

Name: *I. M. Solutions*

Student ID #: *123-45-6789*

**University of California at Berkeley**  
**Electrical Engineering and Computer Science**  
**EE105 Midterm Examination #1**  
**Feb. 25, 2016**  
**(80 minutes)**

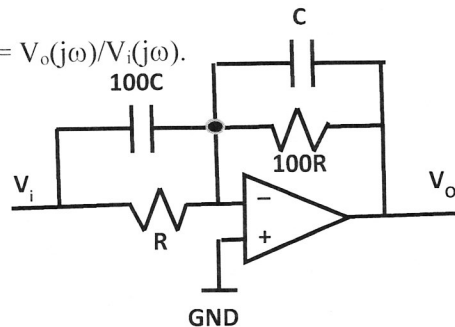
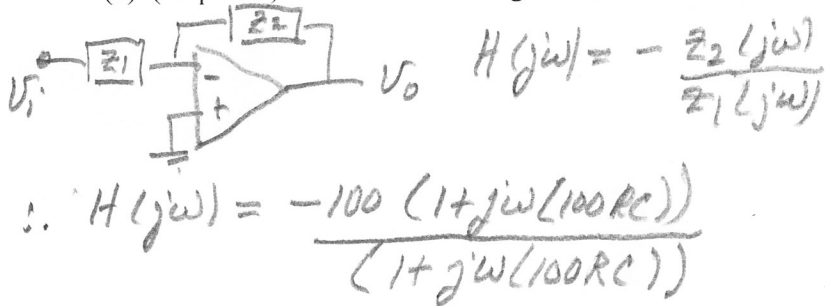
**CLOSED BOOK;** One standard 8.5" x 11" sheet of notes (both sides) permitted

- Read each problem completely and thoroughly before beginning to work on it
- Summarize all your answers in the boxes provided on these exam sheets
- Show your work in the space provided so we can check your work and scan for partial credit
- Remember to put your name in the space above

Problem #	Points Possible	Score
1	30	<i>30</i>
2	35	<i>35</i>
3	35	<i>35</i>
<b>Total</b>	100	<del><i>35</i></del> <i>100</i>

1. (30 points) Frequency response. Assume an ideal opamp.

(a) (10 points) Derive the small-signal transfer function,  $H(j\omega) = V_o(j\omega)/V_i(j\omega)$ .



$$Z_1(j\omega) = \frac{R}{1+j\omega(100RC)}$$

$$Z_2(j\omega) = \frac{100R}{1+j\omega(100RC)}$$

$$H(j\omega) = -100 \left[ \frac{1+j\omega(100RC)}{1+j\omega(100RC)} \right] = -100 \text{ (pole-zero cancel)}$$

