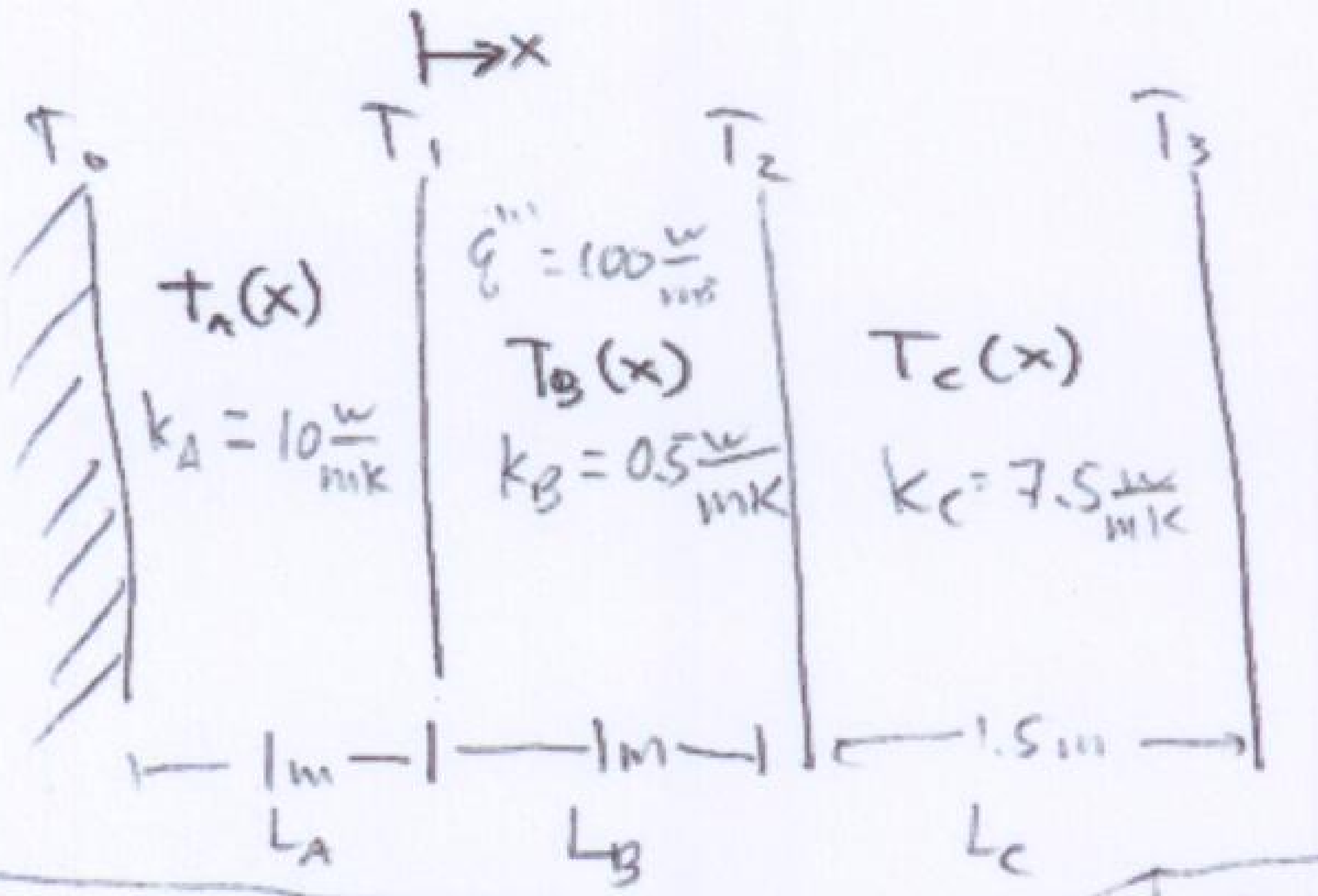
MIDTERM 1 Pr. 1

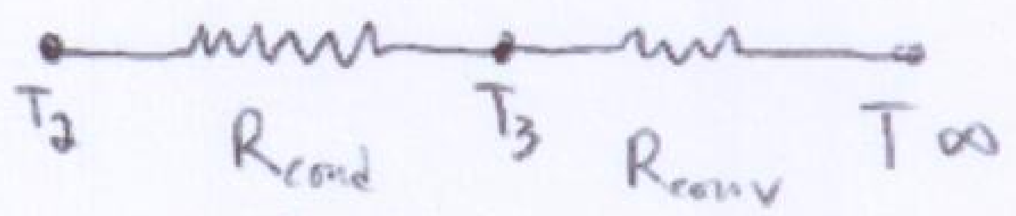


$h = 5 \frac{W}{m^2 K}$

$T_\infty = 10°C$

Finding T_2

$\overrightarrow{q''' A L_B}$



$\frac{L_C}{K_C A}$ $\quad$ $\frac{1}{hA}$

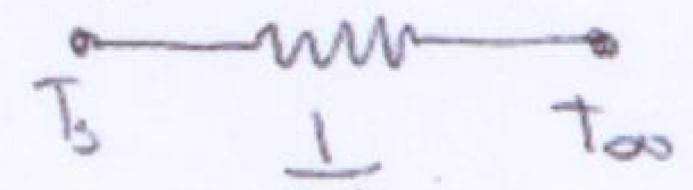
$$q''' A L_B = \frac{T_2 - T_\infty}{\frac{1}{A}\left(\frac{L_C}{k_C} + \frac{1}{h}\right)}$$

$$q''' L_B\left(\frac{L_C}{k_C} + \frac{1}{h}\right) + T_\infty = T_2$$

$$(100)(1)\left(\frac{1.5}{7.5} + \frac{1}{5}\right) + 10°C = 50°C = T_2$$

Finding T_3

$\overrightarrow{q''' A L_B}$



