Math 16A First Midterm 25 Sep 2008

NAME (1 pt): ______________________________

TA (1 pt): ______________________________

Name of Neighbor to your left (1 pt): ______________________________

Name of Neighbor to your right (1 pt): ______________________________

Instructions: This is a closed book, closed notes, closed calculator, closed phone, closed computer, closed network, open brain exam.

You get one point each for filling in the 4 lines at the top of this page. If you are sitting next to the steps or the wall, "steps" or "wall" are acceptable answers for the last 2 lines. Fill in these lines, and then wait for us to tell you to turn the page and start.

All other questions are worth the number of points shown. You may only use techniques in the book through the end of Section 1.6 in order to solve the questions. For full (or partial) credit you need to show your work. You may of course check your answers using any other facts you know.

Write all your answers on this exam. If you need scratch paper, ask for it, write your name on each sheet, and attach it when you turn it in (we have a stapler).

At the end of the exam, you must line up and turn in your exam personally to your Teaching Assistant. Have your photo ID out so that the Teaching Assistant can look at it. You must have your photo ID in order to turn in your exam.

1	
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Total	

ANSWERS

Question 1.) (15 points) Mark the following statements "True" or "False".
(<u>DO NOT GUESS</u>: -2 points for each wrong answer!)

(a) $\frac{d}{dx}(2x^2) = 2x$ T F
Answer: False
(b) The lines $x + 2y = 8$ and $4x - 2y = 12$ are perpendicular. T F
Answer: True.
(c) $\frac{d}{dy}(\frac{5y}{3y^2-12y+1}) = 5y\frac{d}{dy}(\frac{1}{3y^2-12y+1})$ T F
Answer: False.
(d) The curve $y = 2x^3 - x$ has a tangent line parallel to $x + y = 1$. T F
Answer: True.
(e) $\frac{d}{dz}(12z^2 + 4z + 5) = \frac{d}{dz}(12z^2 + 4z) + 5$ T F
Answer: False.

Question 1.) (15 points) Mark the following statements "True" or "False".
(<u>DO NOT GUESS</u>: -2 points for each wrong answer!)

(a) The lines $4x + 2y = 8$ and $2x - y = 7$ are parallel. T F

Answer: False.

(b) $\frac{d}{dy}(\frac{8y}{3\sqrt{y^2-15y}}) = 8\frac{d}{dy}(\frac{y}{3\sqrt{y^2-15y}})$ T F

Answer: True.

(c) The curve $y = 3x^3 - 3x^2 + 7$ has a tangent line parallel to $y = 8$. T F

Answer: True.

(d) $\frac{d}{dx}(3^2) = 6$ T F

Answer: False.

(e) $\frac{d}{dt}(8t^3 + 15t + 7) = \frac{d}{dt}(8t^3 + 15t) + \frac{d}{dt}(7)$ T F

Answer: True.

Question 1.) (15 points) Mark the following statements "True" or "False".
(DO NOT GUESS: -2 points for each wrong answer!)

(a) $\frac{d}{dt}(4t^3 + 7t^2 - 15t) = \frac{d}{dt}(4t^3 + 7t^2) - 15\frac{d}{dt}(t)$. T F

Answer: True.

(b) $\frac{d}{dz}(z^3) = 3z$. T F

Answer: False.

(c) The curve $y = 4x^3 - 5x^2 + 3x + 7$ has a tangent line parallel to $3y = 9x + 7$. T F

Answer: True.

(d) $\frac{d}{dx}\left(\frac{9x+18}{(x+3)(x-4)(x-1)}\right) = \left(\frac{d}{dx}(9)\right)\left(\frac{d}{dx}\left(\frac{x+2}{(x+3)(x-4)(x-1)}\right)\right)$. T F

Answer: False.

(e) The lines $3x + y = 5$ and $x + 3y = 7$ are perpendicular. T F

Answer: False.

Question 2.) (16 points) Compute the following limits. You may only use techniques from Chapter 1.

