UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
Spring 2011

Structural Engineering,
Mechanics and Materials
Professor: S. Govindjee

CE 130N
Midterm Exam
March 10, 2010
50 minutes

Closed Book
Closed Notes
No Calculators

Problem	Score
#1	/30
#2	/50
#3	/20
Total	/100

SOLUTION!

Name

SID

1. True or False, Fill in the Blank (NO justifications required!). [3 pts each].

   (a) True or False: A conservative mechanical system is in an equilibrium state if and only if its potential energy is stationary.

   (b) True or False: In beam bending, the delta function is used to represent any applied point moments along the length of the beam.

   (c) True or False: The governing differential equation for the tension-compression bar is $\frac{d^2}{dx^2}(AE\frac{d^2u}{dx^2}) + b = 0$

   (d) A point mass moving in the (x, y)-plane is acted upon by a conservative force whose potential is given as $\Pi(x, y)$. The force acting on the mass is $\boldsymbol{F}$ = $\partial \Pi(x,y)/\partial x\ \vec{i} + \partial \Pi(x,y)/\partial y\ \vec{j}$

   (e) True or False: Given a built-in beam at $x = 0$, the appropriate boundary conditions are $v(0) = v'''(0) = 0$.

   (f) True or False: To use `bvp4c` in MATLAB, a problem must first be converted to 2nd order form.

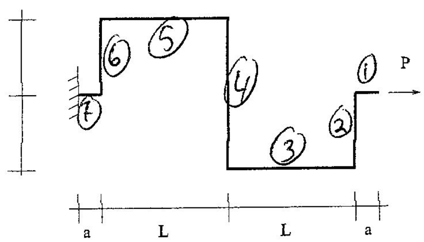
   (g) The (scalar) dimensions of the matrix $\boldsymbol{A}$ in the equation $\boldsymbol{\varepsilon} = \lceil 1/L \rfloor \boldsymbol{A}\boldsymbol{u}$ are #BARS rows and 2× # NODES columns.

   (h) True or False: A linear elastic cube with side length a mm under a pure shear state of loading $\tau_{xy} = 4$ N/mm^2 and deformation $\gamma_{xy} = 0.001$ has stored elastic energy $W_s = 2 \times 10^{-3} a^3$ N · mm.

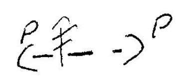
   (i) True or False: The potential energy associated with a dead-load force is $-\frac{1}{2}P\Delta$.

   (j) True or False: Consider a force $F(t) = F_o t$ acting at a point on an elastic bar that moves with velocity $v(t) = v_o t$ where F_o, v_o are given constants. The power of the load is $\frac{1}{3}v_o F_o t^3$

2. A *slender* metal band with constant EI, GJ, and $\frac{1}{\alpha}AG$, as shown, is subjected to a force P. Find the horizontal deflection at the point where the load is applied using conservation of energy.



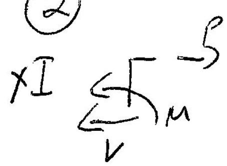
①



$$W_{I_{①}} = \int_0^a \frac{1}{2}\frac{P^2}{EA}\,dx = \boxed{\frac{P^2 a}{2EA}} \approx 0$$

SLENDER

②



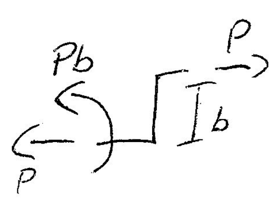
$V = P$

$M = Px$

$$W_{I_{②}} = \alpha\int_0^b \frac{1}{2}\frac{P^2}{AG}\,dx + \int_0^b \frac{1}{2}\frac{(Px)^2}{EI}\,dx$$

$$\boxed{W_I = \alpha\frac{P^2 b}{2AG} + \frac{P^2 b^3}{6EI}}$$

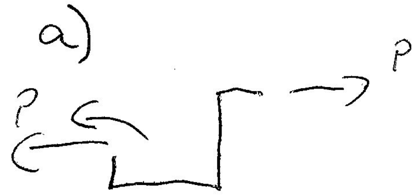
③

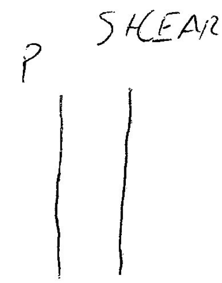


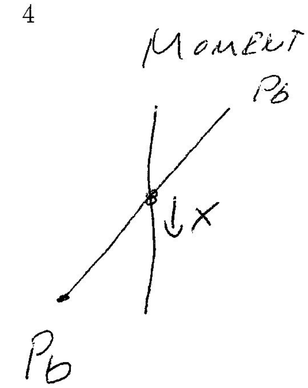
$$W_I = \int_0^L \frac{1}{2}\frac{P^2}{EA}\,dx + \int_0^L \frac{P^2 b^2}{2EI}\,dx$$