$\frac{1}{hA}$

$$q''' A L_B = \frac{T_3 - T_\infty}{\frac{1}{hA}}$$

$$\frac{q''' L_B}{h} + T_\infty = T_3$$

$$\frac{100(1)}{5} + 10°C = 30°C = T_3$$

$T_C(x) = C_1 x + C_2$

BC $T_C(L_B + L_C) = 30°C = C_1(2.5) + C_2$

BC $T_C(L_B) = 50°C = C_1(1) + C_2$

$-20°C = 1.5 C_1$

$-13.333 = C_1$

$30°C = (-13.333)(2.5) + C_2$

$63.333 = C_2$

$$\boxed{T_C = -13.333 \tfrac{k}{m} x + 63.333°C}$$

$$T_B(x) = -\frac{\dot{q}}{2k_B} x^2 + C_3$$

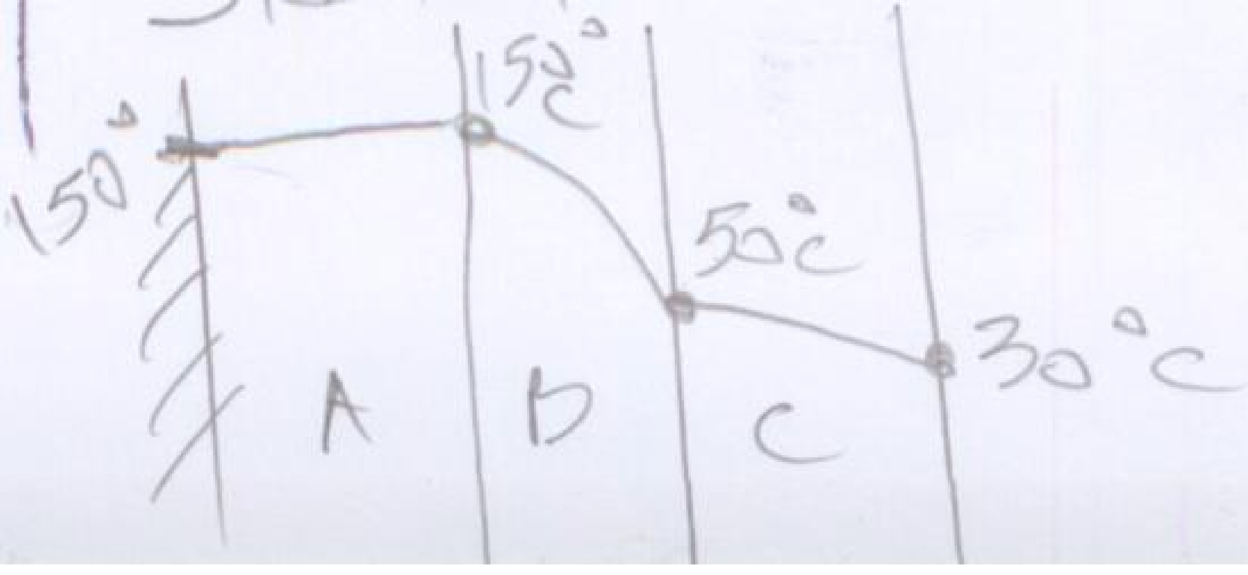
BC $T_B(L_B) = 50°C = -\frac{\dot{q}}{2k_B} L_B^2 + C_3$

