FINAL EXAM, PART A (100 Points, Show All Work)

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Phy 7B
8-14-02

8 Points Each

1. The molar mass of hydrogen is 1.008 g/mole. Calculate the mass of one hydrogen atom.

12 Points Each

2. Figure 1 shows a uniform ring charge of radius a, and total charge +Q. Calculate the electric field, magnitude & direction, at a point P on the axis of the ring at a distance x from the center of the ring.

3. Find the currents I_1, I_2, and I_3 as labeled in Figure 2.

16 Points Each

4. An infinite plane of surface charge density $\sigma = +8\ nC/m^2$ lies in the yz plane at the origin ($x = 0$), and a second infinite plane of surface charge density $\sigma = -8\ nC/m^2$ lies in a plane parallel to the yz plane at $x = 3$ m. Find the electric field at:
   a) $x = 1.5$ m.
   b) $x = 6$ m.

5. A parallel plate capacitor has square plates of side 10 cm and a separation of 4 mm. A dielectric slab of constant $k = 2$ has the same area as the plates, but has a thickness of 3 mm.
   a) What is the capacitance without the dielectric?
   b) What is the capacitance with the dielectric?

6. A 4 μF capacitor is charged to 24 V and then connected across a 200 Ω resistor.
   a) Find the initial charge on the capacitor (at the time of connection to the 200 Ω resistor).
   b) Find the initial current through the 200 Ω resistor.
   c) Find the time constant.
   d) Find the charge on the capacitor 4 ms after the capacitor is connected to the 200 Ω resistor.

20 Points

7. Figure 3 shows a curved path in which a gas is taken from state a to state c and 80 J of heat leave the system and 55 J of work are done <u>on</u> the system.
   a) Determine the change in internal energy, $U_c - U_a$.
   b) When the gas is taken along the path cda, the work done <u>by</u> the gas is 38 J. How much heat Q is added to the gas in the process cda?
   c) If $P_a = 2.5\ P_d$, how much work is done by the gas in the process abc?
   d) What is Q for the path abc?
   e) If $U_a - U_b = 10$ J, what is Q for the process bc?

Possibly Useful Constants
$k = 9 \times 10^9\ N\text{-}m^2/C^2$
$\mu_o = 4\pi \times 10^{-7}$ T-m/A
$\varepsilon_o = 8.85 \times 10^{-12}\ C^2/N\text{-}m^2$
R = 8.31 J/mole-K
$N_A = 6.02 \times 10^{23}$ atoms/mole

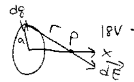

Figure 1

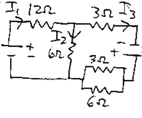

Figure 2

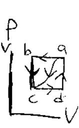

Figure 3

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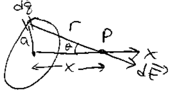
Final Exam Part A

① Hydrogen Atom Mass

$$m = \frac{M}{N_A} = \frac{1.008\ g/mole}{6.02\times10^{23}\ atoms/mole} = \boxed{1.67\times10^{-24}\ \frac{g}{atom} = m}$$

② $\vec{E}$ on the Axis of a Charged Ring



From Symmetry

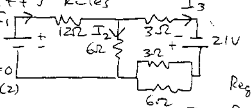
$\vec{E} = E_x\,\hat{i}$

$$E_x = \int dE_x = \int \frac{k\,dq}{r^2}\cos\theta = \int \frac{k\,dq}{r^2}\frac{x}{r} = \int \frac{k\,dq\,x}{(x^2+a^2)^{3/2}}$$

since $r^2 = x^2+a^2$, $\cos\theta = \frac{x}{r}$

$$\Rightarrow E_x = \frac{kx}{(x^2+a^2)^{3/2}}\int dq = \boxed{\frac{kxQ}{(x^2+a^2)^{3/2}} = E_x}$$

③ DC Circuit – Kirchoff's Rules



$18V - 12\Omega(I_1) - 6\Omega I_2 = 0$ (1)

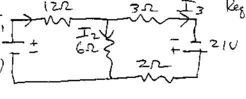
$21V - 2\Omega(I_3) + 6\Omega I_2 - 3\Omega I_3 = 0$ (2)

$I_1 = I_2 + I_3$ (3)

$R_{eq} = \frac{R_1R_2}{R_1+R_2}$

$R_{eq} = \frac{(3\Omega)(6\Omega)}{3\Omega+6\Omega}$

$R_{eq} = 2\Omega$



(3) into (1) ⇒ (4)

$18V - I_2(18\Omega) - I_3(12\Omega) = 0$

$21V + I_2(6\Omega) - I_3(5\Omega) = 0$ (5)

Continued on p2.

