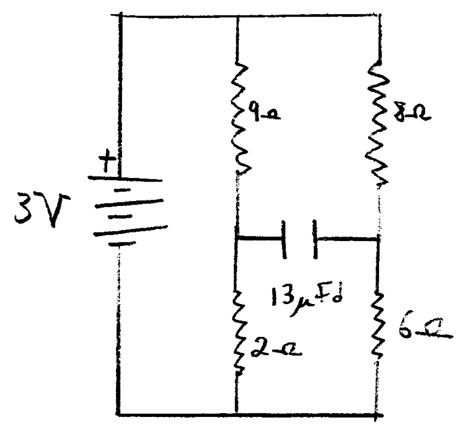
10 pts. b) The directions of magnetic force on the four sides of the rectangle are shown in the diagram. By symmetry, the forces on the left and right sides cancel each other. The forces on the top and bottom are

$$\vec{F}_{top} = I_1 \vec{\ell} \times \vec{B} = I_1 \ell \frac{\mu_0 I_2}{2\pi r_2} \text{ directed up}$$

$$\vec{F}_{bottom} = I_1 \vec{\ell} \times \vec{B} = I_1 \ell \frac{\mu_0 I_2}{2\pi r_1} \text{ directed down}$$

The net force is $\frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \boxed{5.28 \times 10^{-5} \text{ N directed down.}}$

5. (15pts) In the circuit shown below, find the energy stored on the capacitor when it is fully charged.



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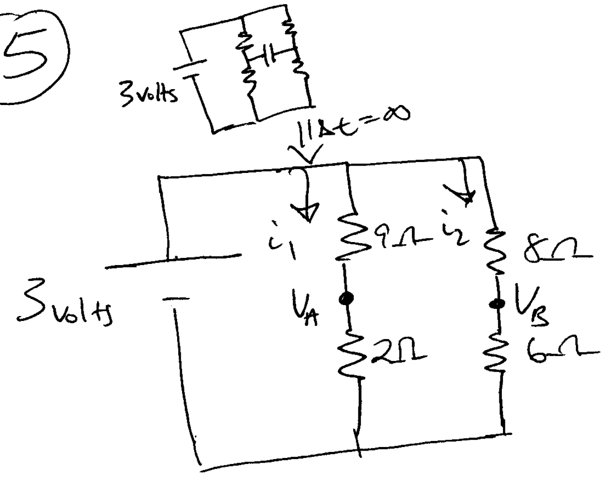

This is what the circuit looks like after an infinite amount of time has passed. i.e. no current flows through capacitor.

① $i_1 = \frac{3 \text{ volts}}{11\,\Omega}$ & $i_2 = \frac{3 \text{ volts}}{14\,\Omega}$

② $V_A = 3 - i_1 9\Omega$ & $V_B = 3 - i_2 8\Omega$

$V_A \simeq .55 \text{ volts}$ $\qquad V_B \simeq 1.29 \text{ volts}$

③ $U = \frac{1}{2} C \underbrace{(V_A - V_B)}_{\text{potential across capacitor}}^2 \simeq \boxed{3.56 \cdot 10^{-6} \text{ J}}$

or

$U = \frac{1}{2} \frac{Q^2}{C}$

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4 points – If you draw above diagram & demonstrate that you need to examine the circuit at $t = \infty$.

4 points – Finding the potential and/or the charge ~~on the~~ across the capacitor.

4 points – Using that potential and/or charge [illegible] in equation 3.

1 point – writing equation 3.

2 points – Correct answer with appropriate units

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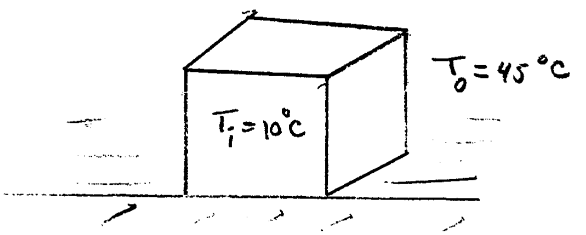
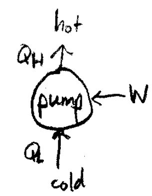
6. (30pts) A cubical food storage container (side length L=3m) has walls and roof of thickness d=0.1 m. The walls and roof are made of material with thermal conductivity κ=0.1J/smC°. On a hot day the outside temperature is 45°C. The owner of the food wants to keep the inside of the container at 10°C by running an electrically powered heat pump cooling system. The heat pump runs at 70% of the Carnot efficiency. If the cost of electricity is \$0.30/kw-hr, how much does it cost to cool the container for ten hours? Ignore the heat loss through the floor.



Heat leaks through the walls and roof at a rate of

$$\frac{Q}{t} = \kappa \frac{A}{d} \Delta T = \kappa \frac{5L^2}{d} \Delta T = 1575 \text{ Watts}$$



The "efficiency" of a cooling system is

$$e = \frac{\text{what you want}}{\text{what you pay for}} = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$$

For a Carnot cycle, $\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$; therefore for this system

$$e = \frac{Q_L}{W} = (0.70) \frac{T_L}{T_H - T_L} = 5.66$$

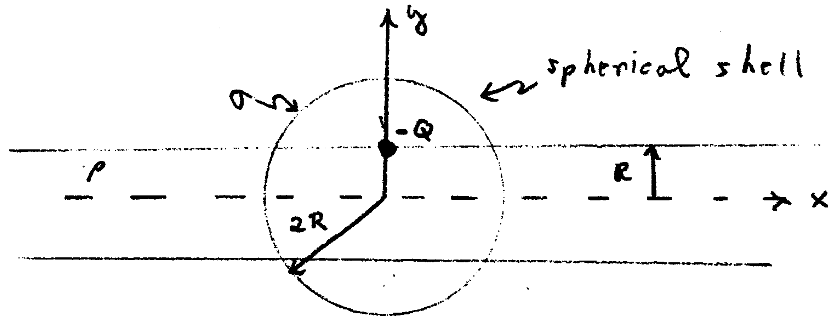
The power needed to run it is

$$\frac{W}{t} = \frac{(Q_L/t)}{e} = 278.3 \text{ Watts}$$

so in 10 hours, 2783 Watt-hours are used, at a cost of

$$(\$0.30/\text{kW-hr})(2.783 \text{ kW-hr}) = \boxed{\$0.83}$$

7. (25pts) The x axis forms the symmetry axis of an infinitely long cylinder of radius R carrying a uniform positive charge density ρ. A shell with uniform surface charge density σ and radius 2R is centered on the origin. A charge –Q and mass m is released from the point x=0, y=R. Find its speed v when it reaches the origin.



5 ① E field in the shell due to the shell

→ charge is 0, No acceleration for Q.

② Conservation of Energy

5 → $Q\Delta V = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{Q\Delta V\,2}{m}}$

$\Delta V = \int_0^R E\cdot dr$

$\oint E\,dA = \frac{Q_{enc}}{\varepsilon_0}$

5 → $2\pi r \ell E = \frac{\pi r^2 \ell \cdot \rho}{\varepsilon_0}$

$E = \frac{r\rho}{2\varepsilon_0}$

5 $\Delta V = \frac{\rho}{4\varepsilon_0}R^2$

$v = \sqrt{\frac{\rho R^2}{4\varepsilon_0}\cdot\frac{Q\,2}{m}} = R\sqrt{\frac{Q\rho}{2\varepsilon_0 m}}$

5 → $\boxed{v = R\sqrt{\frac{Q\rho}{2m\varepsilon_0}}}$ ###

If gravity is considered, no point will be deducted.