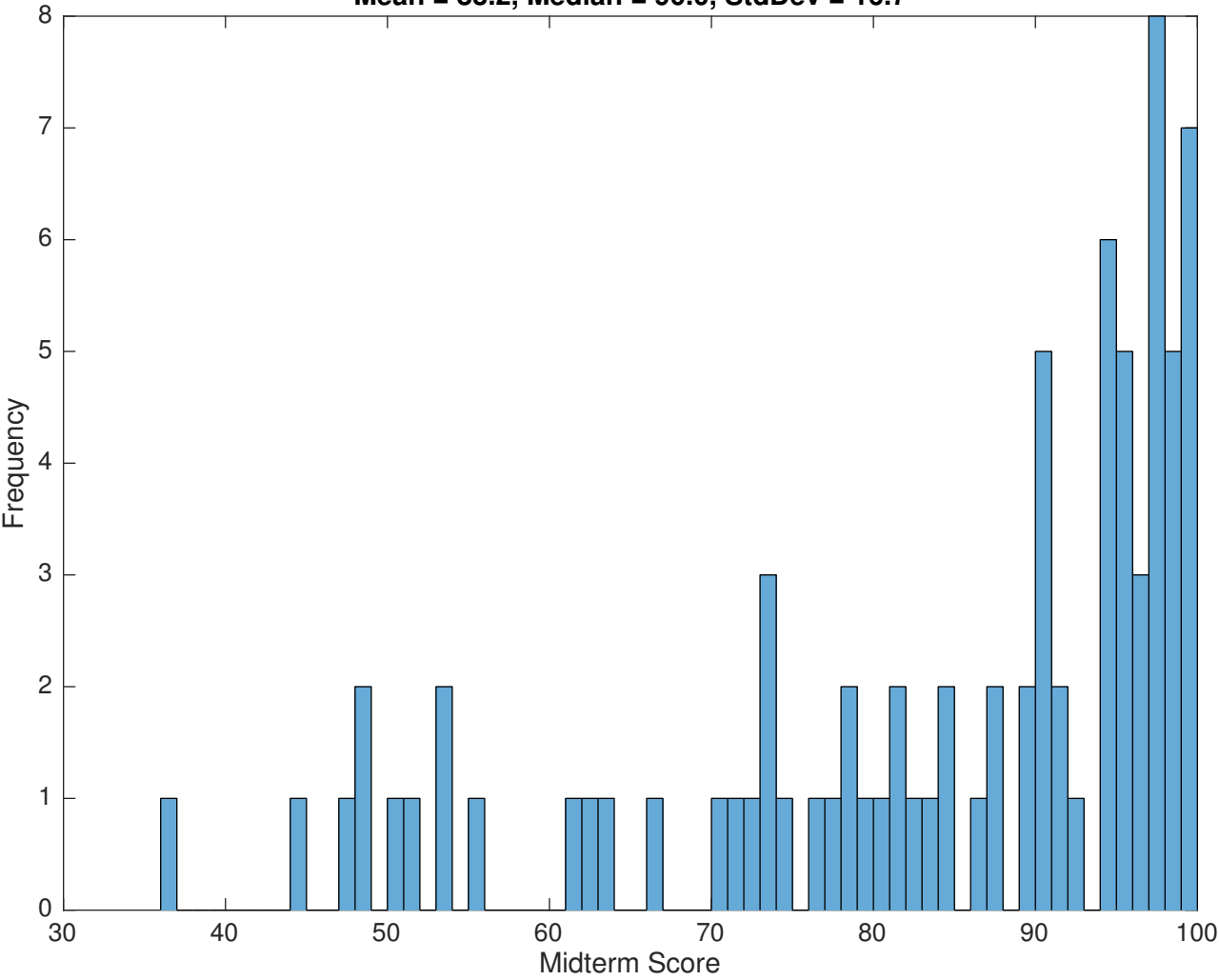


Mean = 83.2, Median = 90.0, StdDev = 16.7



Soln

1a. $S' = \{ u \mid u(0) = 0 \}$

1b. $\mathcal{V} = \{ w \mid w(0) = 0 \}$

1c. $\int_0^a w u'' - w u + 2w dx = 0$

$\int_0^a (w u')' - w' u' - w u + 2w dx = 0$

$w u'|_0^a + \int -w' u' - w u + 2w dx = 0$

$w(0) = 0 \ \& \ u'(a) = 0 \Rightarrow w u'|_0^a = 0$

$\Rightarrow \underbrace{\int_0^a w' u' + w u dx}_{B(w, u)} = \underbrace{\int_0^a 2w dx}_{l(w)}$

1d.

$\underline{k_{i,j}} = B(x^i, x^j) = \int_0^a i j x^{i+j-2} + x^{i+j} dx = \frac{i j}{i+j-1} a^{i+j-1} + \frac{a^{j+i+1}}{i+j+1}$

$\underline{F_j} = l(x^j) = \int_0^a 2 x^j dx = 2 \frac{a^{j+1}}{j+1}$

2a. 8x8

2b. $LM = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 1 \end{bmatrix} \Rightarrow$

80	-50	5	0	0	0	5	-50
-50	100	-50	0	0	0	0	0
5	-50	80	-50	5	0	0	0
0	0	-50	100	-50	0	0	0
0	0	5	-50	80	-50	5	0
0	0	0	0	-50	100	-50	0
5	0	0	0	5	-50	80	-50
-50	0	0	0	0	0	-50	100

3a.
$$\mathcal{V} = \left\{ \omega \mid \omega(0) = 0 \ \& \ \omega'(0) = 0 \right\}$$

3b.
$$\begin{aligned} \delta \Pi &= \left. \frac{d}{d\epsilon} \right|_{\epsilon=0} \Pi(u + \epsilon \omega) = 0 \\ &= \left. \frac{d}{d\epsilon} \right|_{\epsilon=0} \left[\int_0^L \frac{1}{2} (u'' + \epsilon \omega'')^2 dx - (u'''(L) + \epsilon \omega'''(L)) \cdot 1 \right] \\ &= \left[\int_0^L (u'' + \epsilon \omega'') \omega'' dx - \omega'''(L) \right]_{\epsilon=0} \\ &\Rightarrow \int_0^L u'' \omega'' dx = \omega'''(L) \end{aligned}$$