3) Continued

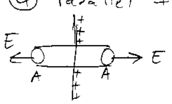
$$(4) + 3(5) \Rightarrow 18V - I_3(12\Omega) + 63V - I_3(15\Omega) = 0$$

$$\Rightarrow I_3 = \frac{81V}{27\Omega} = \boxed{3A = I_3}$$

$$(2) \Rightarrow I_2 = \frac{-21V + 5\Omega(3A)}{6\Omega} = \frac{-6V}{6\Omega} = \boxed{-1A = I_2}$$

$$(3) \Rightarrow I_1 = I_2 + I_3 = -1A + 3A = \boxed{2A = I_1}$$

④ Parallel Infinite Planes of Charge.



Single ∞ Plane of Charge

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow EA + EA = \frac{\sigma A}{\epsilon_0} \Rightarrow E = \frac{\sigma}{2\epsilon_0}$$ (due to single ∞ plane.)

a) $E_1 = E_2 = \frac{\sigma}{2\epsilon_0} = \frac{8 \times 10^{-9} C/m^2}{2(8.85 \times 10^{-12} C^2/Nm^2)} = 452 \frac{N}{C}$

$$E_x = E_1 + E_2 = 452\frac{N}{C} + 452\frac{N}{C} = \boxed{904\frac{N}{C} = E_x}$$

b) for $x = 6m$ (to right of both planes.

$$E_x = E_1 - E_2 = 452\frac{N}{C} - 452\frac{N}{C} = \boxed{0 = E_x}$$

(5) Parallel Plate Capacitor with Dielectric

a) $C_o = \frac{\epsilon_o A}{d} = \frac{(8.85\times10^{-12} C^2/Nm^2)(0.1m)^2}{4\times10^{-3} m} = \boxed{22.1\times10^{-12} F = C}$

b) $V = E_o\left(\frac{d}{4}\right) + \frac{E_o}{K}\left(\frac{3}{4}d\right) = E_o d\left(\frac{1}{4} + \frac{3}{4K}\right) = V_o\left(\frac{K+3}{4K}\right)$

$\Rightarrow V = V_o\left(\frac{2+3}{8}\right) = \frac{5}{8}V_o \Rightarrow C = \frac{Q}{V} = \frac{Q}{\left(\frac{5}{8}V_o\right)} = \frac{8}{5}\frac{Q}{V_o}$

$\Rightarrow C = \frac{8}{5}C_o = \frac{8}{5}(22.1\times10^{-12}F) = \boxed{35.4\times10^{-12} F = C}$

(6) Discharge of a Capacitor

a) $Q_o = CV_o = (4\mu F)(24V) = \boxed{96\mu C = Q_o}$

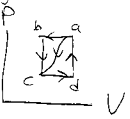
b) $I_o = \frac{V_o}{R} = \frac{24V}{200\Omega} = \boxed{0.12 A = I_o}$

c) $\tau = RC = (200\Omega)(4\mu F) = 800\mu s = \boxed{0.8ms = \tau}$

d) $Q = Q_o e^{-t/\tau} = (96\mu C)e^{-\frac{4ms}{0.8ms}} = (96\mu C)e^{-5}$

$\Rightarrow \boxed{Q = 0.647\mu C}$

⑦ First Law of Thermodynamics



a) $U_c - U_a = Q_{ac} - W_{ac}$

$\Rightarrow U_c - U_a = -80J - (-55J)$

$\Rightarrow \boxed{U_c - U_a = -25J}$

b) $Q_{cda} = (U_a - U_c) + W_{cda} = 25J + 38J$

$\Rightarrow \boxed{Q_{cda} = 63J}$

c) $W_{abc} = \int_{V_a}^{V_b} P\,dV = P_a(V_b - V_a) = 2.5\,P_d(V_c - V_d)$

$\Rightarrow W_{abc} = 2.5\,W_{dc} = 2.5(-W_{cd}) = -2.5(38J)$

$\Rightarrow \boxed{W_{abc} = -95J}$

d) $Q_{abc} = (U_c - U_a) + W_{abc} = -25J + (-95J)$

$\Rightarrow \boxed{Q_{abc} = -120J}$

e) $Q_{bc} = \Delta U_{bc} + W_{bc}$, $W_{bc} = 0$

$\Rightarrow Q_{bc} = U_c - U_b = (U_c - U_a) + (U_a - U_b)$

$\Rightarrow Q_{bc} = -25J + 10J = \boxed{-15J = Q_{bc}}$