FINAL EXAM – Section 3

Instructor: Prof. A. LANZARA

TOTAL POINTS: 134

TOTAL PROBLEMS: 7

Read the whole exam before starting to solve problems. Start with the ones you are more familiar with to secure points.
Show all work, and take particular care to explain what you are doing. Show a logical progression of steps from equations on the equation sheet to your final answer. Partial credit is given. Please use the symbols described in the problems, define any new symbol that you introduce and label any drawings that you make. If you get stuck, skip to the next problem and return to the difficult section later in the exam period. All answers should be in terms of variables.

GOOD LUCK!

Problem 1 (tot 14 pts)

Problem 1.1 (2pts):
An aluminum plate has a circular hole. If the temperature of the plate increases, what happens to the size of the hole? Think about the demo we discussed in class.

a) increases
b) decreases
c) stays the same
d) increases the top half of the hole
e) more information is required

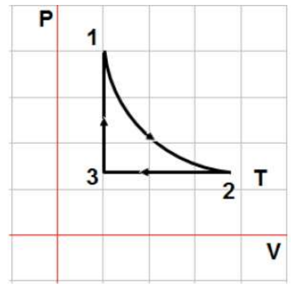
Problem 1.2 (3 points)
The frequency of an LC oscillator is f_0. The plates of the parallel-plate capacitor are then pulled apart to twice the original separation. What is the new frequency of oscillations?

a. $2f_0$
b. f_0 sqrt(2)
c. f_0 / sqrt (2)
d. $f_0/2$

Problem 1.3 (3 points)

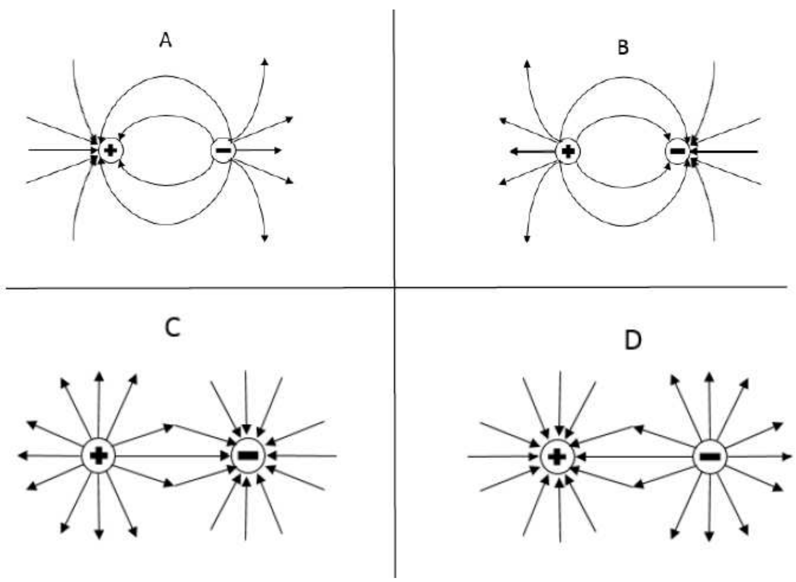
A sample of an ideal gas taken through a closed cycle is presented by the PV diagram. The process 1-2 is perfectly isothermal. Which of the following is true about the change in internal energy and work done by the gas during process 1-2? Explain your reasoning

a) $\Delta U = 0$ $\quad W_{\text{by the gas}} > 0$
b) $\Delta U > 0$ $\quad W_{\text{by the gas}} = 0$
c) $\Delta U < 0$ $\quad W_{\text{by the gas}} < 0$
d) $\Delta U = 0$ $\quad W_{\text{by the gas}} = 0$
e) $\Delta U = 0$ $\quad W_{\text{by the gas}} < 0$



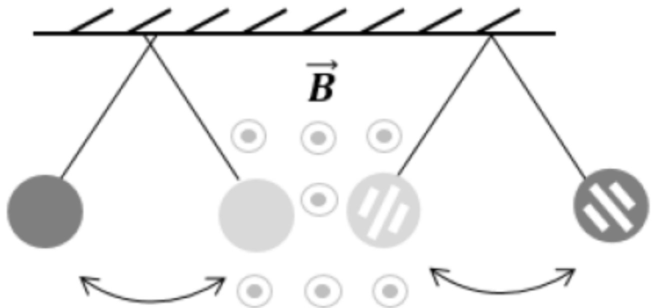
Problem 1.4 (3 points)

Which one of the following figures shows the correct electric field lines for an electric dipole (1pts) ? What is wrong with the other three diagrams?



Problem 1.5 (3 points)

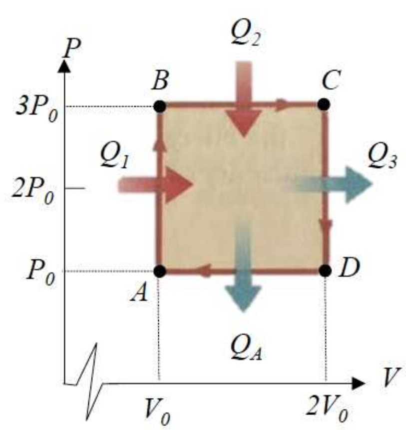
In class we discussed the demo showed by the figure below, where two metal disks of the same size and same material are oscillating in and out of a region with a magnetic field. One disk is solid; the other has a series of slots. Is the retarding effect of Eddy currents on the solid disk greater than, less than or equal to the retarding effect on the slotted disk? Explain your answer.