(a) $\lim_{x\to 25} \frac{25-x}{\sqrt{x}-5}$

(b) $\lim_{t\to 2} \frac{\frac{1}{4}-\frac{1}{2t}}{2-t}$

(c) $\lim_{x\to -\infty} \frac{2x^2-x-1}{x^2+x-2}$

(d) $\lim_{y\to -2} \frac{2y^2+5y+2}{y^2+3y+2}$

Answer:

(a)

$$\begin{aligned}\lim_{x\to 25} \frac{25-x}{\sqrt{x}-5} &= \lim_{x\to 25} \frac{(25-x)(\sqrt{x}+5)}{(\sqrt{x}-5)(\sqrt{x}+5)} \\ &= \lim_{x\to 25} \frac{(25-x)(\sqrt{x}+5)}{x-25} = \lim_{x\to 25} -(\sqrt{x}+5) = -10\end{aligned}$$

(b) $\lim_{t\to 2} \frac{\frac{1}{4}-\frac{1}{2t}}{2-t} = \lim_{t\to 2} \frac{4t(\frac{1}{4}-\frac{1}{2t})}{4t(2-t)} = \lim_{t\to 2} \frac{t-2}{4t(2-t)} = \lim_{t\to 2} \frac{1}{-4t} = -\frac{1}{8}$

(c) $\lim_{x\to -\infty} \frac{2x^2-x-1}{x^2+x-2} = \lim_{x\to -\infty} \frac{2-\frac{1}{x}-\frac{1}{x^2}}{1+\frac{1}{x}-\frac{2}{x^2}} = \frac{2-0-0}{1+0-0} = 2$

(d) $\lim_{y\to -2} \frac{2y^2+5y+2}{y^2+3y+2} = \lim_{y\to -2} \frac{(y+2)(2y+1)}{(y+2)(y+1)} = \lim_{y\to -2} \frac{(2y+1)}{(y+1)} = 3$

Question 2.) (16 points) Compute the following limits. You may only use techniques from Chapter 1.

(a) $\lim_{x\to-2} \frac{\frac{1}{6}+\frac{1}{3x}}{x+2}$

(b) $\lim_{t\to4} \frac{\sqrt{t}-2}{4-t}$

(c) $\lim_{y\to-1} \frac{y^2-y-2}{2y^2+y-1}$

(d) $\lim_{t\to-\infty} \frac{3t^2+t+2}{2t^2-t+9}$

Answer:

(a) $\lim_{x\to-2} \frac{\frac{1}{6}+\frac{1}{3x}}{x+2} = \lim_{x\to-2} \frac{6x(\frac{1}{6}+\frac{1}{3x})}{6x(x+2)} = \lim_{x\to-2} \frac{(x+2)}{6x(x+2)} = \lim_{x\to-2} \frac{1}{6x} = -\frac{1}{12}$

(b)

$$\begin{aligned}\lim_{t\to4} \frac{\sqrt{t}-2}{4-t} &= \lim_{t\to4} \frac{(\sqrt{t}-2)(\sqrt{t}+2)}{(4-t)(\sqrt{t}+2)} \\ &= \lim_{t\to4} \frac{t-4}{(4-t)(\sqrt{t}+2)} = \lim_{t\to4} \frac{-1}{(\sqrt{t}+2)} = -\frac{1}{4}\end{aligned}$$

(c) $\lim_{y\to-1} \frac{y^2-y-2}{2y^2+y-1} = \lim_{y\to-1} \frac{(y+1)(y-2)}{(y+1)(2y-1)} = \lim_{y\to-1} \frac{(y-2)}{(2y-1)} = 1$

(d) $\lim_{t\to-\infty} \frac{3t^2+t+2}{2t^2-t+9} = \lim_{t\to-\infty} \frac{3+\frac{1}{t}+\frac{2}{t^2}}{2-\frac{1}{t}+\frac{9}{t^2}} = \frac{3+0+0}{2-0+0} = \frac{3}{2}$

Question 2.) (16 points) Compute the following limits. You may only use techniques from Chapter 1.

