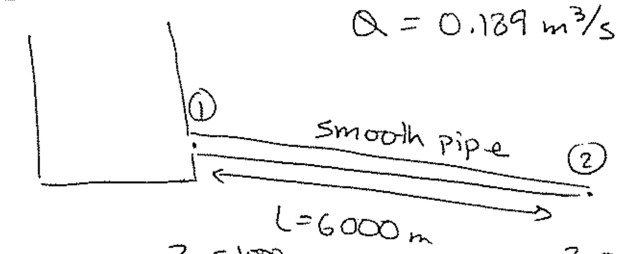
CE 100 Final exam solutions Fall 2006

Problem 1

$Q = 0.189\ m^3/s$



$z_1 = 1000 m$ $\qquad$ $z_2 = 0$

$P_1 = 1000\ kPa$ $\qquad$ $P_2 = 0$

Constant pipe diameter

$\therefore\ V_1 = V_2$

a) $$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\gamma}}_{0} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2}_{0} + h_L$$

$$h_L = \frac{P_1}{\gamma} + z_1 = \frac{1000\,000\ N/m^2}{9810\ N/m^3} + 1000\ m = \underline{\underline{1101.9\ m}}$$ total head loss

b) $$h_L = f\frac{L}{D}\frac{V_2^2}{2g} \quad \text{and} \quad Q = V_2\left(\frac{\pi D^2}{4}\right)$$

$$Re = \frac{\rho V D}{\mu} = \frac{4\rho Q D}{\pi D^2 \mu}$$

$$Re = \frac{4(1000)(0.189)}{\pi\,(1.12\times10^{-3})\,D}$$

$$Re = \frac{214859}{D}$$

$$\therefore\ h_L = f\frac{L}{D}\frac{1}{2g}\frac{Q^2}{\left(\frac{\pi D^2}{4}\right)^2} = 8f\frac{L}{D}\frac{Q^2}{g\pi^2 D^4}$$

$$D^5 = \frac{8}{\pi^2} f \frac{L}{g}\frac{Q^2}{h_L}$$

$$D = \left(\frac{8}{\pi^2}\frac{6000\ m}{9.81\ m/s^2}\frac{(0.189\ m^3/s)^2}{1101.9\ m} f\right)^{1/5} = (0.0161\ f)^{1/5}$$

Since $f = \phi(Re, \varepsilon/D)$, must iterate

① Choose $f = 0.02$, then $D = 0.20 m \rightarrow Re = \frac{214859}{D} = 1.07\times10^6$

For this value of Re, the smooth pipe curve gives $f \approx 0.0125$

② With $f = 0.0125$, again $\underline{\underline{D = 0.18 m}}$ so we have converged

$$Re = \frac{214859}{D} = 1.19\times10^6$$

Problem 2

a) A - blunt orientation $L = 1.5\,m$, $W = 0.5\,m$

C_D graph gives $C_D = 3$ (no Re # dependence)

$$F_D^A = \frac{1}{2}\rho U^2 C_D A = \frac{1}{2}\rho U^2 C_D L W = \frac{1}{2}(1.2\,kg/m^3)(19.4\,m/s)^2(3)(1.5\,m)(0.5\,m)$$

$$F_D^A = \underline{\underline{508.1\,N}}$$

B - streamlined orientation

C_D depends on Re $\rightarrow$ $Re = \frac{UD}{\nu} = \frac{(19.4\,m/s)(1.5\,m)}{1.5\times10^{-5}\,m^2/s} = 1.94\times10^{6}$

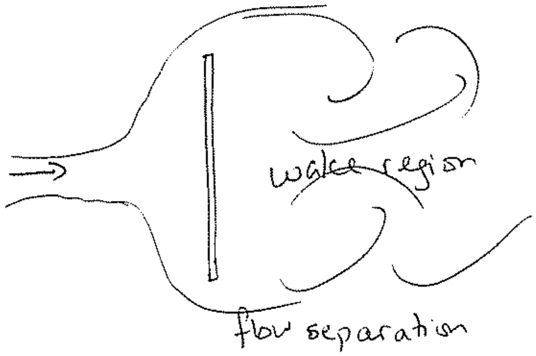
so $C_D \sim 0.007$

$$F_D^B = \frac{1}{2}\rho U^2 C_D L W = \frac{1}{2}(1.2)(19.4)^2(0.007)(1.5)(0.5) = \underline{\underline{1.19\,N}}$$

