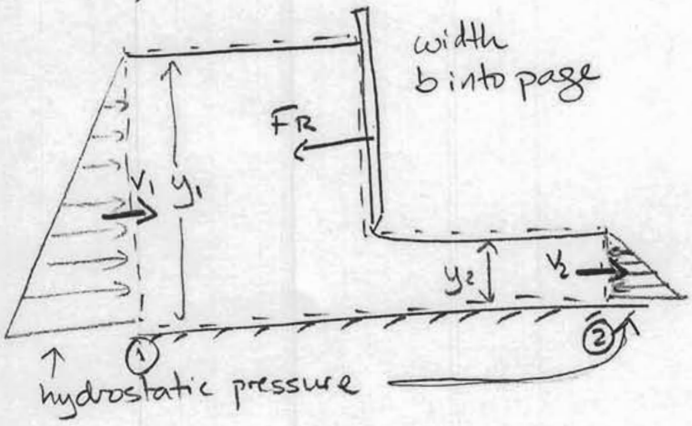
CE 100 Final Exam Solutions - Fall 2005

1 a)



$y_1 = 2m$ $\quad V_1 = ?$

$y_2 = 0.5\,m$ $\quad V_2 = ?$

Find force on sluice gate.

Use energy equation for open channel flow:

$$E_1 = E_2 \qquad \left(E = y + \frac{q^2}{2gy^2} = \text{specific energy}\right)$$

$$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2}$$

Solve for q^2: $\quad q^2 = \dfrac{y_2 - y_1}{\left(\dfrac{y_2^2 - y_1^2}{2gy_1^2 y_2^2}\right)} = \dfrac{2gy_1^2 y_2^2}{y_2 + y_1} = \dfrac{2(9.81)(2)^2(0.5)^2}{2+0.5}$

$$q^2 = 7.8\ m^4/s^2 \implies q = 2.8\ m^2/s$$

Now get velocities: $\quad V_1 = q/y_1 = 1.4\ m/s$

$V_2 = q/y_2 = 5.6\ m/s$ $\quad$ (from cons. of mass $q_1 = q_2 = y_1V_1 = y_2V_2$)

Get force on gate from conservation of momentum:

$$\Sigma F_x = \sum_{out} \dot{m}V - \sum_{in} \dot{m}V$$

$$P_1A_1 - P_2A_2 - F_R = V_2(\rho V_2 y_2 b) - V_1(\rho V_2 y_2 b)$$

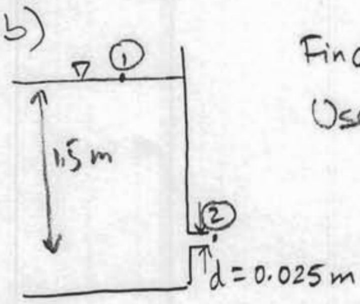
$$\tfrac{1}{2}\gamma y_1^2 b - \tfrac{1}{2}\gamma y_2^2 b - F_R = (V_2 - V_1)(\rho V_2 y_2 b)$$

$$-F_R = \tfrac{1}{2}\gamma b(y_2^2 - y_1^2) + (V_2 - V_1)(\rho V_2 y_2 b)$$

$$F_R = -\tfrac{1}{2}(9810)(1)(0.5^2 - 2^2) - (5.6 - 1.4)(1000 \cdot 5.6 \cdot 0.5 \cdot 1) = 6.63 \times 10^3 N$$

Force on gate equal and opposite: $\underline{\underline{6.63\ kN}}$ (to the right)

1 b)



Find flowrate out of nozzle.

Use extended Bernoulli eqn (energy eqn.):

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + z_2 + h_L$$

0 atmospheric, 0 large tank, 0 atmospheric

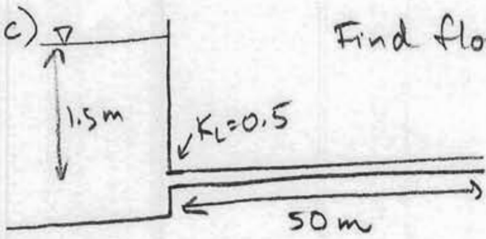
$$V_2^2 = 2g(z_1 - z_2) - h_L$$

$$V_2^2 = 2g(z_1 - z_2) - K_L \frac{V_2^2}{2g}$$

$$V_2^2 = \frac{2g\,\Delta z}{1 + K_L} = \frac{2(9.81)(1.5)}{1 + 0.5} = 19.6\ m^2/s^2$$

