EECS 16B Designing Information Devices and Systems II
Spring 2016 Anant Sahai and Michel Maharbiz **Midterm 1**

Exam location: The Faery Land Of Unicorns and Rainbows

PRINT your student ID: ____

PRINT AND SIGN your name: ________________, ________________ ____________________
(last) (first) (signature)

PRINT your Unix account login: ee16b-_____________

PRINT your discussion section and GSI (the one you attend): ____

Name of the person to your left: ____

Name of the person to your right: ____

Name of the person in front of you: ____

Name of the person behind you: ____

Section 0: Pre-exam questions (3 points)

1. **What has been the most useful concept you learned from EE16A? (1 pt)**

2. **What TV show, book or movie has given you a good laugh? (Feel free to write the title in any language.) (2 pts)**

Do not turn this page until the proctor tells you to do so. You can work on Section 0 above before time starts.

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[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

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Section 1: Warm-up questions (48 points)

3. True or False (2 pts for each question) For each question below, circle **T** on the left of each statement if you think the statement is true; else circle **F** (for false).

(a) **[T / F]** An ideal capacitor dissipates energy from the circuit in the form of heat.

Solutions: False. Ideal capacitors do not have non-idealities which dissipate heat, like series resistance.

(b) **[T / F]** An ideal "golden rules" op-amp behaves as though it has infinite gain.

Solutions: True. The golden rule where the positive and negative terminals have identical voltage in negative feedback holds true in the limit that $A \to \infty$.

(c) **[T / F]** A series RLC circuit connected with a DC input voltage/current in a single loop cannot exhibit voltage or current oscillations in time.

Solutions: We accept both *True* and *False* for this question, because we do not clarify the definition of a DC source during the exam. If you think a DC source should never change the value, then the answer is *True*.

However, if you think a DC voltage source can be $1V$ for $t < 0$ and $0V$ for $t \geq 0$, consider the circuit in Question 7(a), it exhibits oscillations. Hence the answer is *False*.

(d) **[T / F]** Given an impedance Z connected across a voltage source $v(t)$, it is possible for $i(t)$ to be in-phase (no phase shift) with a sinusoidal $v(t)$.

Solutions: True. A simple resistor has an impedance of R, which by Ohm's law results in $v(t) = i(t)R$, where the voltage and the current are in phase.

(e) **[T / F]** Since the current across an open circuit must be zero, the voltage across the open circuit must also be zero by Ohm's law.

Solutions: False. Ohm's law applies only to resistors, and so the voltage across an open circuit could be anything. Furthermore, if we model the open circuit as a resistor with resistance $R \to \infty$, then we have $V = \infty * 0$ which is undefined.

(f) **[T / F]** The voltage across a constant current source must be zero.

Solutions: False. Consider a constant current source in series with a resistor. Then the voltage across the constant current source is no longer zero.

(g) **[T / F]** An electrical impedance across two terminals $Z = j\omega k$ (where ω is a positive angular frequency in rad/s and k is a positive real number) can be implemented using only capacitors.

Solutions: False. Without active elements or inductors, only impedances in the following form can be produced:

$$
\begin{aligned}
Z &= \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} + \ldots + \frac{1}{j\omega C_n} \\
Z &= \frac{1}{j\omega}\left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}\right) \\
Z &= \frac{k'}{j\omega}
\end{aligned}
$$

This is not in the form $Z = j\omega k$.

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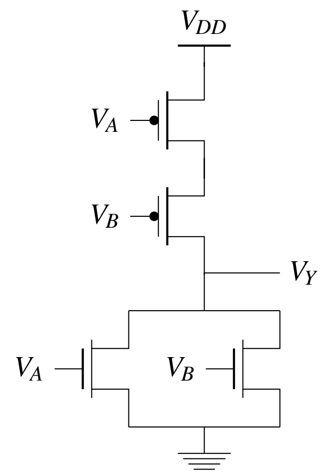
4. Digital Circuits (9 pts)

Consider the circuit below:

(a) (3 pts) The circuit below is a legal CMOS gate. A, B and Y are the Boolean values of the voltages, V_A, V_B and V_Y, respectively. Write down Y as a Boolean formula involving A and B.



