Physics 7B, Speliotopoulos
Final Exam, Spring 2014
Berkeley, CA

Rules: *This final exam is closed book and closed notes. In particular, calculators are not allowed during this exam. Cell phones must be turned off during the exam, and placed in your backpacks, or bags.* ***They cannot be on your person.***

Please make sure that you do the following during the midterm:

- ***Show all your work in your blue book***
- *Write your name, discussion number, ID number on all documents you hand in.*
- *Make sure that the grader knows what s/he should grade by circling your final answer.*
- *Cross out any parts of your solutions that you do not want the grader to grade.*

Each problem is worth 20 points. We will give partial credit on this final*, so if you are not altogether sure how to do a problem, or if you do not have time to complete a problem, be sure to write down as much information as you can on the problem. This includes any or all of the following: Drawing a clear diagram of the problem, telling us how you would do the problem if you had the time, telling us why you believe (in terms of physics) the answer you got to a problem is incorrect, and telling us how you would mathematically solve an equation or set of equations once the physics is given and the equations have been derived. Don't get too bogged down in the mathematics; we are looking to see how much physics you know, not how well you can solve math problems.*

If at any point in the exam you have any questions, just raise your hand, and we will see if we are able to answer them.

Copy and fill in the following information on the front of your bluebook:

Name: ______________________________ *Disc Sec Number:* _______________________

Signature: ___________________________ *Disc Sec GSI:* __________________________

Student ID Number: ____________________

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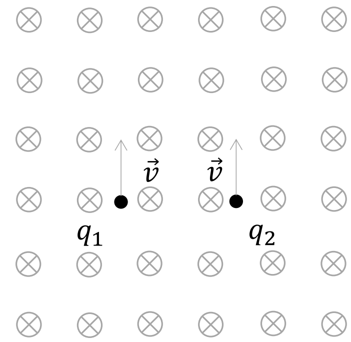
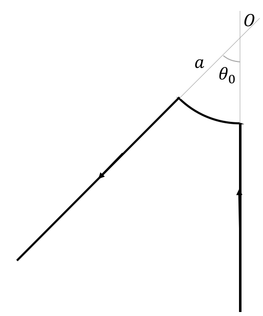
1. Two particles move through a constant magnetic field, B, pointed into the page (see figure). The particle on the left has charge, $q_1 > 0$, and the one on the right has charge, $q_2 < 0$. They both have a *constant velocity*, $\vec{v}$, perpendicular to B. What is the separation, d, between the two charges? All the forces on the charges are either electric or magnetic in nature. (Because the charges move much slower than the speed of light, you can neglect the magnetic fields generated by the moving charges.)

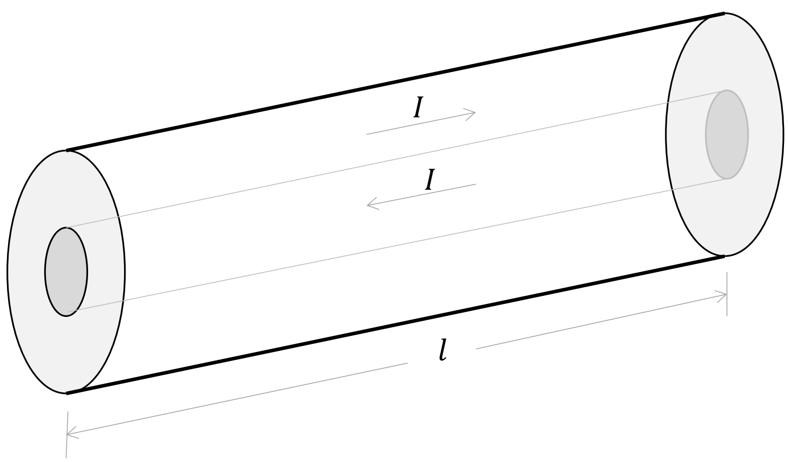
   (If you are bored *over the summer* (not now!), you can ask whether or not someone moving with the charges would conclude that their separation can be constant, and if not, how the electric and magnetic fields must change between moving and stationary reference frames. And that will lead you to special relativity.)



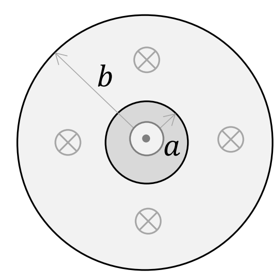
2. The figure to the right shows a section of a wire with current, I, running through it. The arc has an opening angle, θ_0, and radius, a. What is the magnetic field (direction and magnitude), B, at the point, O, shown?