Therefore $V_2 = 4.43\ m/s$ and $Q = V_2 \frac{\pi d^2}{4} = \underline{\underline{2.2 \times 10^{-3}\ m^3/s}}$

1 c)



Find flow rate out of 50-m long pipe:

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$\Delta z = \frac{V_2^2}{2g}\left(1 + f\frac{L}{D} + K_L\right)$$

frictional losses in pipe

$$\varepsilon/D = \frac{0.5\ mm}{25\ mm} = 0.02$$

Need to get f from Moody chart, but $f = \phi(Re, \varepsilon/D)$ and we don't know Re yet, so iterate. Choose $f = 0.048$ from fully turbulent (flat line) section of diagram. Then $V_2 = 0.55\ m/s$ and $Re = \frac{\rho V d}{\mu} = 1.2 \times 10^4$. For this Re, $f = 0.051$, which gives $V_2 = 0.53\ m/s$ and $Re = 1.2 \times 10^4$ so we have converged.

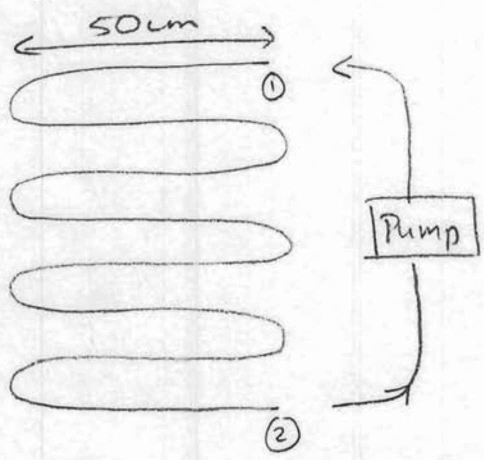
$$\leftarrow V_2^2 = \frac{2g\,\Delta z}{1 + f L/D + K_L}$$

$V_2 \approx 0.53\ m/s$

$\Rightarrow Q = \frac{\pi d^2}{4} V_2 = \underline{\underline{2.6 \times 10^{-4}\ m/s}}$

(much less than part b due to frictional losses in pipe)

2 a)



Find velocity to get Reynolds #.

$$Q = 4\ L/min = \frac{0.004\ m^3}{60\ s} = 6.7 \times 10^{-5}\ m^3/s$$

$$V = \frac{Q}{\pi D^2/4} = \frac{6.7\times10^{-5}}{\pi (0.01)^2/4} = 0.85\ m/s$$

$$Re = \frac{\rho V D}{\mu} = \frac{1327 \cdot 0.85 \cdot 0.01}{2.62\times10^{-4}} = 43000$$

turbulent flow

$\varepsilon = 1\times10^{-6}\ m$, $D = 0.01\ m \rightarrow \varepsilon/D = 0.0001$

To get f (friction factor), use Moody chart: look up $\varepsilon/D = 0.0001$ curve and Re = 43000 to find $f \approx 0.022$

b) Find total head loss through radiator coils by applying energy equation from 1 to 2:

$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2} + h_L$$

neglect elevation changes
velocities equal by conservation of mass

$$\frac{P_1 - P_2}{\gamma} = h_L = f\frac{L}{D}\frac{V^2}{2g} + \sum K_L \frac{V^2}{2g}$$

$$h_L = \frac{V^2}{2g}\left(f\frac{L}{D} + 7K_{L_{bend}} + K_{L_{inlet}} + K_{L_{outlet}}\right)$$

$$h_L = \frac{(0.85\ m/s)^2}{2(9.81\ m/s^2)}\left(0.022\,\frac{4m}{0.01m} + 7(1.5) + 0.5 + 0.5\right)$$

$$h_L = 0.75\ m$$

2b continued:

By applying the energy equation from point 2 to point 1 across the pump, we see that the head loss must be equal to the head from the pump:

$$\frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2} + h_p = \frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1}$$

$$h_p = \frac{P_1 - P_2}{\gamma}$$

From above we have $h_L = \frac{P_1 - P_2}{\gamma}$ so $h_p = h_L$

and the power consumption is $\dot{W}_{pump} = \gamma Q h_L$

$$= 9.81 \cdot 1327 \cdot 6.7\times10^{-5} \cdot 0.75$$

$$\dot{W}_{pump} = \underline{0.65 \text{ Watts}}$$

c) New design: $L = 8\ m$, $D = 0.005\ m$ $\quad \varepsilon/D = \frac{1\times10^{-6} m}{0.005} = 0.0002$

