CE 100 - Final Exam

December 13, 2005

Name ____

Student I.D. ______________________________________

This exam is closed book. You are allowed three sheets of paper (8.5” x 11”, both sides) of your own notes. You **must** turn in your note sheets with your exam. Make sure your name is on all sheets.

You will be given three hours to complete four problems. Write out your solution with symbols before plugging in any numbers and clearly state any assumptions you make, and please box your answer! Read through the whole exam first and skip ahead to the easy parts when you get stuck.

Good Luck!

On all problems, you may assume that the fluid is water, unless otherwise noted. For your reference:

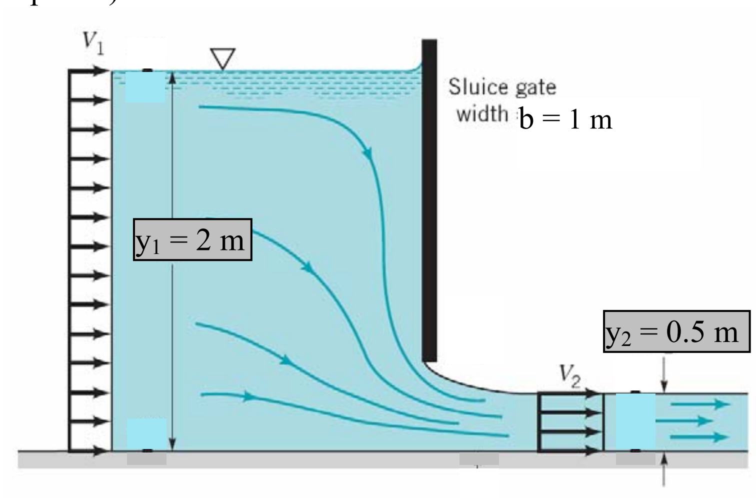
Density of water = ρ_w = 1000 kg/m^3
Dynamic viscosity of water = μ = 1.12 x 10^{-3} N·s/m^2
Atmospheric Pressure = p_{atm} = 100 kPa
Gravitational Acceleration = g = 9.81 m/s^2

The Moody chart can be found on the last page of the exam.

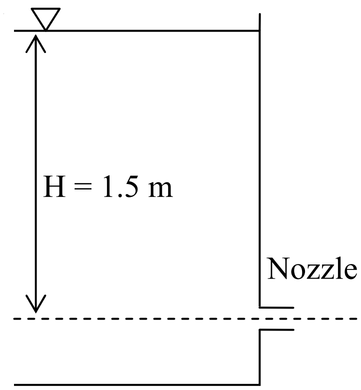
Problem 1 (30 points): Roman aqueducts

Ancient Rome was one of the first cities to develop a central water supply system. Aqueducts were used to transfer the water from the hills to reservoirs inside Rome, after which pipes were used to distribute water further. We will consider this water supply system piece by piece.

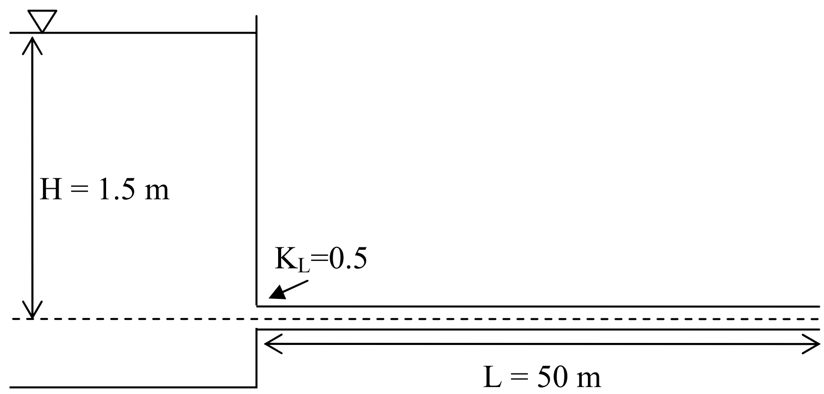
a) A sluice gate was used to regulate the flow along the aqueduct or to close the channel for repairs. Calculate the force on the sluice gate for the conditions shown below. (17 points)



b) Now consider flow from the reservoir tank inside Rome. If the tank discharges directly to the atmosphere through a nozzle, calculate the volume flow rate if the loss coefficient is K_L=0.5 given a 1.5 m head in the tank and an exit diameter of 25 mm. The reservoir is very large compared to the diameter of the nozzle. (5 points)