3. A wire with length, l, is made out of two concentric cylinders, one with radius, a, and the other with radius, b. (The two cylinders are separated by an insulator that has negligible thickness.) There is a constant current, I, flowing out of the page though the inner cylinder, while the same current, I, flows into the page between the inner and outer cylinders (see figure).
   a. Using the energy density of a magnetic field, what is the energy, U, stored in the wire? Take $l \gg b$, so that you can neglect end effects.
   b. Then what is the inductance per length, L/l, of the wire?



Perspective View



End View

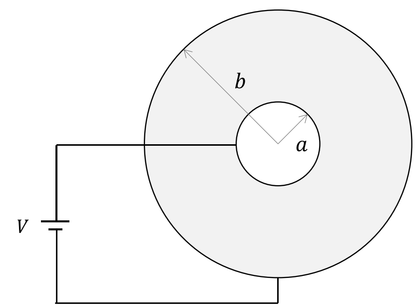
4. A resistor is made from two concentric *spheres* with radii a and b (see figure) between which there is a material with resistivity that varies with radius as,

$$\rho(r) = \rho_0 \left(\frac{r}{a}\right)^s .$$

Current is injected into the resistor from the inner sphere, and removed from the outer cylinder.

   a. What must the exponent, s, be if the electric field between the sphere is constant?

   b. What is the current, I, in the circuit? (If you cannot get part a, take $s = 3$ for this part.)



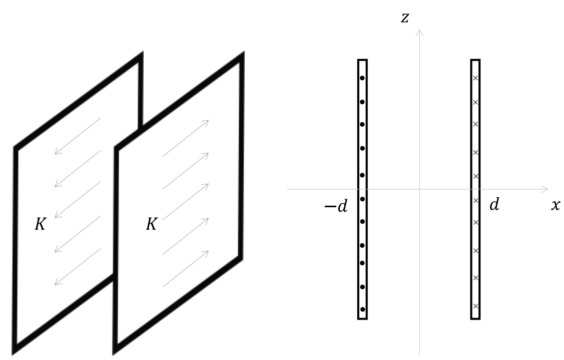
5. The figure to the right shows two infinite sheets of current, each with surface charge density, K, (in amperes per meter), but flowing in opposite directions. The two sheets are separated by a distance, $2d$. The magnetic field $\vec{B}(x)$ is zero for $|x| \geq d$

   a. What is the magnetic field $\vec{B}(x)$ for $|x| < d$?

   b. If the surface current densities increase slowly with time as

$$K(t) = K_0 \left(\frac{t}{\tau}\right),$$

   what is the electric field, $\vec{E}$, everywhere?



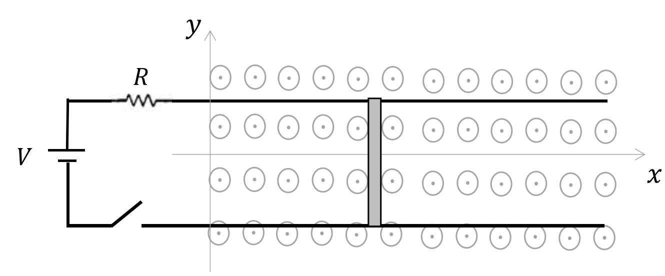
Perspective View

Side View

6. The rail guns we looked at on the homework are limited by inductive effects. To include these effects, we consider the two long rails in a constant magnetic field, B, (see figure on right). A rod with mass, m, and length, l, lies on top of the rails. The rod is initially at rest, and at $t = 0$ the switch is closed. The rails and the rod have negligible resistance.

   a. What is the velocity of the rod, $v(t)$, after the switch is closed? (Hint: You will have to use Kirchoff's loop rule *and* Newton's 2nd.)

   b. Eventually the velocity of the rod approaches a constant, v_T. What is v_T? Explain in physical terms the conditions under which this happens. (You can do this part of the problem without first doing part a.)

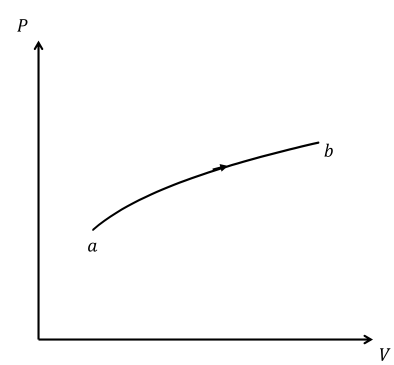


7. While for an adiabatic processes, $dQ = 0$, the figure on the right shows a gas of n moles of monatomic molecules that undergoes instead a process for which

$$dQ = \frac{1}{2} dE.$$

   a. If the temperature at the beginning and end points of the process is T_a and T_b, respectively, what is the change in entropy, ΔS_{ab} for the process? Express it in terms of n, R, T_A and T_B.

   b. For each point along the PV diagram for the process, $PV^\beta = \text{Constant}$. Using the first law of thermodynamics and the equation of state for the gas, determine β.



