EXAM 2 (100 Points, Show All Work)

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Phy 7B
8-6-02

8 Points

1. The current in an electromagnet connected to a 240 V line is 14.5 A. At what rate must cooling water pass over the coils if the water temperature is to rise by no more than 6.5° C?

12 Points Each

2. A copper rod if length, r, rotates at constant angular frequency, ω, in a uniform magnetic field, $\vec{B}$, as shown in Figure 1. Derive the emf, $\mathcal{E}$, developed between the two ends of the rotating rod.

3. The capacitor shown in Figure 2 is initially uncharged. Find the current through the battery
   a) Immediately after the switch is closed.
   b) A long time after the switch is closed.

4. Determine the net resistance of the network shown in Figure 3 between points a and c. Assume that R' = R. Hint: Use symmetry.

5. Starting with the result below for the magnetic field along the axis of a single loop, derive the magnetic field inside a very long solenoid (radius R). Please don't use Ampere's law. $dB = \frac{\mu_o n I}{2} dx \frac{R^2}{(R^2+x^2)^{3/2}}$

6. A smooth conducting sphere of radius r_o carries a charge Q. Half of the energy stored in its electric field is contained in a volume of what radius?

7. Find the direction of the following, as shown in Figure 4:
   a) What direction is the force on a long current carrying wire to right if $\vec{B}$ is out of page?
   b) What direction is the magnetic field if a negative charge is moving down and force to left?
   c) What direction is the magnetic field at a point to the right of the wire and in the plane of the page due to a long current carrying wire with a current direction up the page?

20 Points

8. A coaxial cable consists of a solid inner conductor of radius R_1, surrounded by a concentric cylindrical tube of inner radius R_2 and outer radius R_3. The conductors carry equal and opposite currents I_o, but the current density j varies linearly with distance from the center: $j_1 = c_1 r$ (inner conductor) and $j_2 = c_2 r$ (outer conductor). Determine the magnetic field for:
   a) $r < R_1$
   b) $R_1 < r < R_2$
   c) $R_2 < r < R_3$
   d) $r > R_3$

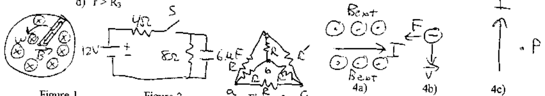

Figure 1 | Figure 2 | Figure 3 | Figure 4

Possibly Useful Constants

$k = 9 \times 10^9$ Nm²/C² $\mu_o = 4\pi \times 10^{-7}$ Tm/A $\varepsilon_o = 8.85 \times 10^{-12}$ C²/Nm² $c_{water} = 4186$ J/Kg-

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Exam 2

① Power Dissipated by Electromagnet

$$P = IV = \frac{mc\Delta T}{t} \Rightarrow \frac{m}{t} = \frac{IV}{c\Delta T} = \frac{(14.5A)(240V)}{4186 \frac{J}{kg^\circ C}(6.5^\circ C)}$$

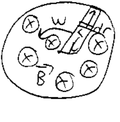
$$\Rightarrow \boxed{m/t = 0.128\ kg/s}$$

② Faraday's Law

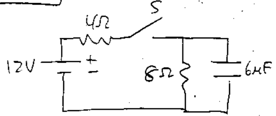
$$d\mathcal{E} = Bv\,dr$$

$$\mathcal{E} = \int d\mathcal{E} = \int_0^r Bv\,dr \quad , v = \omega r$$

$$\Rightarrow \mathcal{E} = \int_0^r B\omega r' dr' = \boxed{\tfrac{1}{2}B\omega r^2 = \mathcal{E}}$$



③ RC Circuit



a) $V_c = 0$ at $t = 0$

$$\Rightarrow 12V - I_0(4\Omega) = 0$$

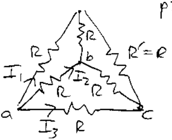
$$\Rightarrow I_0 = 12V/4\Omega = \boxed{3A = I_0}$$

b) $V_c = V_{max}$, $I_c = 0$

$$\Rightarrow 12V - I_f(4\Omega) - I_f(8\Omega) = 0$$

$$I_f = \frac{12V}{12\Omega} = \boxed{1A = I_f}$$

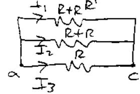
(4) Equivalent Resistance



Since all resistors are equal

Symmetry ⇒ I_1, I_2, I_3 leaving a, must be same as I_1, I, I_3 entering c.

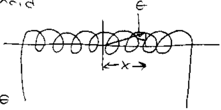
$\Rightarrow I_{top} = 0$



$$\frac{1}{R_{ac}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{2R} = \frac{4}{2R}$$

$$\Rightarrow \boxed{R_{ac} = R/2}$$

(5) Magnetic Field Inside Solenoid



let $dB = \frac{\mu_0 n I}{2} dx \frac{R^2}{(R^2+x^2)^{3/2}}$

use $x = R\tan\theta$, $dx = R\sec^2\theta\, d\theta$

$$\Rightarrow dB = \frac{\mu_0 n I}{2}\left(R\sec^2\theta\, d\theta\right)\frac{R^2}{(R^2+R^2\tan^2\theta)^{3/2}}$$

$$\Rightarrow dB = \frac{\mu_0 n I}{2}\frac{\sec^2\theta\, d\theta}{\sec^3\theta} = \frac{\mu_0 n I}{2}\cos\theta\, d\theta$$

long solenoid $\Rightarrow B = \int dB = \frac{\mu_0 n I}{2}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = \frac{\mu_0 n I}{2}\sin\theta\Big|_{-\pi/2}^{\pi/2}$

$$\Rightarrow \boxed{B = \mu_0 n I}$$

⑥ Energy Stored in Electric Field

$E = 0$ inside $(r < r_0)$ (Conductor)