(a) $\lim_{x\to-\infty} \frac{2-7x-2x^2}{3-x+9x^2}$

(b) $\lim_{t\to-3} \frac{t^2+2t-3}{2t^2+7t+3}$

(c) $\lim_{x\to3} \frac{-\frac{1}{3x}+\frac{1}{9}}{3x-9}$

(d) $\lim_{x\to16} \frac{x-16}{4-\sqrt{x}}$

Answer:

(a) $\lim_{x\to-\infty} \frac{2-7x-2x^2}{3-x+9x^2} = \lim_{x\to-\infty} \frac{\frac{2}{x^2}-\frac{7}{x}-2}{\frac{3}{x^2}-\frac{1}{x}+9} = \frac{0-0-2}{0-0+9} = -\frac{2}{9}$

(b) $\lim_{t\to-3} \frac{t^2+2t-3}{2t^2+7t+3} = \lim_{t\to-3} \frac{(t+3)(t-1)}{(t+3)(2t+1)} = \lim_{t\to-3} \frac{(t-1)}{(2t+1)} = \frac{4}{5}$

(c) $\lim_{x\to3} \frac{-\frac{1}{3x}+\frac{1}{9}}{3x-9} = \lim_{x\to3} \frac{9x(-\frac{1}{3x}+\frac{1}{9})}{9x(3x-9)} = \lim_{x\to3} \frac{(-3+x)}{9x(3x-9)} = \lim_{x\to3} \frac{x-3}{27x(x-3)} = \lim_{x\to3} \frac{1}{27x} = \frac{1}{81}$

(d)

$$\begin{aligned}\lim_{x\to16} \frac{x-16}{4-\sqrt{x}} &= \lim_{x\to16} \frac{(x-16)(4+\sqrt{x})}{(4-\sqrt{x})(4+\sqrt{x})} \\ &= \lim_{x\to16} \frac{(x-16)(4+\sqrt{x})}{16-x} = \lim_{x\to16} -(4+\sqrt{x}) = -8\end{aligned}$$

Question 3.) (15 points) Compute the following derivatives **using only methods of Chapter 1.**

(a) $\frac{d}{dt}\left(\frac{(2t)^3}{t^5(3t^8+4t)t^{-1}}\right)$

Answer: $\frac{d}{dt}\left(\frac{(2t)^3}{t^5(3t^8+4t)t^{-1}}\right) = \frac{d}{dt}\left(\frac{8}{t(3t^8+4t)}\right) = \frac{d}{dt}\left(\frac{8}{3t^9+4t^2}\right) = \frac{d}{dt}\left(8(3t^9+4t^2)^{-1}\right) = -8(3t^9 + 4t^2)^{-2}(27t^8+8t)$

(b) $\frac{d}{dt}\left(4t^2 - \frac{1}{t}\right)^{5/6}$

Answer: $\frac{d}{dt}\left(4t^2 - \frac{1}{t}\right)^{5/6} = \frac{5}{6}\left(4t^2 - \frac{1}{t}\right)^{\frac{-1}{6}}(8t + \frac{1}{t^2})$

(c) $f'(1)$ where $f(t) = 2t(2\sqrt{t^3} - 5t^{-2/5})$

Answer: $f(t) = 2t(2\sqrt{t^3} - 5t^{-2/5}) = 4t^{\frac{5}{2}} - 10t^{\frac{3}{5}}$ *so* $f'(t) = 10t^{\frac{3}{2}} - 6t^{\frac{-2}{5}}$ *and* $f'(1) = 10 - 6 = 4$

Question 3.) (15 points) Compute the following derivatives **using only methods of Chapter 1.**

(a) $\frac{d}{dy}\left(\frac{2}{(4y)^{-2}(y^6+3y^3)}\right)$

Answer: $\frac{d}{dy}\left(\frac{2}{(4y)^{-2}(y^6+3y^3)}\right) = \frac{d}{dy}\left(\frac{32y^2}{y^3(y^3+3)}\right) = \frac{d}{dy}\left(\frac{32}{y^4+3y}\right) = \frac{d}{dy}\left(32(y^4+3y)^{-1}\right) = -32(y^4+3y)^{-2}(4y^3+3)$

(b) $\frac{d}{dy}\left(\sqrt[3]{y} - \frac{y}{3}\right)^{11/2}$

Answer: $\frac{d}{dy}\left(\sqrt[3]{y} - \frac{y}{3}\right)^{11/2} = \frac{11}{2}\left(\sqrt[3]{y} - \frac{y}{3}\right)^{9/2}\left(\frac{1}{3}y^{\frac{-2}{3}} - \frac{1}{3}\right)$

(c) $f'(1)$ where $f(y) = 2y(2y^{3/2} + \frac{6}{y^3})$

Answer: $f(y) = 2y(2y^{3/2} + \frac{6}{y^3}) = 4y^{5/2} + 12y^{-2}$ *so* $f'(y) = 10y^{\frac{3}{2}} - 24y^{-3}$ *and* $f'(1) = 10(1) - 24(1) = -14$