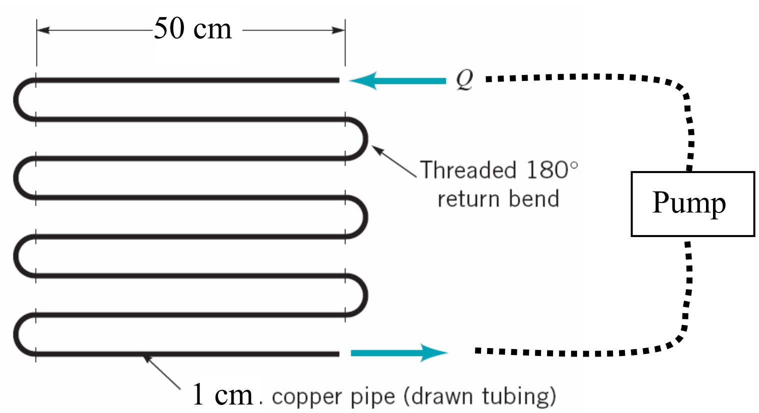
c) Now consider the flow when a 50 m long pipe is connected to the reservoir after the nozzle, as drawn below. The diameter of the pipe is 25 mm. Assume the pipe has an equivalent roughness of 0.5 mm. What is the new volume flow rate exiting the pipe? (8 points)



Problem 2 (25 points): Fluid flow in refrigeration coil

At the back of any common refrigerator you will find a set of radiator coils, as shown below. The condensed liquid refrigerant is pumped through this coil at a steady flow rate and is cooled, thus releasing heat. The key design goal from a thermodynamic point of view is to achieve a large surface area over which waste heat can be radiated to the surroundings in a compact design. From a fluid mechanics perspective, this design results in large major and minor losses resulting from viscous flow in the pipe forming the radiator coil.

Suppose a refrigerator requires 4 liters/min of Freon (R-12) to be pumped through a 4 m long pipe of drawn copper tubing (equivalent roughness $\varepsilon = 1 \times 10^{-6}$ m) with 1 cm diameter arranged in a pattern of 8 straight sections of length 50 cm connected by 180° bends. You are asked to calculate the power consumption of the electric pump required to circulate the refrigerant. The viscosity of Freon is $\mu = 2.62 \times 10^{-4}$ N·s/m^2 and its density is 1327 kg/m^3. You can ignore thermal variations in these properties and elevation differences.



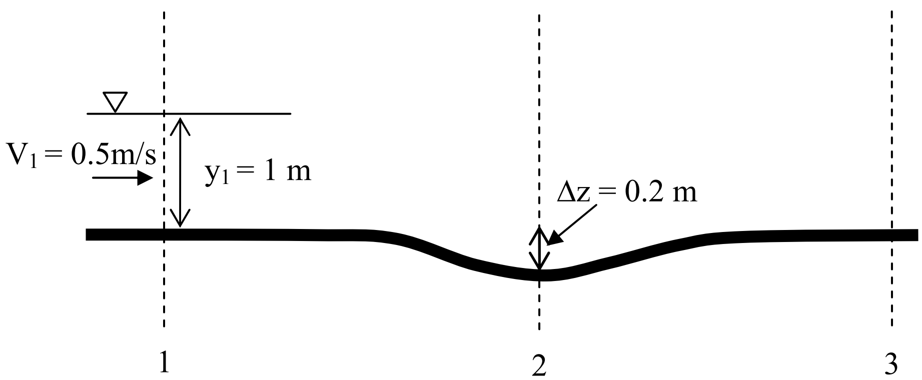
a) First calculate the Reynolds number through the pipe and decide if the flow is laminar or turbulent. Determine the friction factor for flow through the pipe. (5 points)

b) The loss coefficient for each 180° bend is $K_L = 1.5$ and for the inlet and outlet from the pump $K_L = 0.5$. Calculate the total head loss through the radiator coils and thus find the total electric power consumption of the pump to keep the flow going (assume it 100% efficient). Neglect the length of pipe needed to connect the pump to the coils. (15 points)

c) A new design suggestion requires replacement of the radiator coil by an 8 m long pipe of 0.5 cm diameter which has the same surface area for heat transfer and therefore requires the same flow rate. Determine what the new power consumption will be based on your analysis above and comment briefly on the reasons for the difference. (5 points)

Problem 3 (25 points): Flow over a trench

Consider the flow transition shown below, where the flow encounters a smooth trench (0.2 m deep). You wish to determine the velocity (V_2) and depth (y_2) over the trench. The channel is of rectangular cross section and uniform width (b = 1 m). You can assume that the velocity profile is uniform in the vertical.