$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$PV = NkT = nRT$$

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

$$f_{Maxwell}(v) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

$$E = \frac{D}{2}NRT$$

$$Q = mc\Delta T = nC\Delta T$$

$$Q = mL \text{ (For a phase transition)}$$

$$dE = PdV + dQ$$

$$W = \int PdV$$

$$C_P - C_V = R = N_A k$$

$$PV^\gamma = \text{const. (For an adiabatic process)}$$

$$\gamma = \frac{C_P}{C_V} = \frac{d+2}{d}$$

$$C_V = \frac{d}{2}R$$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$$e = \frac{W}{Q_h}$$

$$e_{ideal} = 1 - \frac{T_L}{T_H}$$

$$dQ = TdS$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$\vec{E} = \int \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r} \text{ (For infinite wire)}$$

$$\rho = \frac{dQ}{dV}$$

$$\sigma = \frac{dQ}{dA}$$

$$\lambda = \frac{dQ}{dl}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Delta U = Q\Delta V$$

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$V = \int \frac{dQ}{4\pi\epsilon_0 r}$$

$$\vec{E} = -\vec{\nabla} V$$

$$Q = CV$$

$$C_{eq} = C_1 + C_2 \text{ (In parallel)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (In series)}$$

$$\epsilon = \kappa\epsilon_0$$

$$U = \frac{Q^2}{2C}$$

$$U = \int \frac{\epsilon_0}{2}|\vec{E}|^2 dV$$

$$I = \frac{dQ}{dt}$$

$$\Delta V = IR$$

$$R = \rho\frac{l}{A}$$

$$\rho(T) = \rho(T_0)(1 + \alpha(T - T_0))$$

$$P = IV$$

$$I = \int \vec{j} \cdot d\vec{A}$$

$$\vec{j} = nQ\vec{v_d} = \frac{\vec{E}}{\rho}$$

$$R_{eq} = R_1 + R_2 \text{ (In series)}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ (In parallel)}$$

$$\sum_{\text{junc.}} I = 0 \text{ (junction rule)}$$

$$\sum_{\text{loop}} V = 0 \text{ (loop rule)}$$

$$d\vec{F}_m = Id\vec{l} \times \vec{B}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{\mu} = NI\vec{A}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \cdot \vec{B}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{l} \times \hat{r}}{r^2}$$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi r} \text{ (For infinite wire)}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$M = N_1 \frac{\Phi_1}{I_2} = N_2 \frac{\Phi_2}{I_1}$$

$$L = N\frac{\Phi_B}{I}$$

$$U = \frac{1}{2} L I^2$$

$$U = \int \frac{1}{2\mu_0} |\vec{B}|^2 dV$$

$$\overline{g(v)} = \int_0^\infty g(v) \frac{f(v)}{N} dv$$

($f(v)$ a speed distribution)

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z} \\ d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z} \end{aligned}$$

(Cylindrical Coordinates)

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi} \\ d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi} \end{aligned}$$

(Spherical Coordinates)

$$y(t) = \frac{B}{A}(1 - e^{-At}) + y(0)e^{-At}$$

$$\text{solves } \frac{dy}{dt} = -Ay + B$$

$$y(t) = y_{max}\cos(\sqrt{A}t + \delta)$$

$$\text{solves } \frac{d^2y}{dt^2} = -Ay$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1 + x^2)^{-1/2} dx = \ln(x + \sqrt{1 + x^2})$$

$$\int (1 + x^2)^{-1} dx = \arctan(x)$$

$$\int (1 + x^2)^{-3/2} dx = \frac{x}{\sqrt{1 + x^2}}$$

$$\int \frac{x}{1 + x^2} dx = \frac{1}{2}\ln(1 + x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\int \sin(x) dx = -\cos(x)$$

$$\int \cos(x) dx = \sin(x)$$

$$\int \frac{dx}{x} = \ln(x)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1 + x)^\alpha \approx 1 + \alpha x + \frac{(\alpha - 1)\alpha}{2} x^2$$

$$\ln(1 + x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$