Question 3.) (15 points) Compute the following derivatives **using only methods of Chapter 1.**

(a) $\frac{d}{dx}\left(\frac{\left(\frac{1}{x}\right)(2x)^2}{x^2(x^4+x)}\right)$

Answer: $\frac{d}{dx}\left(\frac{\left(\frac{1}{x}\right)(2x)^2}{x^2(x^4+x)}\right) = \frac{d}{dx}\left(\frac{4x}{x^2(x^4+x)}\right) = \frac{d}{dx}\left(\frac{4}{x^5+x^2}\right) = \frac{d}{dx}(4(x^5+x^2)^{-1}) = -4(x^5 + x^2)^{-2}(5x^4+2x)$

(b) $\frac{d}{dx}\left((x+2\sqrt{x^3})^{-1/2}\right)$

Answer: $\frac{d}{dx}\left((x+2\sqrt{x^3})^{-1/2}\right) = -\frac{1}{2}\left(x+2x^{\frac{3}{2}}\right)^{\frac{-3}{2}}\left(1+3x^{\frac{1}{2}}\right)$

(c) $f'(1)$ where $f(x) = 3\sqrt[6]{x}(x^{3/2}+3x^{-3/2})$

Answer: $f(x) = 3\sqrt[6]{x}(x^{3/2}+3x^{-3/2}) = 3x^{\frac{5}{3}} + 9x^{\frac{-4}{3}}$ *so* $f'(x) = 5x^{\frac{2}{3}} - 12x^{\frac{-7}{3}}$ *and* $f'(1) = 5(1) - 12(1) = -7$

Question 4.) (14 points)

(a) Starting with the definition of the derivative as a limit, differentiate $f(x) = \frac{1}{\sqrt{x}}$ **(using only the rules for limits from Chapter 1.)**

Answer:

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \lim_{h\to 0} \frac{\frac{\sqrt{x}}{\sqrt{x}\sqrt{x+h}} - \frac{\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} = \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \\
&= \lim_{h\to 0} \frac{x - (x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} = \lim_{h\to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \\
&= \frac{-1}{2x^{3/2}}
\end{aligned}
$$

(b) Find the best linear function $y = mx + b$ that approximates $f(x) = \frac{1}{\sqrt{x}}$ near the point $x = 1$. Use this approximation to estimate $f(1.1)$. Note: Your answer should not be $\frac{1}{\sqrt{1.1}}$.

Answer: From the first part, $f'(1) = \frac{-1}{2}$. *So the best linear approximation is* $y - 1 = \frac{-1}{2}(x-1)$ *or* $y = \frac{-1}{2}x + \frac{3}{2}$. *Letting* $x = 1.1$, *we have* $f(1.1) \approx .95$.

Question 4.) (14 points)

(a) Starting with the definition of the derivative as a limit, differentiate $g(t) = 2t^{-1/2}$ **(using only the rules for limits from Chapter 1.)**

Answer:

$$
\begin{aligned}
g'(t) &= \lim_{h\to 0} \frac{\frac{2}{\sqrt{t+h}} - \frac{2}{\sqrt{t}}}{h} = 2\lim_{h\to 0} \frac{\frac{\sqrt{t}}{\sqrt{t}\sqrt{t+h}} - \frac{\sqrt{t+h}}{\sqrt{t}\sqrt{t+h}}}{h} \\
&= 2\lim_{h\to 0} \frac{\sqrt{t} - \sqrt{t+h}}{h\sqrt{t}\sqrt{t+h}} = 2\lim_{h\to 0} \frac{\sqrt{t} - \sqrt{t+h}}{h\sqrt{t}\sqrt{t+h}} \frac{\sqrt{t} + \sqrt{t+h}}{\sqrt{t} + \sqrt{t+h}} \\
&= 2\lim_{h\to 0} \frac{t - (t+h)}{h\sqrt{t}\sqrt{t+h}(\sqrt{t} + \sqrt{t+h})} = 2\lim_{h\to 0} \frac{-1}{\sqrt{t}\sqrt{t+h}(\sqrt{t} + \sqrt{t+h})} \\
&= \frac{-1}{t^{3/2}}
\end{aligned}
$$

(b) Find the best linear function $y = mt + b$ that approximates $g(t) = 2t^{-1/2}$ near the point $t = 1$. Use this approximation to estimate $g(.99)$. Note: Your answer should not be $\frac{2}{\sqrt{.99}}$.