$Q = 6.7\times10^{-5}\ m^3/s \rightarrow V = \frac{Q}{\pi D^2/4} = \frac{6.7\times10^{-5}}{\pi \frac{0.005^2}{4}} = 3.4\ m/s$

$\therefore Re = \frac{\rho V D}{\mu} = 8.6\times10^4$, so the new friction factor from the Moody chart is $f \approx 0.0195$.

$$h_L = \frac{3.4^2}{2g}\left(0.0195 \cdot \frac{8m}{0.005} + 7(1.5) + 0.5 + 0.5\right) = 25.2\ m$$

$W_{pump} = \gamma Q h_L = \underline{21.9 \text{ Watts}}$

At lower Reynolds #, the friction factor is higher, leading to greater losses. Also the pipe length is 2x bigger and the width is 2x smaller so frictional losses are much greater.

3a) $y_1 = 1\,m$, $V_1 = 0.5$ $\rightarrow$ $Fr_1 = \dfrac{V_1}{\sqrt{g y_1}} = 0.16 < 1$

$\therefore$ flow is subcritical

b) Use energy equation for open channel flow:

$$E_1 = E_2 - \Delta z$$

$$E_1 = y_1 + \frac{q^2}{2 g y_1^2} = y_2 + \frac{q^2}{2 g y_2^2} - \Delta z$$

E_1 and Δz are given, $q = y_1 V_1 = 0.5\ m^2/s$, so solve for y_2:

$$(E_1 + \Delta z) = y_2 + \frac{q^2}{2 g y_2^2}$$

$$\boxed{y_2^3 - \underbrace{(E_1+\Delta z)}_{1.2127} y_2^2 + \underbrace{\frac{q^2}{2g}}_{.0127} = 0}$$ cubic equation for y_2

Also use cons. of mass to get V_2 $\rightarrow$ $V_2 = q/y_2$

c) The roots of the equation are $[1.204,\ 0.1074,\ -0.0986]$

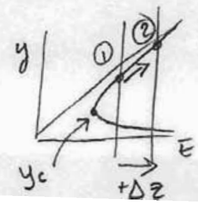
Since the flow at 1 is subcritical, it will select the subcritical depth at 2 since the flow does not pass through critical.

The negative root is unphysical. Of the other 2 choices, the resulting Froude #s are $Fr_{2a} = \dfrac{q/1.204}{\sqrt{g \cdot 1.204}} = 0.12$ and $Fr_{2b} = \dfrac{q/0.1074}{\sqrt{g \cdot 0.1074}} = 4.5$

subcritical   supercritical

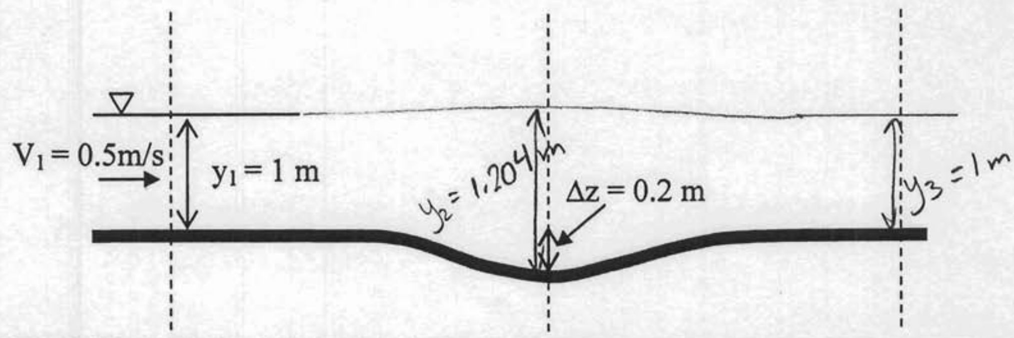
so $y_2 = 1.204$ m is the subcritical depth

