Name KEY Lab Section Number / GSI ______

Chemistry 4A Midterm 1

You are given the following effective potential for an electron orbiting around the nucleus of the H-atom with angular momentum l and radial velocity v_r:

$$E = mv_r^2/2 + l^2/2mr^2 - q^2/(4\pi\epsilon_0 r)$$

where you can take $4\pi\epsilon_0 = 10^{-10}$ C^2/J-m and q = 10^{-19} C.

The radii of the circular orbits (in meters) of this effective potential are:

$$r_0 = l^2(4\pi\epsilon_0/mq^2) = l^2(10^{58})$$

Unless otherwise specified, use $\hbar = 1 * 10^{-34}$ J-s.

To simplify the calculations, the speed of light c = $3 * 10^8$ m/s and the mass of the electron m= $9 * 10^{-31}$ kg.

Some formulas to help you with powers of 10:

$$10^a 10^b = 10^{a+b}$$

$$10^c/10^d = 10^{c-d}$$

To receive full credit, please write the equation that you use to answer the question, and then give your answers in numerial values. Fractions are acceptable, but all powers of 10 should be simplified.

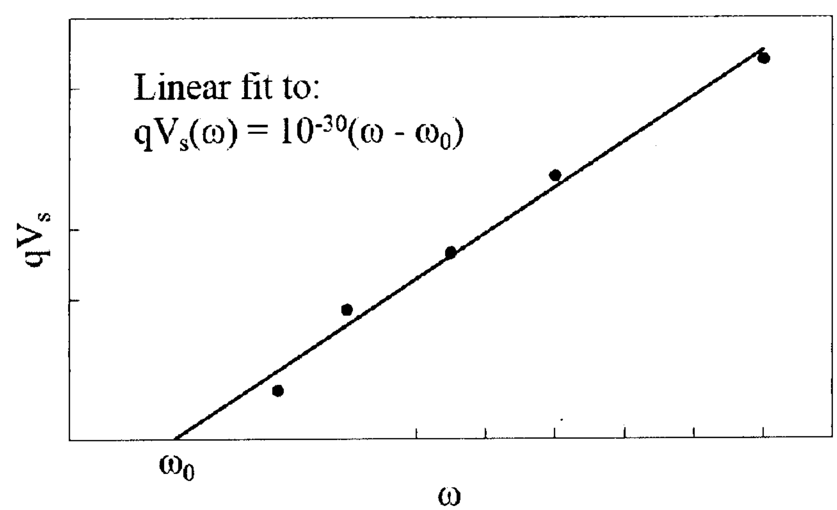

Figure 1

1. (20 pts) Imagine we live in a world where the photoelectric effect resulted in the graph shown in Fig. 1. In this hypothetical world, what is

(a) Planck's constant $\hbar$? Explain how you got this from what is given in the graph.

10^{-30} Js

slope

(b) The angular momentum of an electron in Bohr's model of the H-atom in the lowest energy state?

10^{-30} Js ($\hbar$)

(c) The classical velocity to which this angular momentum corresponds?

$mvr = \hbar$; $\hbar = 10^{-30}$ Js; $m = 9\times10^{-31}$ kg ; $r = 1^2 10^{-58}$ m $= 10^{-60}10^{58}$ m $= 10^{-2}$ m

$$v = \frac{\hbar}{mr} = \frac{10^{-30}\,\text{Js}}{9\cdot10^{-31}\,\text{kg}\cdot10^{-2}\,\text{m}} = \frac{1\,\text{kg}\cdot\text{m}^2/\text{s}}{9\cdot10^{-3}\,\text{kg m}} = \boxed{\frac{1}{9}10^{3}\,\text{m/s} = 111\,\text{m/s}}$$

(d) The electron wavelength to which this angular momentum corresponds?

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{\hbar 2\pi}{mv} = \frac{10^{-30}\,\text{kg m}^2/\text{s}\cdot 2\pi}{111\,\text{m/s}\cdot 9\cdot10^{-31}\,\text{kg}}$$
$$\boxed{= \frac{2\pi}{999}\times10^{1}\,\text{m} = 0.063\,\text{m}}$$

(e) The condition on the grating/slit spacing needed to observe a diffraction pattern for the electron wavelength obtained in (d)?

d on the order of λ

2. (26 pts) Consider that an electron can take on an angular momentum of either $\hbar$ or $2\hbar$ in Bohr's model of the H-atom. Here, $\hbar = 1 * 10^{-34}$ J-s.