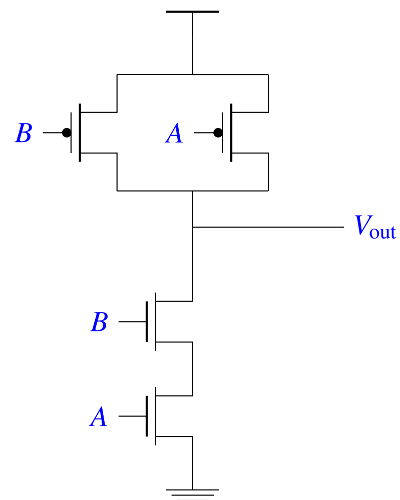
Solutions: The circuit implements the NOR function. It pulls up when both A and B are zero. It pulls down on all other inputs.

Operator	Meaning
$\neg$	NOT
$\vee$	OR
$\wedge$	AND
$\oplus$	XOR

Table 1: Reminder: Logical Operators

Implement each of the following Boolean functions with a single CMOS gate (i.e. implemented using a pull-up network consisting of PMOS transistors connected to a pull-down network consisting of NMOS transistors) by drawing it, or state why the function cannot be implemented as a single CMOS gate in 1-3 sentences. You only have available V_A and V_B as inputs.

(b) (3 pts) $\neg(A \wedge B)$. **Solutions:** The function is the NAND function.



(c) (3 pts) $A \wedge B$. **Solutions:** Given the assignment $A = B = 0$, the pull-up network, being composed of PMOS transistors only, has to pull up the output to 1, no matter how the transistos are arranged. However, the output of $A \wedge B$ on this assignment is 0. Therefore this function cannot be implemented as a single CMOS gate.

5. Can you control me? (8pts)

We have a discrete time system that evolves according to $\vec{x}(t+1) = A\vec{x}(t) + B\vec{u}(t)$. For each part, **answer whether there exists a sequence of control vectors $\vec{u}(t)$ that will bring the state to the origin $\vec{0}$ in a finite number of steps no matter where it starts.**

(a) (4 pts) $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Solutions: The controllability matrix

$$[B, AB] = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

is not full-rank. This system is not controllable.

(b) (4 pts) $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Solutions: The controllability matrix

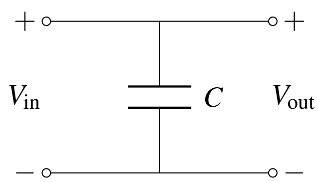
$$[B, AB] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

is full-rank. This system is controllable.

6. Transfer Functions (9 pts)

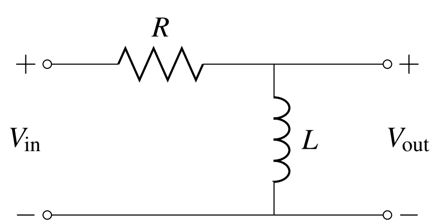
Consider the circuit diagrams below. We define $H(\omega) = V_{out}/V_{in}$ as the voltage transfer function for each circuit. Here, assume that the input is connected to an ideal voltage source that applies a sinusoidal voltage. For each circuit, **provide an expression for** $H(\omega)$ where ω is the frequency of the applied sinusoidal voltage in radians per second. Here the transfer functions should be expressed as functions of j, ω, constants and the physical constants (R, C, L) of the systems.

(a) (3 pts) $H(\omega) = ?$



Solutions: Clearly, $V_{out} = V_{in}$, The transfer function is $H(\omega) = 1$.