Problem 2 (20pts)

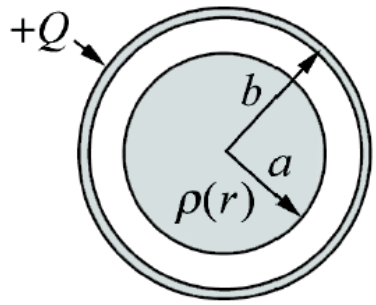
n moles of a monoatomic ideal gas are taken through the cycle shown in the figure below. At point A the pressure, volume and temperature are P_0, V_0, T_0. In terms of n, R and T_0 find:

a) (5pts) The total energy entering the system through heat per cycle
b) (5pts) The total energy leaving the system through heat per cycle
c) (5pts) The efficiency of an engine operating in this cycle
d) (5pts) The efficiency of an engine operating in a Carnot cycle between the same temperature extremes.



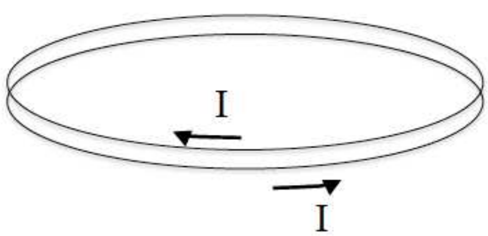
Problem 3 (20pts)
Consider a solid non-conducting sphere of radius *a* carrying a non-uniform positive charge density given by $\rho(r) = \rho_0 (r^2 / a^2)$, where ρ_0 is a positive constant. A very thin non-conducting concentric spherical shell of radius *b*, with *b* > *a* carries a positive charge Q uniformly distributed on the surface.
Determine the direction and magnitude of the electric field in each of the regions:

a) (7pts) r < a
b) (7pts) a < r < b
c) (6pts) r > b



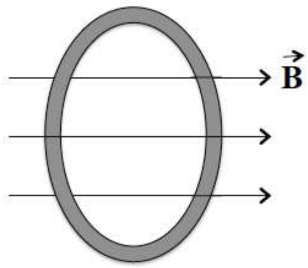
Problem 4 (20pts)
Two circular loops are parallel, coaxial and almost in contact, separated by a tiny distance d. Each loop has a radius of R=10d. The top loop carries a clockwise current I, and the bottom loop carries a counterclockwise current I.

a) (15pts) Calculate the magnitude and direction of the magnetic force exerted by the bottom loop on the top loop.
b) (5pts) The upper loop has a mass m. Calculate the magnitude and direction of its initial acceleration, assuming that the only forces acting on it are the force in part a) and the gravitational force.

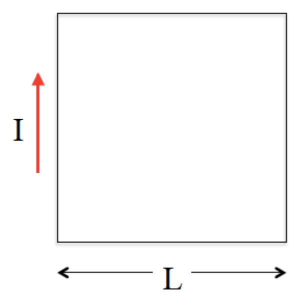


Problem 5 (20pts)
A circular loop of wire of radius L is in a uniform magnetic field, with the plane of the loop perpendicular to the direction of the field. The magnetic field varies with time according to $B(t) = a + bt$, where a and b are constants.

a) (5pts) Calculate the magnetic flux through the loop at t=0
b) (5pts) Calculate the emf induced in the loop.
c) (5pts) If the resistance of the loop is R, what is the induced current?
d) (5pts) At what rate is energy being delivered to the resistance of the loop?



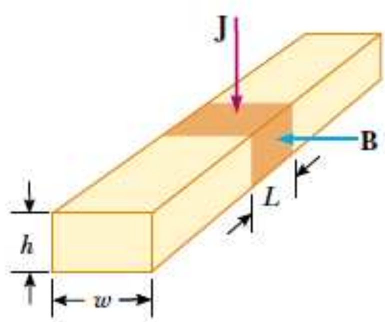
Problem 6 (20pts)
A conductor in the shape of a square loop of edge length L carries a current I as shown in the figure below.
a) (12pts) Calculate the magnitude and direction of the magnetic field at the center of the square.
b) (8pts) If this conductor is formed into a single circular turn and carriers the same current, what is the value of the magnetic field at the center?



Problem 7 (20pts)

Sodium melts at a temperature T_M. Liquid sodium, an excellent thermal conductor, is used in some nuclear reactors to cool the reactor core. The liquid sodium is moved through pipes by pumps that exploit the force on a moving charge in a magnetic field. The principle is as follows. Assume the liquid metal to be in an electrically insulating pipe having a rectangular cross section of width *w* and height *h*. A uniform magnetic field perpendicular to the pipe affects a section of length L. An electric current directed perpendicular to the pipe and to the magnetic field produces a current density J in the liquid sodium.

a) (8pts) Explain why this arrangement produces on the liquid a force that is directed along the length of the pipe.
b) (12pts) Show that the section of liquid in the magnetic field experiences a pressure increase JLB.



$PV^\gamma = \text{const.}$ (For an adiabatic process)

$$W = -\frac{d}{2}(P_f V_f - P_0 V_0)$$
(For an adiabatic process)

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \hat{r}}{r^2}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$
$$d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$$
(Cartesian Coordinates)

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z}$$
$$d\vec{l} = dr\hat{r} + r d\theta\hat{\theta} + dz\hat{z}$$
(Cylindrical Coordinates)

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi}$$
$$d\vec{l} = dr\hat{r} + r d\theta\hat{\theta} + r\sin(\theta) d\phi\hat{\phi}$$
(Spherical Coordinates)

$$y(t) = \frac{B}{A}(1 - e^{-At}) + y(0)e^{-At}$$
solves $\frac{dy}{dt} = -Ay + B$

$$y(t) = y_{max}\cos(\sqrt{A}t + \delta)$$
solves $\frac{d^2y}{dt^2} = -Ay$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}}\sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int_0^\pi \sin^3(x) dx = \frac{4}{3}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\int \sin(x) dx = -\cos(x)$$

$$\int \cos(x) dx = \sin(x)$$

$$\int \frac{dx}{x} = \ln(x)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$