$$50°C + \frac{\dot{q}}{2k_B} L_B^2 = 50 + \frac{100}{2(0.5)}(1^2) = 150°C = C_3$$

$$\boxed{T_B = -100 \tfrac{k}{m^2} x^2 + 150°C}$$

$T_A = T_1 = T_B(0) = 150°C$

SKETCH



MIDTERM 1
#2

2

a) $-hA_s(T-T_\infty) = \rho V c \frac{dT}{dt}$ ← EITHER

5

$q'' A_{s,h} + \dot{E}_g - [h(T-T_\infty) + \varepsilon\sigma(T^4 - T_{SUR}^4)] A_{s(c,r)} = \rho V c \frac{dT}{dt}$

JUSTIFICATION

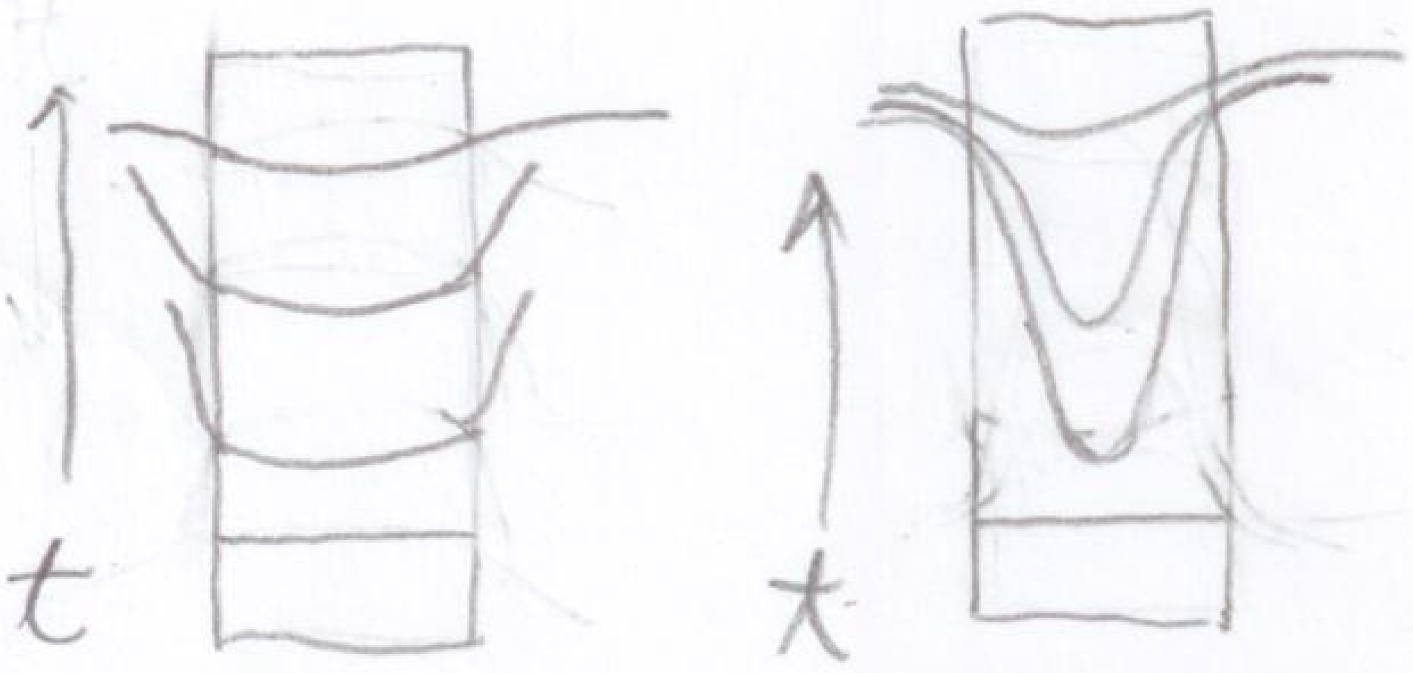
$Bi = \frac{hL}{k_s} = \frac{(400\ W/m^2K)(.0007m)}{10\ W/mK} = .028 \ll 0.1$