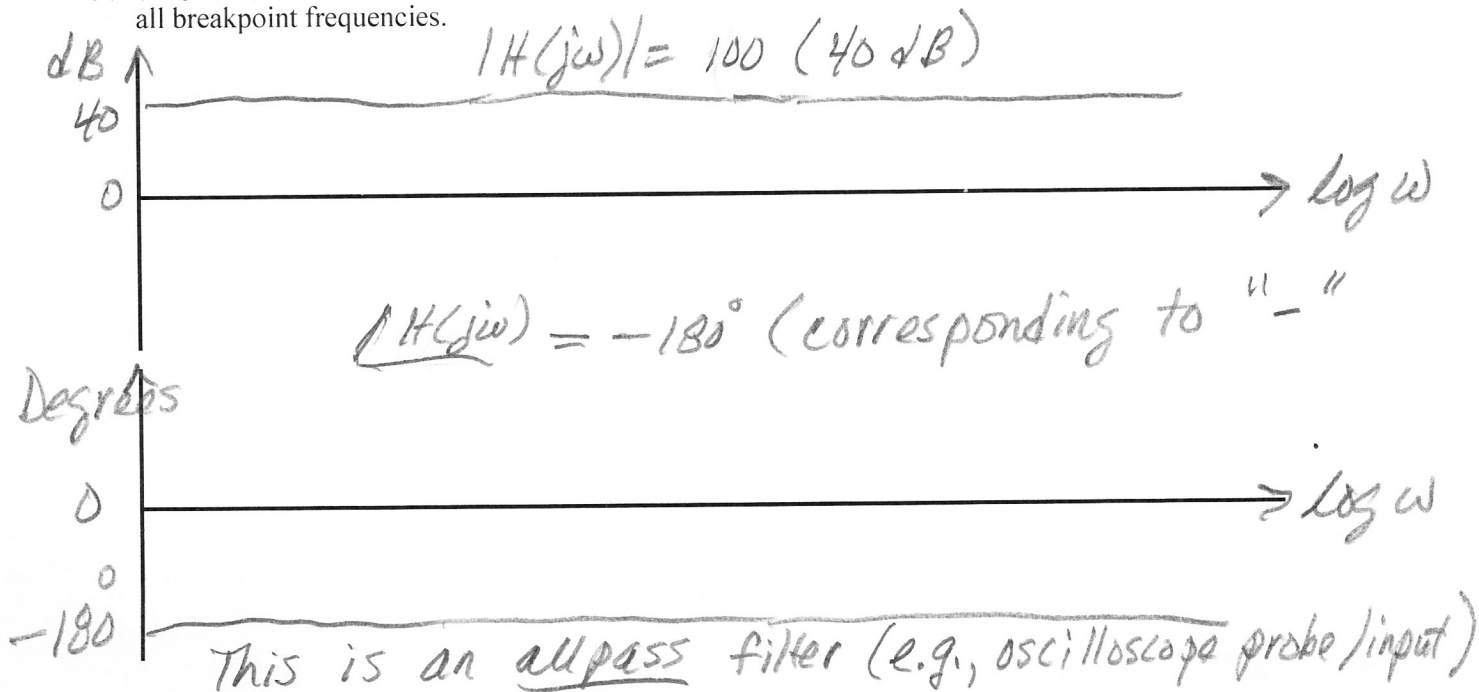
(b) (10 points) Determine the pole(s) and zero(s) frequencies. (Note: Be careful with  $\omega$  versus  $f$ )

Zero freq:  $\omega_z(100RC) = 1 \Rightarrow \omega_z = \frac{1}{100RC} \text{ rad/sec}$

Pole freq:  $\omega_p(100RC) = 1 \Rightarrow \omega_p = \frac{1}{100RC} \text{ rad/sec}$

Pole-zero cancellation  $\Rightarrow H(j\omega) = -100 \text{ (40 dB)}$

(c) (10 points) Sketch the Bode magnitude and phase plots (label all constant gain and phase levels and all breakpoint frequencies).



2. (35 points). CMOS bias calculations. For the NMOS,  $V_{tn} = 0.7 \text{ V}$ ,  $k_n = 1.5 \text{ mA/V}^2$ , and  $\lambda_n = 0$ . For the PMOS,  $V_{tp} = -0.7 \text{ V}$ ,  $k_p = 0.5 \text{ mA/V}^2$ , and  $\lambda_p = 0$ .

Find the values of  $I$ ,  $V_1$ ,  $V_2$ , and  $V_3$ .

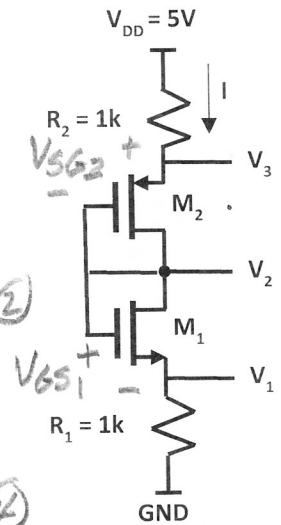
Overall, 4 equations in 4 unknowns:

$$I = \frac{V_1}{R_1} = \frac{V_1}{1k} \dots \textcircled{1}; \quad I = \frac{k_n}{2} (V_{GS1} - V_{tn})^2$$

$$I = \frac{k_p}{2} (V_{SG2} - |V_{tp}|)^2 = \frac{k_n}{2} (V_2 - V_1 - 0.7)^2 \dots \textcircled{2}$$

$$= \frac{k_p}{2} (V_3 - V_2 - 0.7)^2 \dots \textcircled{3}; \quad I = \frac{V_{DD} - V_3}{R_2}$$

$$\text{Now, consider } \textcircled{2} \text{ and } \textcircled{3}: \quad I = \frac{V_{DD} - V_3}{1k} \dots \textcircled{4}$$



$$I = \frac{k_n}{2} (V_{GS1} - 0.7)^2 = \frac{k_p}{2} (V_{SG2} - 0.7)^2 \Rightarrow V_{SG2} = 1.73 V_{GS1} - 0.51 \dots \textcircled{5}$$

$$\text{KVL} \Rightarrow IR_1 + V_{GS1} + V_{SG2} + IR_2 = V_{DD}$$

$$\text{Thus, } I = \frac{V_{DD} - V_{GS1} - V_{SG2}}{2k} = 10^{-3} (2.76 - 1.37 V_{GS1}) \dots \textcircled{6}$$