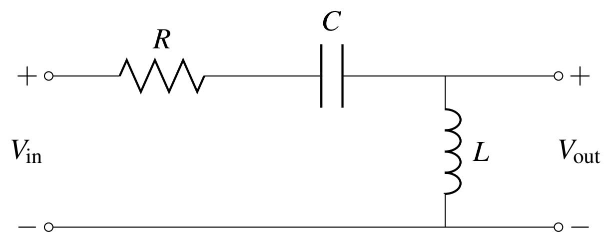
(b) (3 pts) $H(\omega) = ?$



Solutions: This is a voltage divider. The transfer function is

$$H(\omega) = \frac{j\omega L}{R + j\omega L}$$

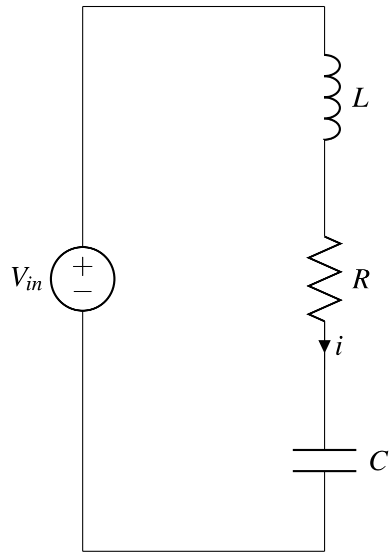
(c) (3 pts) $H(\omega) = ?$



Solutions: This is a voltage divider. The transfer function is

$$H(\omega) = \frac{j\omega L}{R + \frac{1}{j\omega C} + j\omega L}$$

7. RLC Transient Matching (8pts)



Throughout this problem, we assume $V_{in} = 1V$ for $t < 0$ and $0V$ for $t \geq 0$.

For this problem you are asked to **match** the transient behavior for **the current,** i, of the RLC circuit for various values of R, L, and C.

Circle your answer. There is no need to give any justification. However, 0 points will be awarded for an incorrect answer, 0.5 point will be awarded for leaving it blank and 4 points will be awarded for the correct answer

Solutions: In this circuit, we have the three equations to describe the current and voltage for each component: $v_R = i \times R$, $C\frac{dv_C}{dt} = i$ and $L\frac{di}{dt} = v_L$. By KVL, we have $V_{in} = v_L + v_R + v_C = L\frac{di}{dt} + i \times R + \frac{\int idt}{C}$. Take derivative, we can get $L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$.

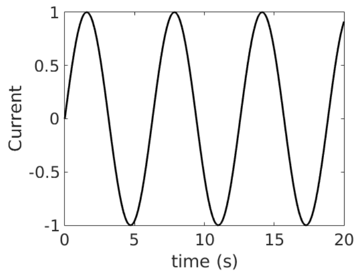
To solve this second order differential equation, we could define $\vec{x} = \begin{pmatrix} i \\ \frac{di}{dt} \end{pmatrix}$. Therefore, we can get the following differential equations in the matrix form:

$$\begin{pmatrix} \frac{di}{dt} \\ \frac{d^2i}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{LC} & \frac{-R}{L} \end{pmatrix} \begin{pmatrix} i \\ \frac{di}{dt} \end{pmatrix}$$

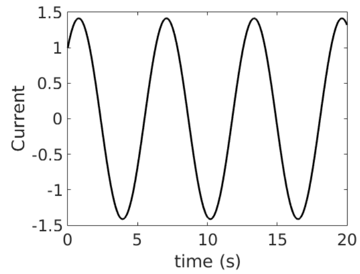
Find the eigenvalues for the above circuit, you will get $\lambda = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$

(a) (4 pts) For $R = 0\Omega$, $L = 1H$, $C = 1F$ Which one is the correct transient response of the current in the circuit?

