Math 55 First Midterm Exam, Prof. Srivastava
February 18, 2016, 3:40pm–5:00pm, F295 Haas Auditorium.

Name: ______________________________

SID: ______________________________

Instructions: Write all answers in the provided space. Please write carefully and clearly, in complete English sentences. This exam includes three pages of scratch paper at the end, which must be submitted, but will not be graded. Do not under any circumstances unstaple the exam.

You are not allowed to use any notes, books, electronic devices, or your own scratch paper.

UC Berkeley Honor Code: *As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.*

Question	Points
1	12
2	8
3	7
4	8
5	7
6	8
Total:	50

Do not turn over this page until your instructor tells you to do so.

Name and SID: ______________________________

1. Circle true (**T**) or false (**F**) for each of the following. There is no need to provide an explanation.

   (a) (3 points) The compound propositions

   $$(p \leftrightarrow q) \to r \qquad \text{and} \qquad r \vee (\neg p \leftrightarrow \neg q)$$

   are logically equivalent. **T** **F**

   (b) (3 points) The proposition

   $$(\exists x P(x)) \wedge (\forall y \forall z (P(y) \wedge P(z) \to y = z))$$

   means that there is exactly one element x in the domain such that $P(x)$ is true. **T** **F**

   (c) (3 points) If A and B are sets such that $A \subseteq \mathbb{Z}$ and $B \subseteq \mathbb{Z}$ then $A \times B = B \times A$. **T** **F**

   (d) (3 points) The set

   $$S = \{x \in \mathbb{Z} : \quad 5x \equiv 3 \pmod 7\}$$

   is countably infinite. **T** **F**

2. Prove that the following statements are **false** (i.e., prove their negations). Recall that $\mathbb{Z}^+ = \{1, 2, \ldots\}$ denotes the set of positive integers.

   (a) (4 points)
   $$\forall a, b \in \mathbb{Z}^+ \quad \exists k \in \mathbb{Z}^+ \quad (a + bk \text{ is prime}).$$

   (b) (4 points)
   $$\exists a, b \in \mathbb{Z}^+ \quad \forall k \in \mathbb{Z}^+ \quad (a + bk \text{ is prime}).$$

3. (7 points) Prove or disprove: if A and B are arbitrary sets, then

$$\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B),$$

where $\mathcal{P}(A) = \{S : S \subseteq A\}$ denotes the power set of A.

4. (8 points) Prove that if a and m are positive integers such that $gcd(a, m) = 1$ then the function

$$f : \{0, \ldots, m-1\} \to \{0, \ldots, m-1\}$$

defined by

$$f(x) = (a \cdot x) \ \mathbf{mod} \ m$$

is a bijection, where **mod** denotes the remainder operation.

5. (7 points) Prove that if a and m are positive integers such that $gcd(a, m) \neq 1$ then a does *not* have an inverse modulo m.

6. (a) (4 points) Calculate the remainder $(-9)^{933} \ \mathbf{mod}\ 13$.

(b) (4 points) Use the Euclidean Algorithm to find the greatest common divisor of 270 and 63.

Name and SID: ______________________________

[Scratch Paper 1]

Name and SID: ______________________________

[Scratch Paper 2]

[Scratch Paper 3]