(a) To what radii do these angular momenta correspond?

$$r_o = l^2 10^{58}$$

$\hbar$: $r_o = \hbar^2 10^{58} = (10^{-34})^2 10^{58} = 10^{-68} 10^{58} = \boxed{10^{-10}\ m}$

$2\hbar$: $r_o = (2\hbar)^2 10^{58} = 4(\hbar^2 10^{58}) = \boxed{4 \times 10^{-10}\ m}$

(b) To what energies do these angular momenta corresond?

$$E = m v_r^2/2 + l^2/2mr^2 - q^2/(4\pi\varepsilon_o r) \quad , v_r = 0 \text{ for circular orbits}$$

$$E = l^2/2mr^2 - q^2/(4\pi\varepsilon_o r)$$

$\hbar$: $\frac{10^{-68}}{2 \cdot 9 \times 10^{-31}(10^{-10})^2} - \frac{(10^{-19})^2}{10^{-10} \cdot 10^{-10}} = \frac{1}{18} \cdot 10^{-17} - 10^{-18}$ $\boxed{\simeq -4.4 \times 10^{-19} J}$

$2\hbar$: $\frac{4 \times 10^{-68}}{2 \cdot 9 \times 10^{-31}(4 \times 10^{-10})^2} - \frac{(10^{-19})^2}{10^{-10}(4 \times 10^{-10})} = \frac{1}{4 \cdot 18} 10^{-17} - \frac{1}{4} 10^{-18}$ $\boxed{\simeq -1 \times 10^{-19} J}$

(c) What is the radial frequency of light (ω) emitted when an electron falls from the higher to lower angular momentum state?

$$\omega = \frac{E}{\hbar} \qquad \Delta E = -1 \times 10^{-19} J + 4.4 \times 10^{-19} J$$

$$= 3.4 \times 10^{-19} J$$

$$\omega = \frac{3.4 \times 10^{-19} J}{1 \times 10^{-34} J \cdot s}$$

$$= \boxed{3.4 \times 10^{15} s^{-1}} = 3.4 \times 10^{15} Hz$$

(d) What is the wavelength of the light emitted in (c)?

$$c = \omega/k \quad k = 2\pi/\lambda \Rightarrow \lambda = \frac{c 2\pi}{\omega}$$

$$= \frac{3 \cdot 10^8 m/s \cdot 2\pi}{3.4 \times 10^{15} s^{-1}}$$

$$= \boxed{\frac{6\pi}{3.4} \times 10^{-7} m = 5.5 \times 10^{-7} m}$$

(e) What is the ionization energy (energy needed to eject the electron from the H-atom) for the $\hbar$ electron?

ejected electron: $E = 0$

$0 J - (-4.4 \times 10^{-19} J)$ from b

$= \boxed{4.4 \times 10^{-19} J}$

3. (12 pts) Consider that the electron can have one of the following quantum mechanical wave functions. In the region where the electron exists, the potential energy is zero.

$$1.\, \sin(kx - \omega t)$$

where k varies continuously.

The following are only defined for $0 < x < L$ and the wavefunction is zero otherwise:

$$2.\, \sin(k_1 x)\cos(\omega_1 t)$$
$$3.\, \sin(k_2 x)\cos(\omega_2 t)$$

where k_n are discrete values that take on $k_n = n\pi/L$.

(a) Which one(s) represent an unbound electron?

1

(b) Which one(s) are standing waves?

2, 3

(c) Which has the highest number of nodes?

3

(d) Of those that represent bound electrons, which has the highest energy?

3

For multiple choice problems 4-6, circle the letter(s) of all true answer choices.

4. (5 pts) Which of the following is true of the cathode ray experiment?

   (a) It showed electrons exist by applying a voltage accross electrodes.

   (b) In the experiment, particles were ejected from the anode (positive electrode).

   (c) It measured the charge of the electron to be 1.6 x 10^{-19} Coulombs.

   (d) It showed that electrons exhibit wave-like properties.

5. (5 pts) Which of the following is true of the Rutherford experiment?

   (a) It showed that the positive charge in matter is concentrated in a dense nucleus.

   (b) It scattered a beam of negatively charged particles off of a metal foil.

   (c) It suggested that a planetary model of the atom was appropriate.

   (d) It concluded that the jellium model of matter, with an evenly distributed positive charge and where the electrons were not moving, is correct.

6. (5 pts) Which of the following is true of the electron diffraction experiment?

   (a) It showed electrons exist by changing their direction in an applied magnetic field.

   (b) It required a grating with slit-spacings that were on the order of atomic spacings.

   (c) It suggested that the energy of light related to its frequency.

   (d) It showed that electrons exhibit wave-like properties.