Equate  $\textcircled{6}$  to  $\textcircled{2}$ :

$$10^{-3} (2.76 - 1.37 V_{GS1}) = 0.75 \times 10^{-3} (V_{GS1} - 0.7)^2$$

$$1.33 (2.76 - 1.37 V_{GS1}) = V_{GS1}^2 - 1.4 V_{GS1} + 0.49$$

$$\Rightarrow V_{GS1}^2 + 0.42 V_{GS1} - 3.18 = 0$$

$$\Rightarrow V_{GS1} = \frac{-0.42 \pm 3.59}{2} = \boxed{1.59 \text{ V}} \text{ or } -2.01 \text{ V}$$

$$V_{SG2} = 1.73 V_{GS1} - 0.51 = \boxed{2.24 \text{ V}}$$

$$I = \frac{V_{DD} - V_{GS1} - V_{SG2}}{2k} = \boxed{0.59 \text{ mA}}$$

$$\boxed{V_1 = IR_1 = 0.59 \text{ V}; \quad V_3 = V_{DD} - IR_2 = 4.41 \text{ V};}$$

$$\boxed{V_2 = V_3 - V_{SG2} = 2.17 \text{ V}}$$

$$\boxed{I = 0.59 \text{ mA}}$$

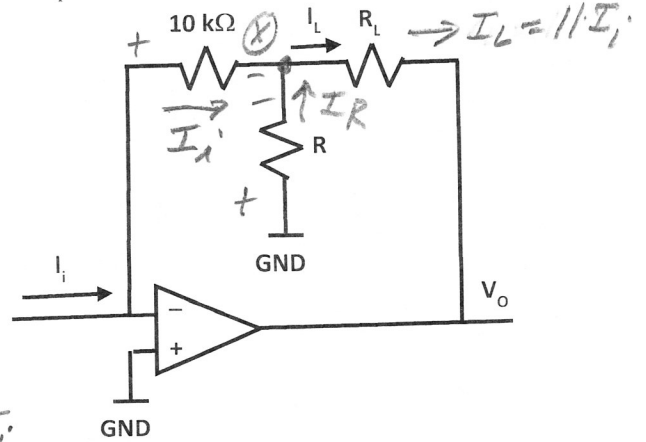
$$\boxed{V_1 = 0.59 \text{ V}}$$

$$\boxed{V_2 = 2.17 \text{ V}}$$

$$\boxed{V_3 = 4.41 \text{ V}}$$

3. (35 points) Assuming an ideal opamp, design the current amplifier shown below to have a current gain of  $I_L/I_i = 11$  A/A.

(a) Find the required value for  $R$ .



Note:  $\frac{I_L}{I_i} = 11 \Rightarrow I_L = 11 I_i$

KCL @  $\textcircled{x}$ :  $I_i + I_R = I_L = 11 I_i$

$\therefore I_R = 10 I_i = \frac{0 - V_x}{R}$

But  $V_x = 0 - I_i (10k) = -(10k) I_i$

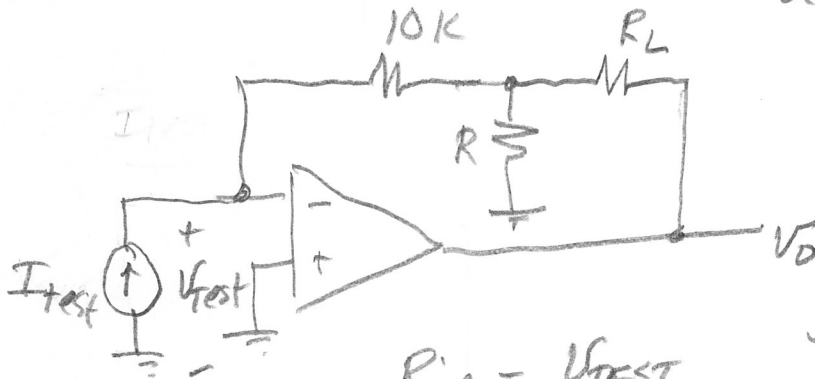
$\therefore 10 R I_i = -(-10k \cdot I_i) \Rightarrow 10 R = 10k$

$\therefore R = \frac{10k}{10} = 1k$

$R =$	$1k\Omega$
-------	------------

(b) What is the input resistance of this current amplifier,  $R_{in}$ ?