$$= \boxed{\frac{P^2 L}{2EA} + \frac{P^2 b^2 L}{2EI}}$$

(4) a)







$$W_I = \alpha \int_0^{2b} \frac{P^2}{2AG}\,dx + 2\cdot\int_0^{b} \frac{(Px)^2}{2EI}\,dx$$

$$= \frac{\cancel{2}\alpha P^2 b}{\cancel{2}AG} + \frac{2P^2 b^3}{6EI}$$

$$= \boxed{\frac{\alpha P^2 b}{AG} + \frac{P^2 b^3}{3EI}}$$

(5) SIMILAR TO (3)

$$W_I = \frac{P^2 L}{2EA} + \frac{P^2 b^2 L}{2EI}$$

(6) SIMILAR TO (2)

$$W_I = \frac{\alpha P^2 b}{2AG} + \frac{P^2 b^3}{6EI}$$

7 IS SIMILAR TO 1

$$W_I = \frac{P^2 a}{2EA}$$

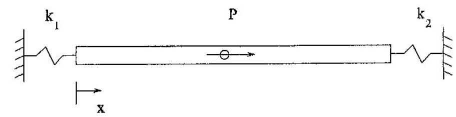
$$W_I = \overset{1}{\frac{P^2 a}{2EA}} + \overbrace{\frac{\alpha P^2 b}{2AG} + \frac{P^2 b^3}{6EI}}^{2} + \overbrace{\frac{P^2 L}{2EA} + \frac{P^2 b^2 L}{2EI}}^{3}$$
$$+ \overbrace{\frac{\alpha P^2 b}{AG} + \frac{P^2 b^3}{3EI}}^{4} + \overbrace{\frac{P^2 L}{2EA} + \frac{P^2 b^2 L}{2EI}}^{5}$$
$$+ \overbrace{\frac{\alpha P^2 b}{2AG} + \frac{P^2 b^3}{6EI}}^{6} + \overbrace{\frac{P^2 a}{2EA}}^{7}$$

$$W_I = \frac{P^2 a}{EA} + \frac{2\alpha P^2 b}{AG} + \frac{2P^2 b^3}{3EI} + \frac{P^2 L}{EA} + \frac{P^2 b^2 L}{EI}$$

$W_E = \frac{1}{2} P \Delta$ AND $W_I = W_E$

$$\Delta = \frac{2Pa}{EA} + \frac{4\alpha P^2 b}{AG} + \frac{4Pb^3}{3EI} + \frac{2PL}{EA} + \frac{2Pb^2 L}{EI}$$

3. Consider an elastic bar with length L and constant AE that is subjected to a point force at $x = a$. Find $u(x)$.



BOUNDARIES

$R(0) = K_1 U(0)$

$R(L) = -K_2 U(L)$

$$AE\frac{d^2U}{dx^2} + b = 0$$

$$AE\frac{d^2U}{dx^2} = -P\delta(x-a)$$

$$AE\frac{dU}{dx} = -P\langle x-a\rangle^0 + C_1$$

$$AE\,U = -P\langle x-a\rangle + C_1 x + C_2$$

$$U(0) = \frac{C_2}{AE} \qquad R(0) = \frac{K_1 C_2}{AE} = C_1 \quad (1)$$

$$U(L) = \frac{-P(L-a) + C_1 L + C_2}{AE} \qquad R(L) = -K_2(U(L)$$

$$-P + C_1 = -K_2\left[\frac{-P(L-a) + C_1 L + C_2}{AE}\right] \quad (2)$$

FROM 1

$$C_2 = \frac{C_1 AE}{K_1}$$ THEN

$$-P + C_1 = \frac{K_2 \cdot P(L-a)}{AE} - \frac{K_2 C_1 L}{AE} - \frac{K_2 C_1}{K_1}$$

$$C_1\left(1 + \frac{K_2 L}{AE} + \frac{K_2}{K_1}\right) = P + \frac{K_2 P(L-a)}{AE}$$

$$C_1 = \frac{P + \frac{K_2 P(L-a)}{AE}}{1 + \frac{K_2 L}{AE} + \frac{K_2}{K_1}}$$

$$C_2 = \frac{AE}{K_1}\left[\frac{P + \frac{K_2 P(L-a)}{AE}}{1 + \frac{K_2 L}{AE} + \frac{K_2}{K_1}}\right]$$

THEN

SEE OTHER PAGE →

$$U(x) = -\frac{P\langle x-a\rangle}{AE} + \overbrace{\left[\frac{P + \frac{K_2 P(L-a)}{AE}}{1 + \frac{K_2 L}{AE} + \frac{K_2}{K_1}}\right]}^{C_1} x + \overbrace{\frac{AE}{K_1}\left[\frac{P + \frac{K_2 P(L-a)}{AE}}{1 + \frac{K_2 L}{AE} + \frac{K_2}{K_1}}\right]}^{C_2}$$

SIMPLER ANSWER ARE POSSIBLE WITH SOME ALGEBRA.