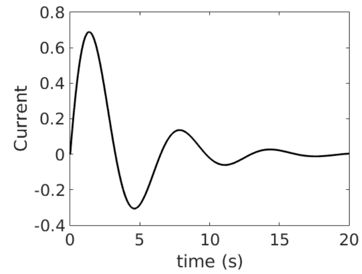
Solutions: For $R = 0$, the eigenvalues are pure imaginary numbers, and this produces sinusoidal oscillation. The oscillation has period of $\frac{1}{\sqrt{LC}} = 1\frac{rad}{s} = 2\pi\frac{1}{s}$. The initial conditions are that the current $i = 0$, therefore, the correct answer is part (A)



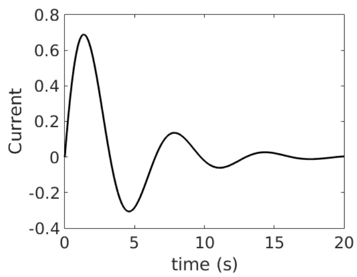
(A)



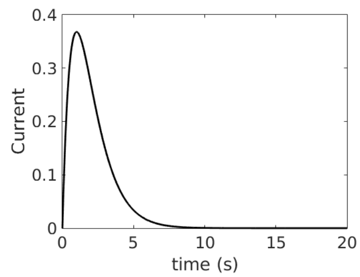
(B)



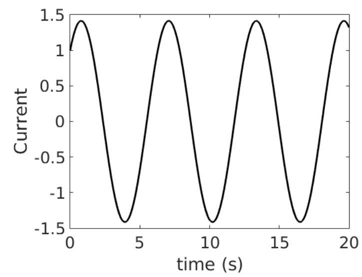
(C)



(A)



(B)



(C)

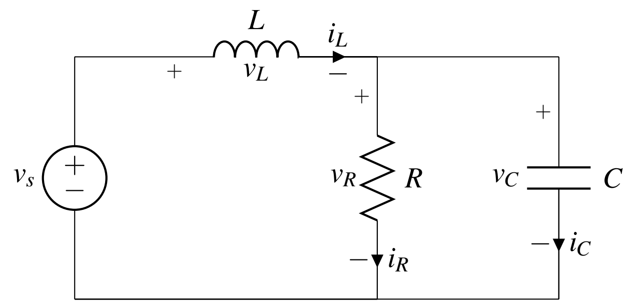
(b) (4 pts) For $R = 0.5\Omega, L = 1H, C = 1F$ Which one is the correct transient response of the current in the circuit?

Solutions: For R=0.5, this circuit is in the underdamped stage. We can see this because the differential equation solution has oscillating terms if and only if $\frac{R}{2L} < \frac{1}{\sqrt{LC}}$. This case exhibits both negative real and imaginary eigenvalues and thus, should oscillate and decay with time. Initial conditions say the circuit should start out with 0 current, therefore part (A) is the correct answer again.

Section 2: In The Zone(59 points)

8. RLC Problem (26 pts)

Consider the circuit below: let's try to analyze it with everything you know about circuits.



(a) (3 pts) Assume $v_s = V_0$ for $t < 0$, and $v_s = 0$ for $t \geq 0$. **What is $v_C(0)$? What is $i_L(0)$?**

Solutions: Here C is an open circuit for the DC source while L works as short, so $v_C = v_R = V_0$. The current going through L is the same as that of R: $i_L = i_R = \frac{v_R}{R} = \frac{V_0}{R}$

(b) (3 pts) If $v_s = 0$ (a constant) for any $t \geq 0$, **what is the steady state value of v_C?** (i.e. $v_C(t \to \infty)$) **What is the steady state value of i_L?**

Solutions: Same as (a), $V_C = 0$, and $i_L = 0$.

(c) (3 pts) **Write down the KCL equation on a node connecting the three passive components in terms of i_L, i_C and i_R.**

Solutions: $i_s = i_L = i_R + i_C$

(d) (3 pts) **Write down a KVL equation for the loop containing the voltage source, inductor and the capacitor in terms of v_s, v_L and v_C.**

Solutions: $v_s = v_L + v_C$

(e) (6 pts) **Write down differential equations for v_C and i_L** using the relationships between the voltage across each component and the current through it, in addition to the equations obtained above. **Convert them into the following matrix form** (notice that $v_s = 0$ for any $t \geq 0$):

$$\begin{pmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{pmatrix} = A \begin{pmatrix} i_L \\ v_C \end{pmatrix}$$