Use test source to find  $R_{in}$ :



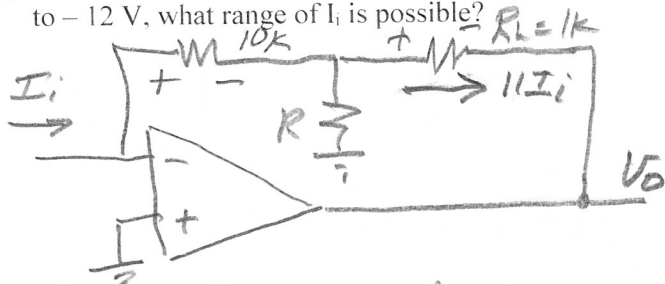
$$R_{in} = \frac{V_{TEST}}{I_{TEST}}$$

$$= \frac{0}{I_{TEST}} = 0$$

IDEAL OPAMP WITH Negative Feedback MEANS  $V_{in}^- = V_{in}^+ = 0$   
 $\therefore V_{TEST} = 0$

$R_{in} =$	$0\Omega$
------------	-----------

(c) If  $R_L = 1 \text{ k}\Omega$  and the opamp operates in an ideal manner as long as  $V_o$  is in the range  $+12 \text{ V}$  to  $-12 \text{ V}$ , what range of  $I_i$  is possible?



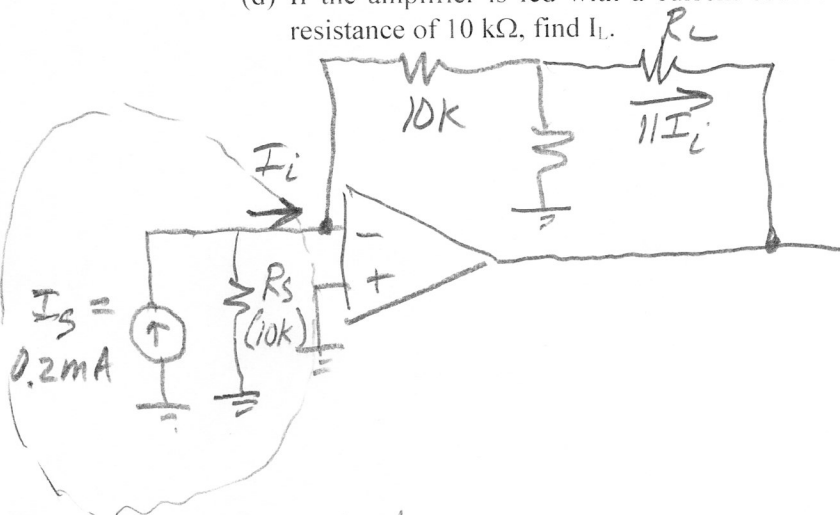
$$\begin{aligned} V_o &= 0 - I_i (10\text{k}) - 11I_i (1\text{k}) \\ &= -10\text{k} I_i - 11\text{k} I_i \\ &= -21\text{k} I_i \end{aligned}$$

$$I_i (\text{max}) = \frac{V_o (\text{max})}{21\text{k}} = \frac{-12\text{V}}{21\text{k}} = -0.57\text{mA}$$

$$I_i (\text{min}) = \frac{-V_o (\text{min})}{21\text{k}} = \frac{-(-12\text{V})}{21\text{k}} = +0.57\text{mA}$$

$I_i$ range	$-0.57\text{mA} \leq I_i \leq 0.57\text{mA}$
-------------	--

(d) If the amplifier is fed with a current source having a current of  $0.2 \text{ mA}$  and a source resistance of  $10 \text{ k}\Omega$ , find  $I_L$ .



Norton equivalent  
input current  
source

Because of ideal  
opamp with  
negative feedback,  
 $V_{in}^+ = V_{in}^- = 0 \text{ V}$   
∴ Current through  
 $R_s$ ,  $I_{R_s} = 0$   
∴  $I_i = I_s = 0.2\text{mA}$   
∴  $I_L = 11 I_i = 2.20\text{mA}$

$I_L =$	$2.20 \text{ mA}$
---------	-------------------