$\Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$ for $r > r_0 \Rightarrow u = \frac{1}{2}\epsilon_0 E^2 = \frac{Q^2}{32\pi^2\epsilon_0 r^4}$

$U = \int u\,dV$

let $R \equiv$ Sphere radius that contains half the energy.

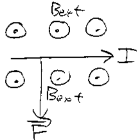
$$\frac{U}{U_{tot}} = \frac{1}{2} \Rightarrow \frac{\int_{r_0}^{R} \frac{Q^2}{32\pi^2\epsilon_0 r^4}(4\pi r^2 dr)}{\int_{r_0}^{\infty} \frac{Q^2}{32\pi^2\epsilon_0 r^4}(4\pi r^2 dr)} = \frac{1}{2}$$

$$\Rightarrow \frac{\int_{r_0}^{R} \frac{dr}{r^2}}{\int_{r_0}^{\infty} \frac{dr}{r^2}} = \frac{1}{2} \Rightarrow \frac{-\frac{1}{r}\Big|_{r_0}^{R}}{-\frac{1}{r}\Big|_{r_0}^{\infty}} = \frac{1}{2} \Rightarrow \frac{\frac{1}{R} - \frac{1}{r_0}}{0 - \frac{1}{r_0}} = \frac{R - r_0}{R} = \frac{1}{2}$$

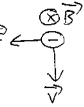
$$\Rightarrow \frac{1}{2}R = r_0 \Rightarrow \boxed{R = 2r_0}$$

⑦ Right Hand Rules

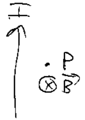
a)



b)



c)



$\vec{F}$ is down page

$\vec{B}$ is into page

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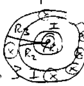

⑧ Ampere's Law – Coaxial Cable

$$I_o = \int_0^{R_1} J_1 (2\pi r\,dr) = 2\pi C_1 \int_0^{R_1} r^2\,dr = \frac{2\pi C_1 R_1^3}{3}$$

since $J_1 = C_1 r \Rightarrow C_1 = \frac{3 I_o}{2\pi R_1^3}$ ↖ (inner wire)

$$I_o = \int_{R_2}^{R_3} J_2 (2\pi r\,dr) = 2\pi C_2 \int_{R_2}^{R_3} r^2\,dr = \frac{2\pi C_2 (R_3^3 - R_2^3)}{3}$$

since $J_2 = C_2 r \Rightarrow C_2 = \frac{3 I_o}{2\pi (R_3^3 - R_2^3)}$ ↖ (outer wire)

Cylindrical Symmetry $\Rightarrow \vec{B}$ circular

a) $r < R_1$

$$\oint \vec{B}\cdot d\vec{s} = \mu_o I_{enc} \Rightarrow B(2\pi r) = \mu_o \int_0^r C_1 r (2\pi r\,dr)$$

$$\Rightarrow C_1 (2\pi \mu_o) r^3/3 = B(2\pi r) \Rightarrow B = \frac{\mu_o C_1 r^2}{3} = \frac{\mu_o r^2}{3}\left(\frac{3 I_o}{2\pi R_1^3}\right)$$

$$\Rightarrow \boxed{B = \left(\frac{\mu_o I_o}{2\pi R_1^3}\right) r^2 \text{ CCW } r < R_1}$$

b) $R_1 < r < R_2$

$$\oint \vec{B}\cdot d\vec{s} = \mu_o I_{enc} \Rightarrow B(2\pi r) = \mu_o I_o \Rightarrow \boxed{R_1 < r < R_2 \quad B = \frac{\mu_o I_o}{2\pi r} \text{ CCW}}$$

c) $R_2 < r < R_3$

$$\oint \vec{B}\cdot d\vec{s} = \mu_o I_{enc} \Rightarrow B(2\pi r) = \mu_o \left[I_o - \int_{R_2}^{r} C_2 r (2\pi r\,dr)\right]$$

$$\Rightarrow B(2\pi r) = \mu_o \left[I_o - \frac{C_2 2\pi}{3}(r^3 - R_2^3)\right] \Rightarrow B = \mu_o I_o \left[1 - \frac{(r^3 - R_2^3)}{R_3^3 - R_2^3}\right]$$

$$\Rightarrow \boxed{B = \frac{\mu_o I_o}{2\pi r} \frac{(R_3^3 - r^3)}{R_3^3 - R_2^3} \text{ CCW } R_2 < r < R_3}$$

d) $\oint \vec{B}\cdot d\vec{s} = \mu_o I_{enc} \Rightarrow B(2\pi r) = 0 \Rightarrow \boxed{B = 0 \quad r > R_3}$