Solutions: We have (1) $v_R = i_R \times R = v_C$, (2) $C\frac{dv_C}{dt} = i_C$ and (3) $v_L = L\frac{di_L}{dt}$. Plug in the KCL equation, $i_L = \frac{v_C}{R} + C\frac{dv_C}{dt}$. Plug in the KVL equation, then $L\frac{di_L}{dt} + v_c = v_s$. because $v_s = 0$, we could drop out the constant term. Finally we could get the following matrix form:

$$\begin{pmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{pmatrix} \begin{pmatrix} i_L \\ v_C \end{pmatrix}$$

(f) (8 pts) For the differential equations above, we know the solution can be obtained from the general solutions $c_1 e^{\lambda_1 t}\vec{v}_1 + c_2 e^{\lambda_2 t}\vec{v}_2$. **What are the values of λ_1 and λ_2? Express them in terms of R, L, C and constants.**

Solutions: λ_1 and λ_2 are eigenvalues of A. To find eigenvalues of A, we could find the roots of $det(A - I\lambda) = -\lambda \times (-\lambda - \frac{1}{RC} + \frac{1}{LC}) = 0$. For $\lambda^2 + \frac{\lambda}{RC} + \frac{1}{LC} = 0$, the solutions should be $\lambda = \frac{-\frac{1}{RC} \pm \sqrt{\frac{1}{(RC)^2} - \frac{4}{LC}}}{2}$.

[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

9. Hold me and linearize me (13 pts)

Consider a non-linear two-dimensional system with states x_0 and x_1 and scalar input u that evolves according to the following coupled differential equations

$$\begin{aligned}\frac{d}{dt}x_0(t) &= \dot{x}_0 = x_1(t)\\ \frac{d}{dt}x_1(t) &= \dot{x}_1 = 4-(\frac{u(t)}{x_0(t)})^2\end{aligned} \tag{1}$$

(a) (5 pts) **Find an input u_e so that if the system starts in state vector $\vec{x}_e = \begin{bmatrix}1\\0\end{bmatrix}$ and we apply the input $u(t) = u_e$, the system will always stay in that same state.** **Solutions:**
The non-linear system is given in vector form as

$$\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x}(t), u(t)) = \begin{bmatrix} x_1(t)\\ 4-(\frac{u(t)}{x_0(t)})^2\end{bmatrix}$$

Now setting $\frac{d}{dt}\vec{x}_e(t) = \vec{f}(\vec{x}_e(t), u_e(t)) = \vec{0}$ we have

$$u_e(t)^2 = 4$$

Thus, we have
$\vec{x}_e = \begin{bmatrix}1\\0\end{bmatrix}$ and $u_e(t) = \pm 2$

(b) (8 pts) **Write linearized state-space equations around $\vec{x}_e$ and u_e. Convert them into the following form and find the matrices A and B.**

$$\frac{d}{dt}\vec{x}(t) = A(\vec{x}-\vec{x}_e) + B(u(t)-u_e)$$

Solutions: We find the Jacobian matrices for both the state vector and the input. Thus,

$$\frac{\partial \vec{f}}{\partial \vec{x}} = \begin{bmatrix} 0 & 1\\ 2\frac{u_e(t)^2}{x_{0_e}(t)^3} & 0\end{bmatrix} = A_e$$

and

$$\frac{\partial \vec{f}}{\partial \vec{u}} = \begin{bmatrix} 0 \\ -2u_e(t) \end{bmatrix} = B_e$$

The system evolves as

$$\frac{d}{dt}\vec{x}(t) = A_e(\vec{x} - \vec{x}_e) + B_e(\vec{u} - \vec{u}_e)$$

For $u_e = 2$,

$$A = \begin{bmatrix} 0 & 1 \\ 8 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

For $u_e = -2$,

$$A = \begin{bmatrix} 0 & 1 \\ 8 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

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[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

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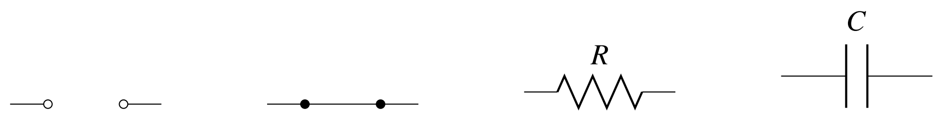
10. Circuit Design (8 pts)

In this problem, you will find a circuit where several components have been left *blank* for you to fill in.