$\therefore$ $V_2 = \dfrac{0.5}{1.204} = 0.41\ m/s$



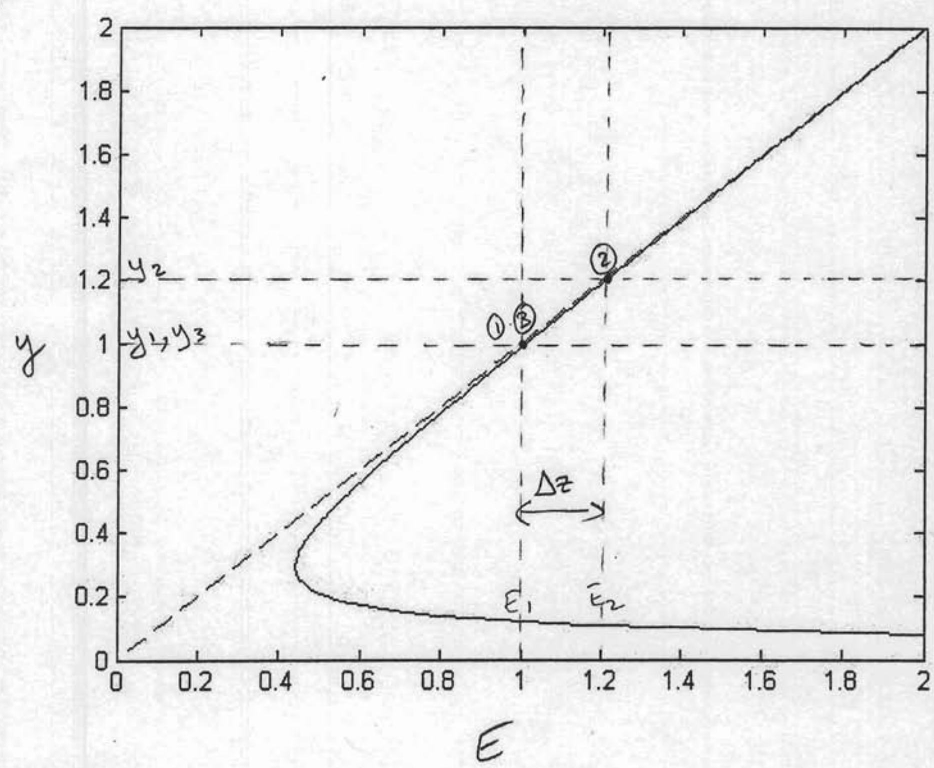
Note: the specific energy increases from 1 to 2 ($E_2 = E_1 + \Delta z$) so there is no way for this flow to pass through critical.

d)



$V_1 = 0.5$m/s
$y_1 = 1$ m
$y_2 = 1.204$ m
$\Delta z = 0.2$ m
$y_3 = 1$ m

e)



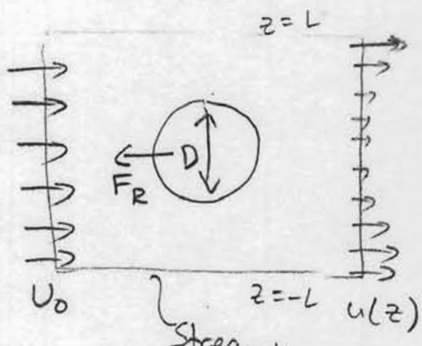
y_2
y_1, y_3
1
3
2
Δz
E_1
E_2
y
E
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

4 a) Cons. of momentum in x-direction to find drag force on cylinder

$$\Sigma F_x = \cancel{\frac{\partial}{\partial t}\int_{cv} \rho V_x d\forall} + \int_{cs} \rho V_x (\underline{v}\cdot\underline{n})\, dA$$

$\underset{\text{steady}}{0}$

$$-F_R = \underbrace{\int_{-L}^{L} -\rho U_o^2 H dz}_{\text{inflow cross-section}} + \underbrace{\int_{-L}^{L} \rho u^2(z) H dz}_{\text{outflow cross-section}}$$



$$F_R = 2\rho U_o^2 HL - H\rho \int_{-L}^{L} u^2(z)\, dz$$

reaction force acting to left

Drag on cylinder is $\boxed{F_D = 2\rho U_o^2 HL - H\rho \int_{-L}^{L} u^2(z)\, dz}$ acting to right

b) Find average velocity at section 2 and relate to U_o

$$u_{avg \text{ at } ②} = \frac{H\int_{-L}^{L} u(z)\, dz}{\underbrace{2HL}_{\text{area of cross section}}}$$

From conservation of mass: $\underbrace{U_o H(2L)}_{\text{inflow}} = \underbrace{H\int_{-L}^{L} u(z)\, dz}_{\text{outflow}}$

$$\therefore\ u_{avg \text{ at } ②} = \frac{U_o H(2L)}{2HL} = \underline{\underline{U_o}}$$

which must be the case since mass is conserved.