Answer: From the first part, $g'(1) = -1$. *So the best linear approximation is* $y - 2 = -1(t-1)$ *or* $y = -t + 3$. *Letting* $t = .99$, *we have* $g(.99) \approx 2.01$.

Question 4.) (14 points)

(a) Starting with the definition of the derivative as a limit, differentiate $y(s) = \frac{3}{s^{1/2}}$ **(using only the rules for limits from Chapter 1.)**

Answer:

$$
\begin{aligned}
y'(s) &= \lim_{h\to 0} \frac{\frac{3}{\sqrt{s+h}} - \frac{3}{\sqrt{s}}}{h} = 3\lim_{h\to 0} \frac{\frac{\sqrt{s}}{\sqrt{s}\sqrt{s+h}} - \frac{\sqrt{s+h}}{\sqrt{s}\sqrt{s+h}}}{h} \\
&= 3\lim_{h\to 0} \frac{\sqrt{s} - \sqrt{s+h}}{h\sqrt{s}\sqrt{s+h}} = 3\lim_{h\to 0} \frac{\sqrt{s} - \sqrt{s+h}}{h\sqrt{s}\sqrt{s+h}} \frac{\sqrt{s} + \sqrt{s+h}}{\sqrt{s} + \sqrt{s+h}} \\
&= 3\lim_{h\to 0} \frac{s-(s+h)}{h\sqrt{s}\sqrt{s+h}(\sqrt{s} + \sqrt{s+h})} = 3\lim_{h\to 0} \frac{-1}{\sqrt{s}\sqrt{s+h}(\sqrt{s} + \sqrt{s+h})} \\
&= \frac{-3/2}{s^{3/2}}
\end{aligned}
$$

(b) Find the best linear function $y = ms + b$ that approximates $y(s) = \frac{3}{s^{1/2}}$ near the point $s = 1$. Use this approximation to estimate $y(1.1)$. Note: Your answer should not be $\frac{3}{\sqrt{1.1}}$.

Answer: From the first part, $y'(1) = -\frac{3}{2}$. *So the best linear approximation is* $y - 3 = -\frac{3}{2}(s-1)$ *or* $y = -\frac{3}{2}s + 4.5$. *Letting* $s = 1.1$, *we have* $y(1.1) \approx 2.85$.

Question 5) (16 points) Let C_1 be the graph of the curve $y = x^2 + 1$, and let C_2 be the graph of the curve $y = -2x^2 - 2$. Find constants m and b so that the straight line $y = mx + b$ is tangent to both C_1 and C_2. Hint: the line will intersect C_1 and C_2 at two different points called (x_1, y_1) and (x_2, y_2). Try to find these points. Note that there are two different tangent lines, and so two possible correct answers; either one will do.

Answer: *We have* $y_1 = x_1^2 + 1$ *since* (x_1, y_1) *is on* C_1*, and* $m = 2x_1$ *since the line is tangent to* C_1 *at* (x_1, y_1) *and has the same slope as* C_1 *there. Similarly,* $y_2 = -2x_2^2 - 2$ *since* (x_2, y_2) *is on* C_2*, and* $m = -4x_2$*. Thus we have three expressions for the slope* m*:* $2x_1$*,* $-4x_2$*, and* $\frac{y_1 - y_2}{x_1 - x_2}$*.* $2x_1 = m = -4x_2$ *implies* $x_1 = -2x_2$*,* $y_1 = x_1^2 + 1 = 4x_2^2 + 1$*, and*

