CE 100 - Final Exam

December 12, 2009

Name ____

Student I.D. ____________________________________

This exam is closed book. You are allowed three sheets of paper (8.5" x 11", both sides) of your own notes.

You will be given three hours to complete four problems. Write out your solution with symbols before plugging in any numbers and clearly state any assumptions you make, and please box your answer! Read through the whole exam first and skip ahead to the easy parts when you get stuck.

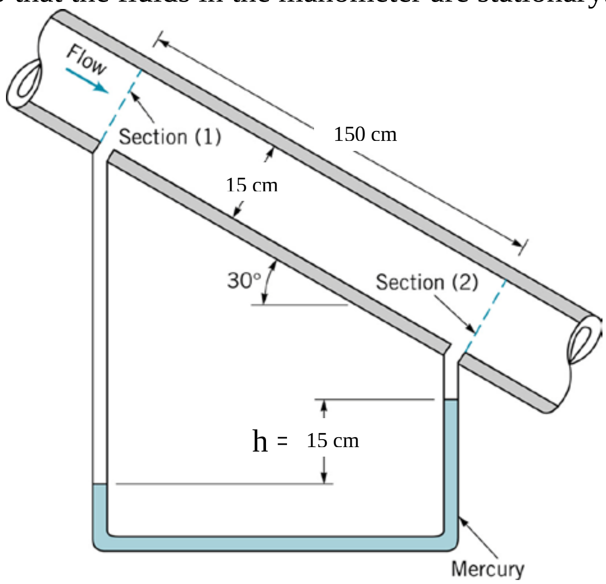
Good Luck!

On all problems, you may assume that the fluid is water, unless otherwise noted. For your reference:

Density of water = ρ_w = 1000 kg/m^3
Dynamic viscosity of water = μ = 1.12 x 10^{-3} N·s/m^2
Density of air = ρ = 1.2 kg/m^3
Kinematic viscosity of air = ν = 1.5 x 10^{-5} m^2/s
Atmospheric Pressure = p_{atm} = 100 kPa
Gravitational Acceleration = g = 9.81 m/s^2

1) Viscous pipe flow (25 points)

Water flows through an inclined pipe. A U-tube manometer with fluid density ρ_m = 13,600 kg/m^3 measures the pressure difference between points 1 and 2 as shown. The pipe flow is steady so that the fluids in the manometer are stationary.



a) Find the pressure difference (P_1-P_2) between sections 1 and 2. (5 points)

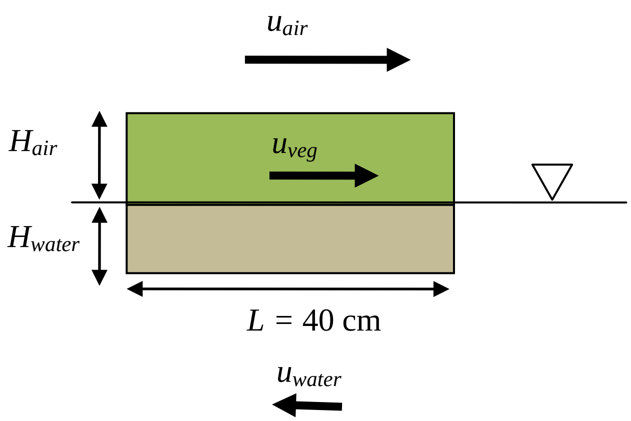
b) Find the head loss (h_L) between sections 1 and 2. (5 points)

c) Find the net axial force exerted by the pipe wall on the flowing water between sections 1 and 2. (10 points)

d) What value would the manometer show for h if there were no flow in the pipe? (5 points)

2) Drag force on vegetation raft (25 points)
Water hyacinth is a floating aquatic plant which tends to clump together to form vegetation rafts, as illustrated below.

a) Assuming we can conceptualize the floating vegetation raft as in the sketch below, determine the depth to which the plant is submerged (H_{water}). The raft has an average density of ρ_f = 400 kg/m^3 and the total height is $H = H_{air} + H_{water}$ = 25 cm. Width into the page is b = 10 cm. (6 points)



b) The vegetation raft experiences drag due to the wind and due to the water. The combined forces cause the raft to move at a velocity of u_{veg}. The drag due to each fluid can be found by using the relative velocities as

$$F_D^{air} = \frac{1}{2}\rho_{air} C_D^{air} A_{air} \left(u_{air} - u_{veg}\right)^2$$

$$F_D^{water} = \frac{1}{2}\rho_{water} C_D^{water} A_{water} \left(u_{water} - u_{veg}\right)^2$$

where A is the frontal area given by $A = Hb$ in each fluid, where b = 10 cm. Set up a force balance to find the vegetation raft velocity under steady state given u_{air} = 10 m/s, u_{water} = -0.5 m/s, C_D^{air} = 1.0, C_D^{water} = 2.0. (10 points)

c) Given a water velocity of u_{water} = -0.5 m/s, find the wind velocity u_{air} required to have u_{veg} = 0. (3 points)