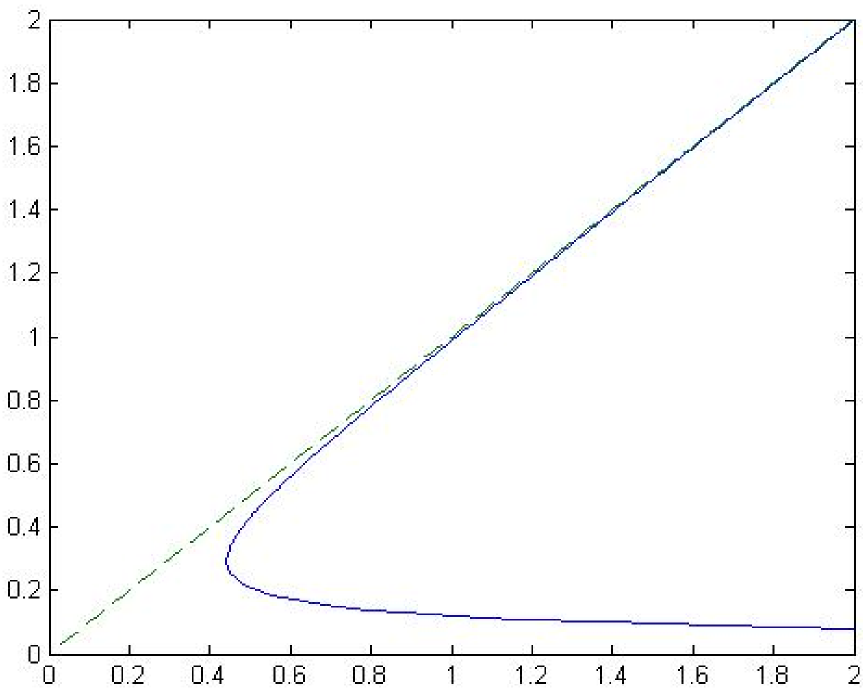
a) Is the flow at section 1 subcritical or supercritical? (2 points)

b) Write the equations you need to solve to determine V_2 and y_2 for the conditions shown. Modify them to produce a single cubic equation for the depth but do not solve the equation. (8 points)

c) The roots to your cubic equation from part (b) are (1.2040, 0.1074 , -0.0986). Select the appropriate root and briefly explain your choice. State your final answer for depth and velocity in the trench. (6 points)

d) Sketch the free surface at sections 2 and 3 on the diagram above (your drawing doesn't need to be to scale). (4 points)

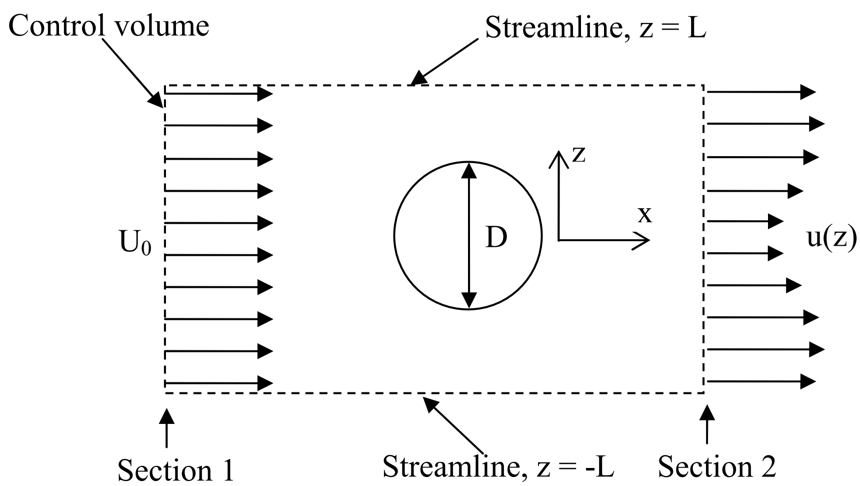
e) Show the depths at points 1, 2 and 3 on the specific energy diagram below. Label Δz and label the axes of the diagram. (5 points)