Assume the op-amp is *ideal*.

You have at your disposal *only one of each* of the following components:



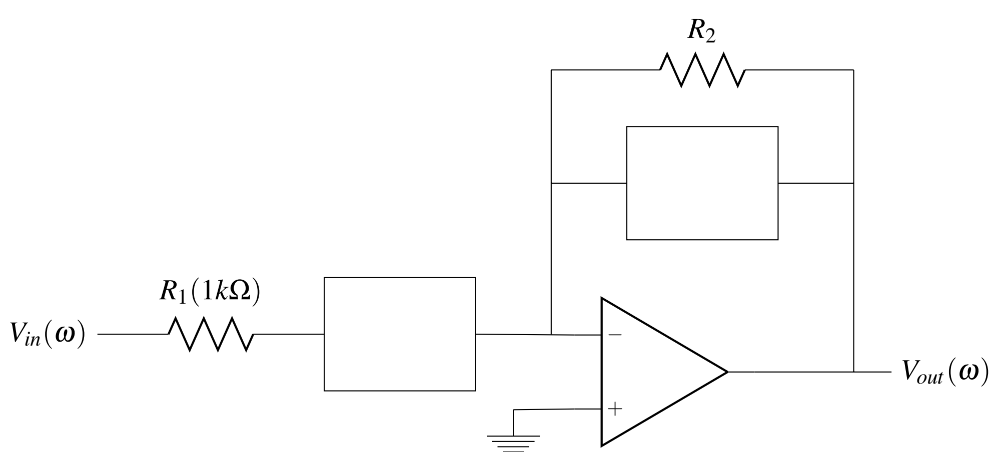
(a) an open circuit

(b) a short circuit

(c) a resistor (you choose from the value $R = 0.5k, 1k, 2k\Omega$)

(d) a capacitor (you choose from the value $C = 0.5\mu, 1\mu, 2\mu F$)

Consider the circuit below. The voltage source $v_{in}(t)$ has the form $v_{in}(t) = v_0\cos(\omega t + \phi)$. The labeled voltages $V_{in}(\omega)$ and $V_{out}(\omega)$ are the phasor representation of $v_{in}(t)$ and $v_{out}(t)$. The transfer function $H(\omega)$ is defined as $H(\omega) = V_{out}(\omega)/V_{in}(\omega)$.



Let R_1 be $1k\Omega$. **Fill in the boxes and determine the value of** R_2 so that

- It is a high-pass filter.
- $|H(\infty)| = 2$.
- $|H(10^3)| = \sqrt{2}$.

Solutions:

Let the left box be Z_1 and the right box be Z_2. The circuit should be a high-pass filter, so Z_2 cannot be a short circuit or a capacitor (otherwise $V_{out}(\infty) = 0$). Since Z_2 cannot be capacitors, Z_1 must be a capacitor (otherwise it is not a filter). Z_2 is either an open circuit or a resistor. Let $R_f = R_2 \parallel Z_2$ and $Z_1 = C$. The transfer function is given by

$$H(\omega) = -\frac{R_f}{R_1 + \frac{1}{j\omega C}} = -\frac{j\omega R_f C}{1 + j\omega R_1 C} = -\frac{R_f}{R_1}\frac{j\omega R_1 C}{1 + j\omega R_1 C}$$