5

b) $\ln \frac{\theta_i}{\theta} = \frac{hA_s t}{\rho V c} = \frac{ht}{\rho c L_c} = \frac{hL_c}{k} \frac{k}{\rho c} \frac{t}{L_c^2} =$

$\frac{hL_c}{k} \frac{\alpha t}{L_c^2} = Bi \cdot Fo$

c)



Bi << 1   Bi >> 1

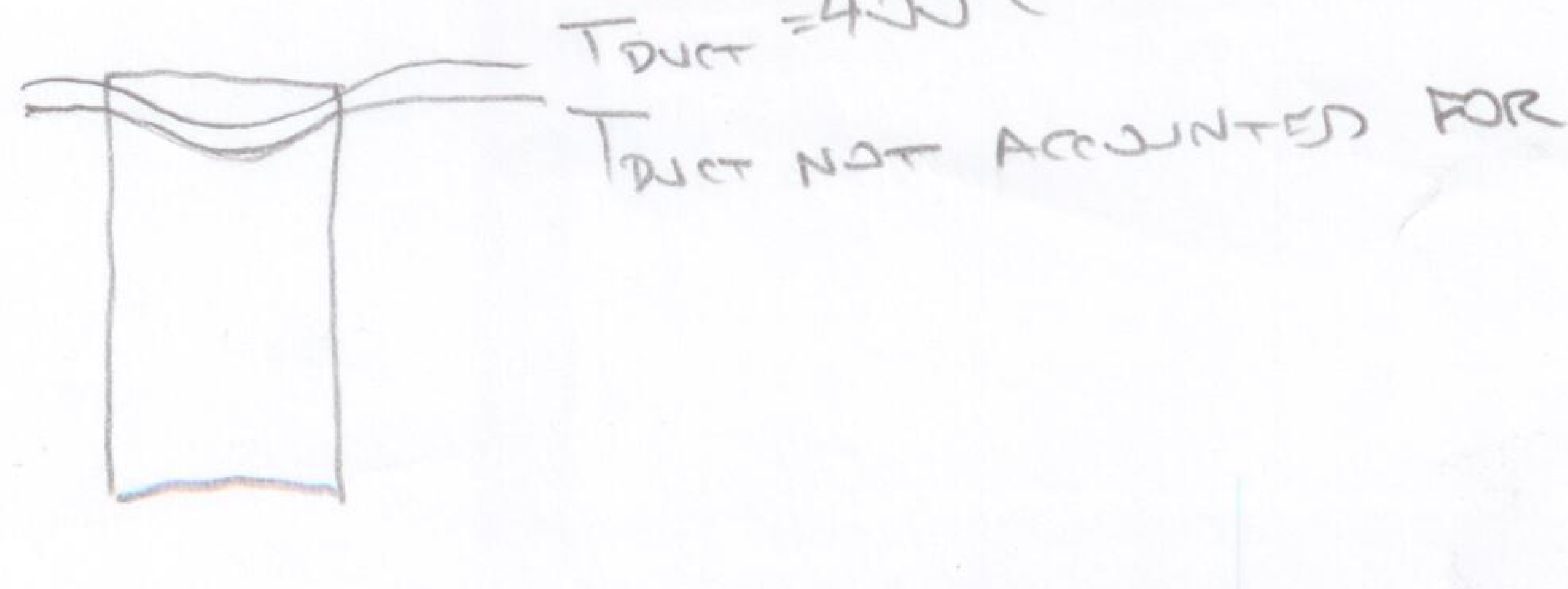
MGM SAVE 5%

5

5

d) $[h(T-T_\infty) + \varepsilon\sigma(T^4 - T_{SUR}^4)] A_s = \rho V c \frac{dT}{dt}$