Problem 4 (20 points): Drag on a bridge pier

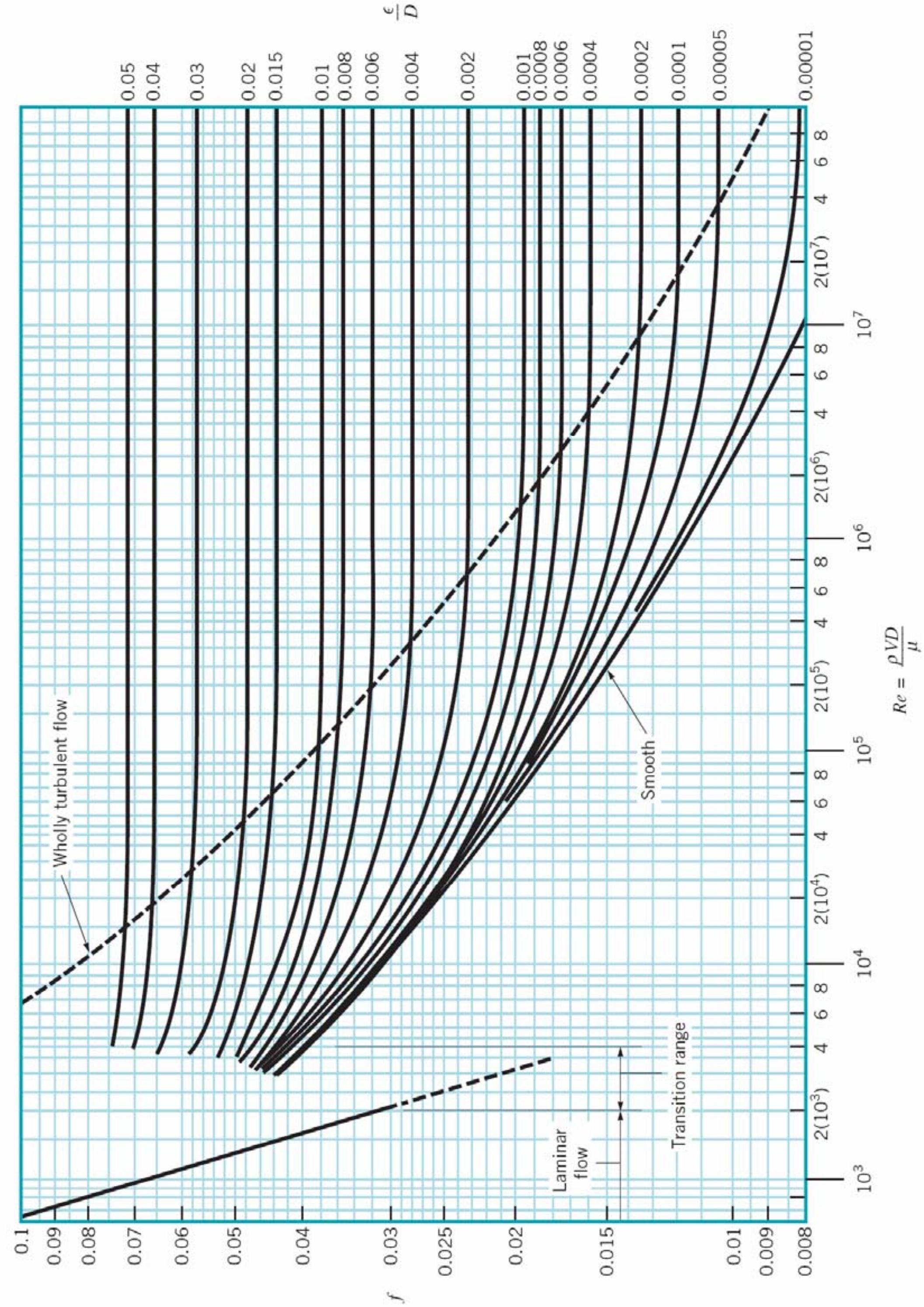
You wish to calculate the drag on the circular cylindrical bridge pier shown below. In lab #5, you calculated the drag force on a cylinder by measuring the pressure distribution around the cylinder. Since it's not easy to put pressure taps on a concrete bridge pier, you decide to perform a momentum balance using the velocity distributions far upstream and downstream of the pier. In your preliminary analysis, assume that the channel walls are far away and do not affect the flow around the pier in any way.

The velocity far upstream is uniform at U_0 and the downstream velocity is measured to be u(z), and varies with z because it is affected by the wake of the cylinder. For simplicity, assume that the total pressure forces at sections 1 and 2 are the same. Also assume that the edges of the control volume at z = L and z = -L are along straight streamlines.



a) Use the setup shown above to perform a momentum balance for a cylinder of height H (into the page) using the control volume shown to find an analytical (integral) expression for the drag force exerted on the bridge pier. In which direction does the drag force act? (15 points)

b) Use conservation of mass to find the average velocity at section 2 and compare it to the upstream velocity. Show your work. (5 points)



$\frac{\epsilon}{D}$
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0.0001
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0.00001
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0.009
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10^3
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10^4
$2(10^4)$
10^5
$2(10^5)$
10^6
$2(10^6)$
10^7
$2(10^7)$
4
6
8
$Re = \frac{\rho VD}{\mu}$
Wholly turbulent flow
Laminar flow
Transition range
Smooth