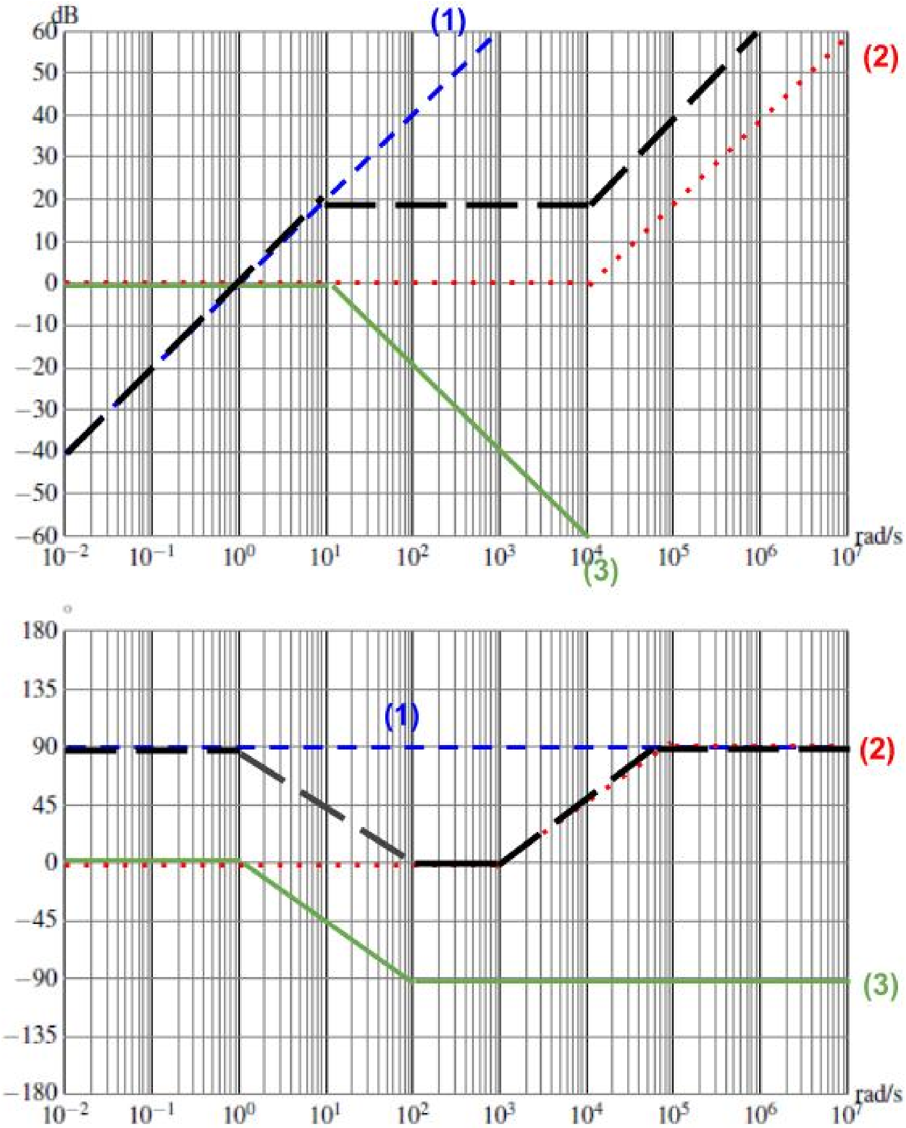
Observing the transfer function, we know $H(0) = 0$ and $H(\infty) = -\frac{R_f}{R_1}$ so it is a high-pass filter. From $|H(\infty)| = 2$, we know $R_f = 2R_1 = 2k\Omega = R_2 \parallel Z_2$. Because of the limited choices of resistors, Z_2 must be an open circuit and $R_2 = 2k\Omega$.

From $H(10^3) = \sqrt{2}$, we have (let $x = 10^3 R_1 C$)

$$\sqrt{2} = 2\frac{\sqrt{x^2}}{\sqrt{1+x^2}} \Rightarrow \frac{1}{2} = \frac{x^2}{1+x^2} \Rightarrow x^2 = 1 \Rightarrow 10^3 R_1 C = 1 \Rightarrow C = \frac{1}{10^3 R_1} = 1\mu F$$

Thus,

- $R_2 = 2k\Omega$.
- The right box : an open circuit.
- The left box : a capacitor with $C = 1\mu F$.