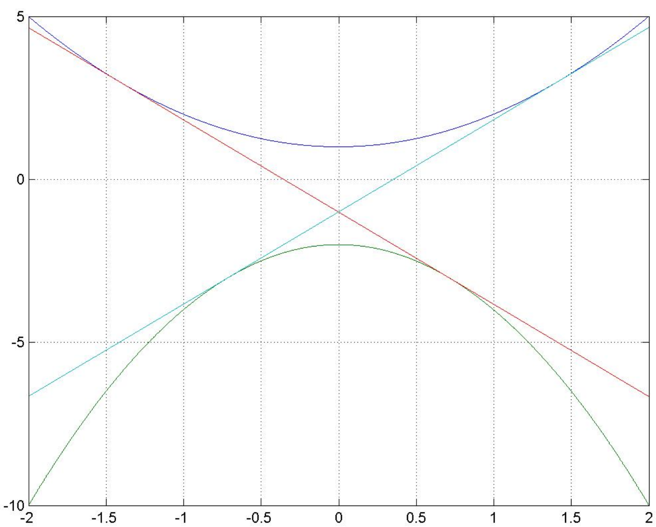
$$-4x_2 = m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{(4x_2^2 + 1) - (-2x_2^2 - 2)}{-2x_2 - x_2} = \frac{6x_2^2 + 3}{-3x_2} = \frac{-2x_2^2 - 1}{x_2}$$

Multiplying the last equation by x_2 *yields* $-4x_2^2 = -2x_2^2 - 1$ *or* $x_2^2 = \frac{1}{2}$*. There are two possible answers.*

First, we could take $x_2 = \sqrt{\frac{1}{2}}$*. Then* $x_1 = -2x_2 = -\sqrt{2}$*,* $y_2 = -2x_2^2 - 2 = -3$*, and* $y_1 = x_1^2 + 1 = 3$*. Thus* $m = 2x_1 = -2\sqrt{2}$*, and the equation of the line is* $y - y_1 = m(x - x_1)$ *or* $y = -2\sqrt{2}x - 1$*.*

Second, we could take $x_2 = -\sqrt{\frac{1}{2}}$*. Then* $x_1 = -2x_2 = \sqrt{2}$*,* $y_2 = -2x_2^2 - 2 = -3$*, and* $y_1 = x_1^2 + 1 = 3$*. Thus* $m = 2x_1 = 2\sqrt{2}$*, and the equation of the line is* $y - y_1 = m(x - x_1)$ *or* $y = 2\sqrt{2}x - 1$*.*

A plot of C_1 *and* C_2*, along with the two tangent lines, is shown below (this was not required as part of the answer, but helps explain it).*



Question 5) (16 points) Let G_1 be the graph of the curve $y = 2t^2 + 2$, and let G_2 be the graph of the curve $y = -t^2 - 1$. Find constants m and b so that the straight line $y = mt + b$ is tangent to both G_1 and G_2. Hint: the line will intersect G_1 and G_2 at two different points called (t_1, y_1) and (t_2, y_2). Try to find these points. Note that there are two different tangent lines, and so two possible correct answers; either one will do.

Answer: *We have* $y_1 = 2t_1^2 + 2$ *since* (t_1, y_1) *is on* G_1*, and* $m = 4t_1$ *since the line is tangent to* G_1 *at* (t_1, y_1) *and has the same slope as* G_1 *there. Similarly,* $y_2 = -t_2^2 - 1$ *since* (t_2, y_2) *is on* G_2*, and* $m = -2t_2$*. Thus we have three expressions for the slope* m*:* $4t_1$*,* $-2t_2$*, and* $\frac{y_1 - y_2}{t_1 - t_2}$*.* $4t_1 = m = -2t_2$ *implies* $t_2 = -2t_1$*,* $y_2 = -t_2^2 - 1 = -4t_1^2 - 1$*, and*

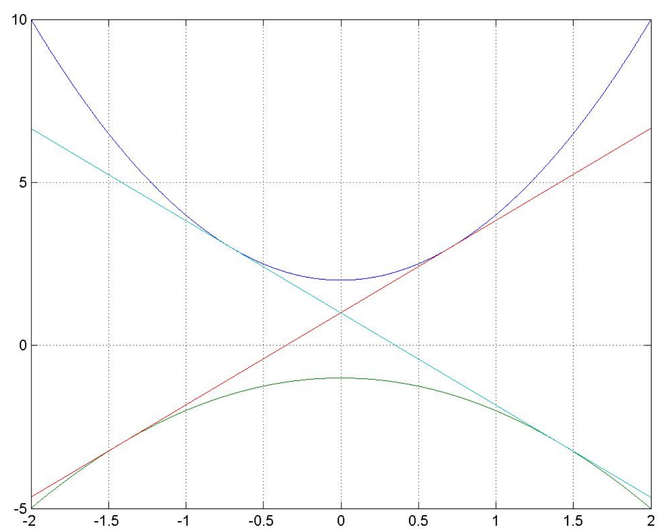
$$4t_1 = m = \frac{y_1 - y_2}{t_1 - t_2} = \frac{(2t_1^2 + 2) - (-4t_1^2 - 1)}{t_1 - (-2t_1)} = \frac{6t_1^2 + 3}{3t_1} = \frac{2t_1^2 + 1}{t_1}$$