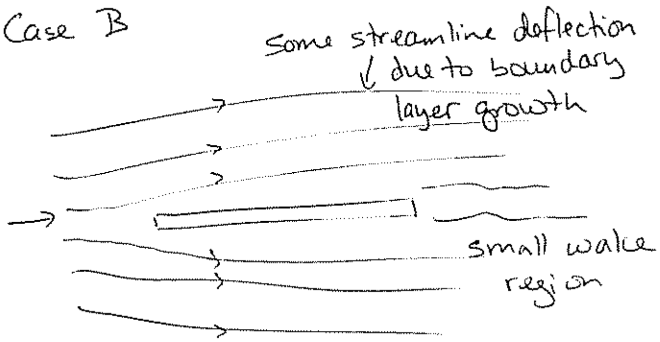
b) Case A (blunt orientation) – pressure drag dominates

Case B (streamlined orientation) - friction/viscous drag dominates

c) Case A



Case B



d) $F_D^{car} = \frac{1}{2}\rho U^2 C_D A = 0.5(1.2)(19.4)^2(0.4)(3.7) = \underline{334.2\,N}$

$$F_{total}^{A} = F_D^A + F_D^{car} = \underline{\underline{842.3\,N}}$$

$$F_{total}^{B} = F_D^B + F_D^{car} = \underline{\underline{335.4\,N}}$$

Problem 2 continued

e) <u>Case A</u>

$P_A = F_{total}^{A} \cdot U = 16.34 \text{ kW}$

<u>Case B</u>

$P_B = F_{total}^{B} \cdot U = 6.51 \text{ kW}$

Cost to operate vehicle in case B is smaller than A

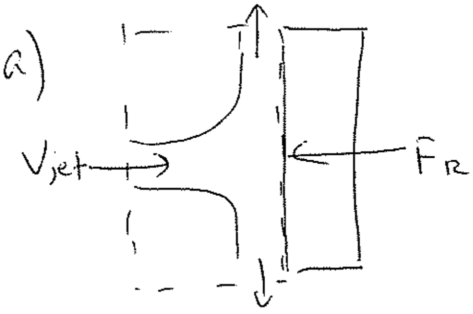
$$\frac{\text{Cost B}}{\text{Cost A}} = \frac{P_B}{P_A} = \frac{6.51 \text{ kW}}{16.34 \text{ kW}} = 0.40$$

Cost in orientation is 40% of cost in orientation B

(or cost B is 2.5 greater than cost A)

Problem 3

a)



P=0 for free jet

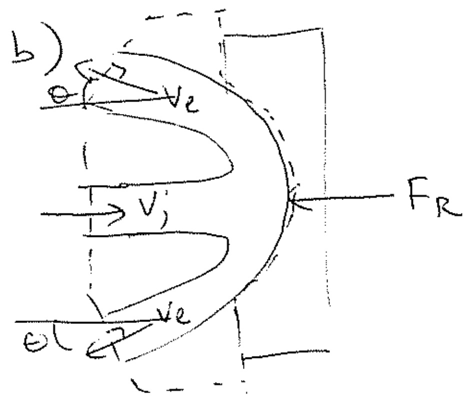
$$\Sigma F_x = \dot{m} V_{out} - \dot{m} V_{in}$$

$$-F_R = -\dot{m} V_{jet} \qquad \dot{m} = \rho Q = \rho V_{jet} A_{jet}$$

$$F_R = \rho V_{jet}^2 \left(\frac{\pi D_{jet}^2}{4}\right) = (1000\,kg/m^3)(8\,m/s)^2\left(\pi \frac{0.1\,m^2}{4}\right)$$

$$\underline{F_R = 502.7\,N} \text{ to the left}$$

b)



draw CV ⊥ to flow

$$\Sigma F_x = \dot{m} V_{out} - \dot{m} V_{in}$$ ← $V_{out,in}$ must be in x direction

$$-F_R = \dot{m}(-V_{out}\cos\theta - V_{in})$$

$$F_R = \rho Q (V_{out}\cos\theta + V_{in})$$

$$F_R = (1000\,kg/m^3)(8\,m/s)\left(\pi\frac{(0.1\,m)^2}{4}\right)(7\,m/s\cos\theta + 8\,m/s)$$

$$\underline{F_R = 439.8\cos\theta + 502.7}$$

For θ = 0 → $\underline{F_R = 942.5\,N}$ to the left
For θ = 45 → $\underline{F_R = 813.7\,N}$ to the left

c) Forces in part b must be larger to cause the increased change of momentum in the water jet (~~force~~ to make the fluid completely reverse its direction (or momentum) takes more force than to deflect it at 90° as in part a). The largest force requirement is for θ=0 where the momentum change is largest.