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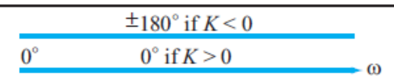
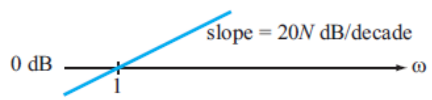
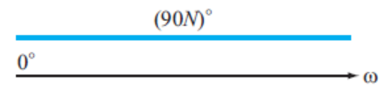
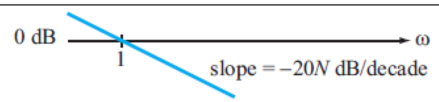
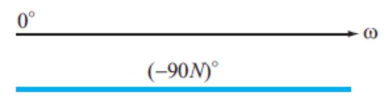
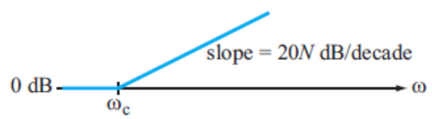
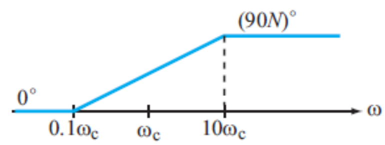
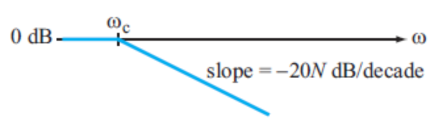
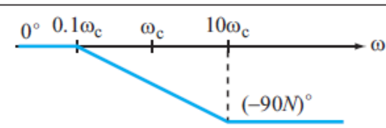
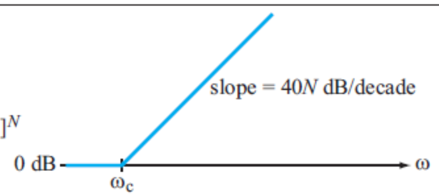
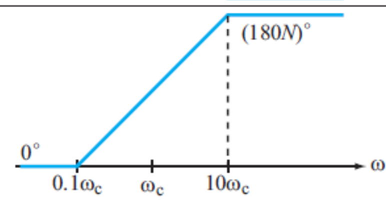
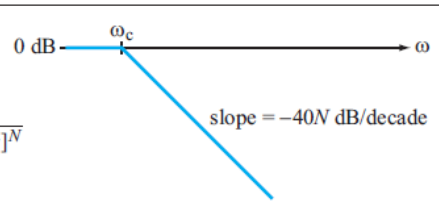
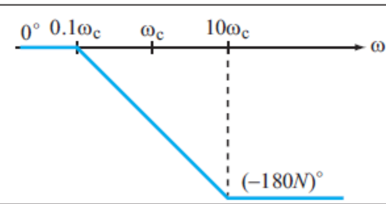
11. Bode plot (12 pts)

Draw the Bode plot for the transfer function $H(\omega) = \frac{(j\omega\times 10)(10+j\omega\times 10^{-3})}{(100+j\omega\times 10)}$ Remember you have the Bode plot table in the next page!

Solutions:

$$H(\omega) = \frac{(j\omega \times 10)(10 + j\omega \times 10^{-3})}{(100 + j\omega \times 10)} = \frac{j\omega(1 + \frac{j\omega}{10^4})}{(1 + \frac{j\omega}{10})}$$

Therefore, we should draw three lines and perform superposition among them. In the following graph, line (1) is $j\omega$, line (2) is $(1 + \frac{j\omega}{10^4})$, and line (3) is $(1 + \frac{j\omega}{10})^{-1}$.

Factor	Bode Magnitude	Bode Phase
Constant K		
Zero @ Origin $(j\omega)^N$		
Pole @ Origin $(j\omega)^{-N}$		
Simple Zero $(1 + j\omega/\omega_c)^N$		
Simple Pole $\left(\frac{1}{1 + j\omega/\omega_c}\right)^N$		
Quadratic Zero $[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N$		
Quadratic Pole $\frac{1}{[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N}$		

[Doodle page! Draw us something if you want or give us suggestions or complaints. You can also use this page to report anything suspicious that you might have noticed.]