4a.
$$\begin{aligned} T &= \underline{N} \underline{T} & \nabla T &= \underline{\nabla N} \underline{T} \\ \omega &= \underline{N} \underline{\omega} & \nabla \omega &= \underline{B} \underline{\omega} \end{aligned}$$

$$2\pi \int (\underline{B}\underline{\omega})^T k \underline{B} \underline{T} r dr dz = 2\pi \int (\underline{N}\underline{\omega})^T \rho \underline{T} r dr dz$$

$$\underline{\omega}^T \left[\underbrace{\left(2\pi \int \underline{B}^T k \underline{B} r dr dz \right)}_{\underline{K}} \underline{T} - \underbrace{2\pi \int \underline{N}^T \rho \underline{T} r dr dz}_{\underline{F}} \right] = 0$$

$$\Rightarrow \underline{K} \underline{T} = \underline{F}$$

$$\underline{K} = 2\pi \int_A \underline{B}^T k \underline{B} r dr dz$$

$$\underline{B} = [\underline{B}_1 \ \underline{B}_2 \ \dots]$$

$$\underline{B}_A = \nabla N_A = \begin{pmatrix} \partial N_A / \partial r \\ \partial N_A / \partial z \end{pmatrix} = \begin{bmatrix} \frac{\partial \xi}{\partial r} & \frac{\partial \eta}{\partial r} \\ \frac{\partial \xi}{\partial z} & \frac{\partial \eta}{\partial z} \end{bmatrix} \begin{pmatrix} \frac{\partial N_A}{\partial \xi} \\ \frac{\partial N_A}{\partial \eta} \end{pmatrix}$$

can omit always zero term if isoparametric