Problem 4 $\quad S_o = 0.0001 \quad n = 0.014\ s/m^{1/3}$

a) Channel 1 $\quad b = 1m,\ H = 3m$

$Q = \frac{A}{n} R_h^{2/3} S_o^{1/2} \quad R_h = A/P_w \quad A = bH = 3m^2 \quad P_w = b + 2H = 7m \quad R_h = 3/7\ m$

$Q = \frac{3m^2}{0.014\ s/m^{1/3}} (3/7\ m)^{2/3} (0.0001)^{1/2} = \underline{\underline{1.22\ m^3/s}}$

Channel 2 $\quad b = 3m,\ H = 1m \quad A = bH = 3m^2 \quad P_w = b + 2H = 5m$

$Q = \frac{A}{n} R_h^{2/3} S_o^{1/2} \quad R_h = 3/5\ m$

$Q = \frac{3}{0.014} (3/5)^{2/3} (0.0001)^{1/2} = \underline{\underline{1.52\ m^3/s}}$

b) For uniform flow the balance of forces is between gravity and friction (surface stresses along channel). In Channel 2 the wetted perimeter is smaller, giving a larger hydraulic radius and hence a greater flow rate.

c) Flow rate is determined by $Q = \frac{A}{n} S_o^{1/2} R_h^{2/3}$ so to maximize flow rate, we maximize R_h (since A is constant/fixed).

$R_h = A/P_w$ so to maximize R_h w/ A constant, we must minimize P_w (this reduces channel bed stress (friction)).

$P_w = b + 2H$ but $A = bH$ so $P_w = A/H + 2H$ (A is constant).

Minimize $P_w \rightarrow \frac{\partial P_w}{\partial H} = -A/H^2 + 2 = 0 \Rightarrow A = 2H^2$

$Hb = 2H^2 \Rightarrow \underline{\underline{b = 2H}}$

or you can minimize with respect to b:

$P_w = b + 2H = b + 2A/b$

$\frac{\partial P_w}{\partial b} = 1 - 2A/b^2 = 0 \Rightarrow b^2 = 2A$

$b^2 = 2Hb$

$\underline{\underline{b = 2H}}$

$\underline{b = 2H \text{ is optimal aspect ratio}}$

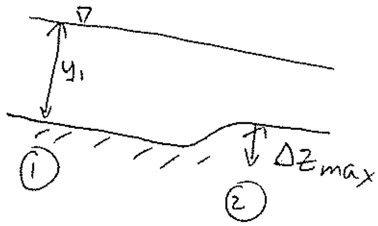
Problem 4

d) for $H = 1m$, we want $b = 2H = 2m$ $\rightarrow R_h = A/P_w$ $P_w = b + 2H = 4m$

thus $Q = \frac{A}{n} S_o^{1/2} R_h^{2/3}$ $R_h = \frac{2m^2}{4m} = 0.5m$

$$Q = \frac{(2)}{0.014} (0.0001)^{1/2} (0.5)^{2/3} = 0.90 \ m^3/s$$

Therefore $q = Q/b = \frac{0.9 \ m^3/s}{2 \ m} = 0.45 \ m^2/s$

e) 

The max allowable step size for this flow rate is Δz_{max} which will make the flow critical over the bump.

Given $q = 0.45 \ m^2/s$, $y_1 = H = 1m$

The critical depth is given by $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{0.45^2}{9.81}\right)^{1/3} = 0.274m$

Specific energy at 1 is $E_1 = y_1 + \frac{q^2}{2gy_1^2} = 1 + \frac{(0.45)^2}{2(9.81)(1)^2} = 1.01 \ m$

Energy at critical depth is $E_{min} = \frac{3y_c}{2} = 0.411 \ m$

$E_1 = E_2 + \Delta z_{max}$

$E_1 = E_{min} + \Delta z_{max}$ $\rightarrow$ $\underline{\Delta z_{max} = 0.60 \ m}$