e)



9) NON-DIMENSIONALIZING ENERGY EQN

1

$$\Theta = T - T_\infty / T_i - T_\infty$$

$$x^* = x/L$$

$$\frac{d^2T}{dx^2} = \frac{1}{\alpha}\frac{dT}{dt}$$

MIDTERM 1
3

$$\frac{\partial \Theta}{\partial x^*} = \frac{\partial \Theta}{\partial\left(\frac{x}{L}\right)} = \frac{L\,dT}{dx\,(T_i - T_\infty)}$$

$$\frac{\partial^2 \Theta}{\partial x^{*2}} = \frac{L^2\,d^2T}{dx^2(T_i - T_\infty)}$$

$$\frac{\cancel{(T_i - T_\infty)}}{L^2}\frac{\partial^2\Theta}{\partial x^{*2}} = \frac{1}{\alpha}\cancel{(T_i - T_\infty)}\frac{\partial\Theta}{\partial t}$$

$$\frac{\partial \Theta}{\partial t} = \frac{\partial T}{T_i - T_\infty\,dt}$$

DON'T KNOW YET

$$\frac{L^2}{\alpha\,\partial t} = \frac{1}{\partial t^*}$$

$$dt^* = \frac{\alpha dt}{L^2} \longrightarrow t^* = \frac{\alpha t}{L^2}$$

(Fo)

$$\frac{\partial^2\Theta}{\partial x^{*2}} = \frac{\partial\Theta}{\partial Fo}$$

* THIS IS WHAT YOU ARE FINDING YOU SHOULDN'T BE SETTING IT AS A NON-DIMENSIONAL VARIABLE TO START THE PROBLEM

b) $T(x,0)=T_i \longrightarrow \Theta^*(x^*,0)=1$ ②

$$\left.\frac{\partial T}{\partial x}\right|_{x=0}=0 \qquad \left.\frac{\partial \Theta^*}{\partial x^*}\right|_{x^*=0}=0$$

$$k\frac{\partial \Theta}{\partial x^*}=k\frac{\partial \Theta}{\partial\left(\frac{x}{L}\right)}=\frac{L\,dT}{dx(T_i-T_\infty)}$$

$$\left.\frac{\partial \Theta^*}{\partial x^*}\right|_{x^*=1}=-Bi\,\Theta^*(1,t^*)$$

$$\frac{dT}{dx}=\frac{k(T_i-T_\infty)}{L}\frac{\partial\Theta}{\partial x^*}=h\Theta$$

c) Bi ⇒ RATIO OF CONVECTION TO CONDUCTION IN THE SOLID. USED TO DETERMINE VALIDITY OF LUMPED THERMAL CAPACITANCE METHOD. SHOWS WHETHER SOLID CAN BE ASSUMED AT UNIFORM TEMP.

$\frac{hL}{k_{SOLID}}$

Fo ⇒ DIMENSIONLESS TIME

$\frac{\alpha t}{L^2} \rightarrow \frac{kt}{\rho C L^2}$ RATIO OF HEAT CONDUCTION TO THERMAL ENERGY STORAGE RATE. i.e.

d) $T=T(x,t,T_i,T_\infty,L,k,\alpha,h)$

THIS PROBLEM

$$\frac{\rho C}{k}\frac{\partial T}{\partial t} \rightarrow \frac{\frac{\rho C}{t}}{k}$$

PROBLEM 2 SAME BUT NO x.

SIGNIFICANCE - TEMPERATURE IN THE SOLID $\neq f(x)$