Multiplying the last equation by t_1 *yields* $4t_1^2 = 2t_1^2 + 1$ *or* $t_1^2 = \frac{1}{2}$*. There are two possible answers.*

First, we could take $t_1 = \sqrt{\frac{1}{2}}$*. Then* $t_2 = -2t_1 = -\sqrt{2}$*,* $y_1 = 2t_1^2 + 2 = 3$*, and* $y_2 = -t_2^2 - 1 = -3$*. Thus* $m = 4t_1 = 2\sqrt{2}$*, and the equation of the line is* $y - y_1 = m(t - t_1)$ *or* $y = 2\sqrt{2}t + 1$*.*

Second, we could take $t_1 = -\sqrt{\frac{1}{2}}$*. Then* $t_2 = -2t_1 = \sqrt{2}$*,* $y_1 = 2t_1^2 + 2 = 3$*, and* $y_2 = -t_2^2 - 1 = -3$*. Thus* $m = 4t_1 = -2\sqrt{2}$*, and the equation of the line is* $y - y_1 = m(t - t_1)$ *or* $y = -2\sqrt{2}t + 1$*.*

A plot of G_1 *and* G_2*, along with the two tangent lines, is shown below (this was not required as part of the answer, but helps explain it).*



Question 5) (16 points) Let C_1 be the graph of the curve $z = -y^2 - 4$, and let C_2 be the graph of the curve $z = 2y^2 - 1$. Find constants m and b so that the straight line $z = my + b$ is tangent to both C_1 and C_2. Hint: the line will intersect C_1 and C_2 at two different points called (y_1, z_1) and (y_2, z_2). Try to find these points. Note that there are two different tangent lines, and so two possible correct answers; either one will do.

Answer: We have $z_1 = -y_1^2 - 4$ since (y_1, z_1) is on C_1, and $m = -2y_1$ since the line is tangent to C_1 at (y_1, z_1) and has the same slope as C_1 there. Similarly, $z_2 = 2y_2^2 - 1$ since (y_2, z_2) is on C_2, and $m = 4y_2$. Thus we have three expressions for the slope m: $-2y_1$, $4y_2$, and $\frac{z_1-z_2}{y_1-y_2}$. $-2y_1 = m = 4y_2$ implies $y_1 = -2y_2$, $z_1 = -y_1^2 - 4 = -4y_2^2 - 4$, and

$$4y_2 = m = \frac{z_1 - z_2}{y_1 - y_2} = \frac{(-4y_2^2 - 4) - (2y_2^2 - 1)}{(-2y_2) - y_2} = \frac{-6y_2^2 - 3}{-3y_2} = \frac{2y_2^2 + 1}{y_2}$$

Multiplying the last equation by y_2 yields $4y_2^2 = 2y_2^2 + 1$ or $y_2^2 = \frac{1}{2}$. There are two possible answers.

First, we could take $y_2 = \sqrt{\frac{1}{2}}$. Then $y_1 = -2y_2 = -\sqrt{2}$, $z_1 = -y_1^2 - 4 = -6$, and $z_2 = 2y_2^2 - 1 = 0$. Thus $m = 4y_2 = 2\sqrt{2}$, and the equation of the line is $z - z_2 = m(y - y_2)$ or $z = 2\sqrt{2}y - 2$.

Second, we could take $y_2 = -\sqrt{\frac{1}{2}}$. Then $y_1 = -2y_2 = \sqrt{2}$, $z_1 = -y_1^2 - 4 = -6$, and $z_2 = 2y_2^2 - 1 = 0$. Thus $m = 4y_2 = -2\sqrt{2}$, and the equation of the line is $z - z_2 = m(y - y_2)$ or $z = -2\sqrt{2}y - 2$.

A plot of C_1 and C_2, along with the two tangent lines, is shown below (this was not required as part of the answer, but helps explain it).