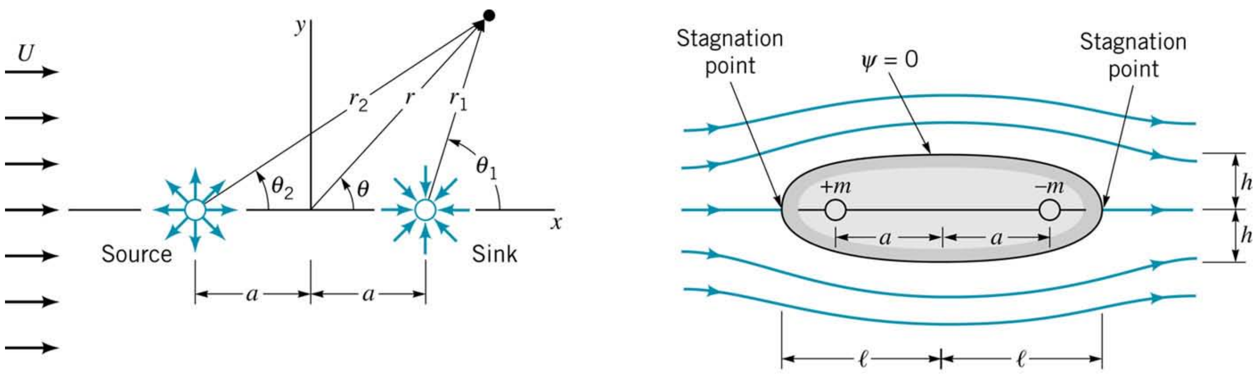
d) If the size of the raft becomes longer (L increases), do you expect friction drag to increase or decrease compared to form drag (also known as pressure drag)? How could you incorporate the new length L in your drag formula given that $A = Hb$ and does not depend on L? (6 points)

3) Groundwater pumping – potential flow (20 points)

Toxic chemical spills can cause groundwater contamination in the subsurface. A common treatment approach is to inject clean water into the ground and pump dirty water out, known as "pump and treat". This creates a capture zone in the flow that can be described by a Rankine oval. Thus any contaminated water inside the oval can be pumped out and will no longer flow downstream.

The streamfunction for flow around the Rankine oval is given by superposition of uniform flow and a source/sink pair:

$$\psi = Ur\sin\theta - \frac{m}{2\pi}\tan^{-1}\left(\frac{2ar\sin\theta}{r^2 - a^2}\right)$$



(a) (b)

a) Sketch streamlines inside the oval (use a new sketch below). (4 points)

b) Find expressions for the velocity components v_r and v_θ. (5 points)

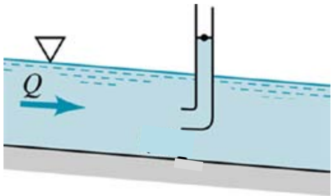
c) How would you approximate the derivative you needed above for v_θ numerically using a central finite difference scheme? (3 points)

d) What is the magnitude of the velocity at $r = 0$? (3 points)

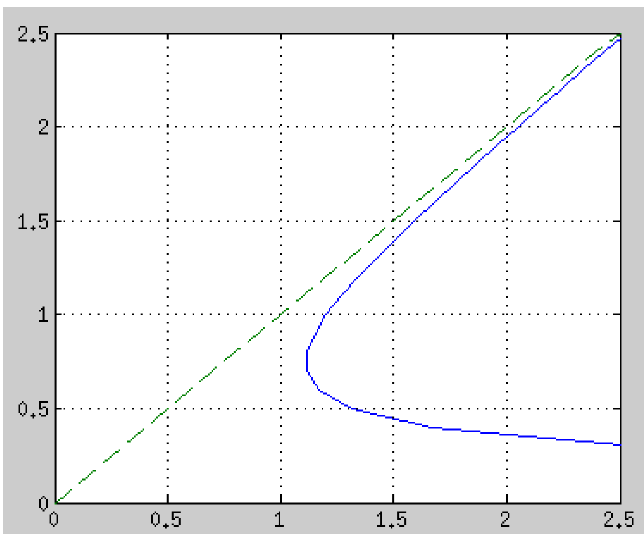
e) Find an expression for the pressure at $r=0$ using p_0 as a reference at an arbitrary point far away. (5 points)

4) Channel design (30 points)

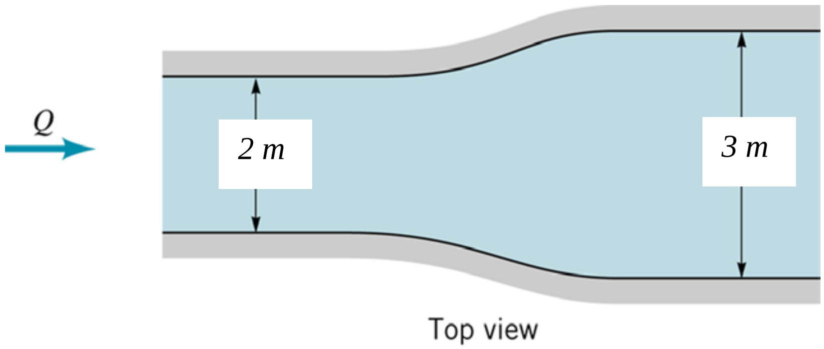
Water flows in a rectangular channel of width b_1 = 2 m at a rate of q = 2 m^2/s. When a Pitot tube is placed in the stream, water in the tube rises to a level of 1.4 m above the channel bottom. Neglect bed slope and friction.



a) Write the equations you need to solve to determine V_1 and y_1 for the conditions shown. Modify them to produce a single cubic equation for the two possible flow depths in the channel but do not solve the equation. The roots to your cubic equation are (1.275, 0.468, -0.342). Label and describe the two possible solutions on the specific energy diagram below. (10 points)



b) Suppose the channel now transitions from a width of b_1 = 2 m to b_2 = 3 m. Calculate the new alternate depths are and show which one will be chosen depending on the choice of upstream depth (i.e. you should have two answers and two explanations). The roots to your new cubic equation are (1.350, 0.285, -0.235). Label these on your diagram above. (10 points)



c) What will happen to the velocity in the channel if it gets wider and wider? Explain your reasoning on the energy diagram. (5 points)

d) Sketch what a velocity probe (like the acoustic doppler velocimeter (ADV) which you used in lab 4) would record if it were placed in the channel. Plot u vs. time. Also plot <u> vs. depth where < > denotes a time average. Label/describe your plots. (5 points)