EECS 16A   Designing Information Devices and Systems I
Spring 2016   Elad Alon, Babak Ayazifar   Midterm 2

Exam location: 1 Pimentel, last SID# 0, 1 + 3

PRINT your student ID: _____

PRINT AND SIGN your name: _____ , _____   _____
(last)   (first)   (signature)

PRINT your Unix account login: ee16a-_____

PRINT your discussion section and GSI (the one you attend): _____

PRINT your lab GSI (the one you attend): _____

Row Number (front row is 1): _____   Seat Number (left most is 1): _____

Name and SID of the person to your left: _____

Name and SID of the person to your right: _____

Name and SID of the person in front of you: _____

Name and SID of the person behind you: _____

Section 0: Pre-exam questions

1. What has been your favorite part of 16A so far? (1 pt)

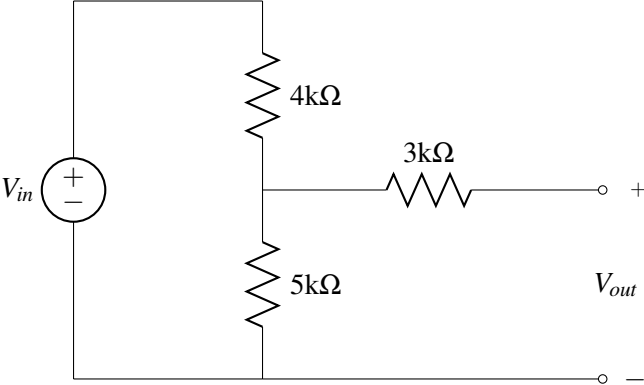**2. What are you looking forward to during Spring Break? (2 pts)**
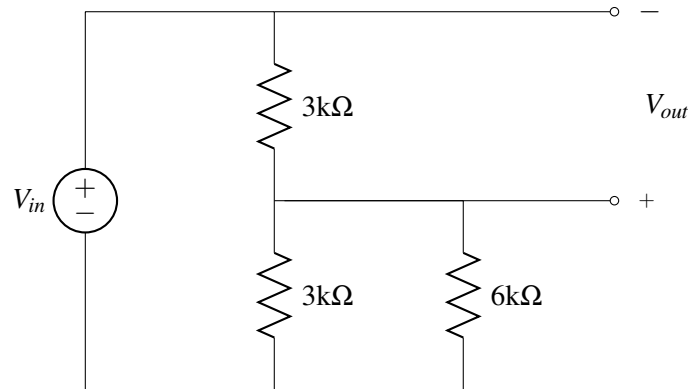
Section 1: Analysis questions

Unless told otherwise, you must show work to get credit. Note that all op-amps can be assumed to be ideal (unless explicitly stated otherwise.)

3. **You Can't Divide Me! (9 pts)**

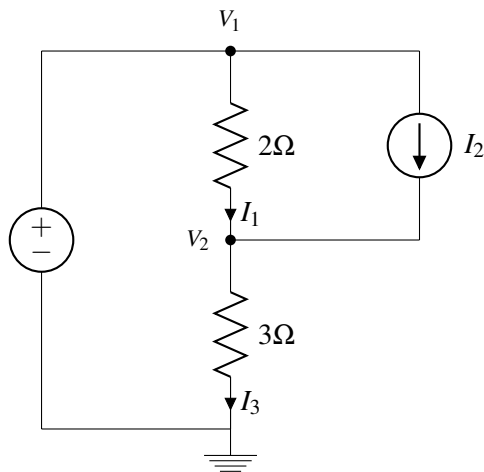   (a) **(4 pts)** Find V_{out} as a function of V_{in}.

(b) **(5 pts)** Find V_{out} as a function of V_{in}.
(Hint: Pay attention to where the positive and negative terminals of V_{out} are.)

4. {}Circuits (14 pts)

(a) **(4 pts)** Given the circuit shown below:

Use **only KCL and Ohm's Law** to set up a system of equations relating each of the labeled currents (I_1, I_2, I_3) and voltages (V_1, V_2). Then construct a 3x5 matrix A representing this system of equations such that $Ax = 0$ where x is:

$$x = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_1 \\ V_2 \end{bmatrix}$$

(b) **(6 pts)** For the matrix A provided below, which may or may not be the matrix you constructed in a), find a basis for the nullspace of A

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 3 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 2 & -5 \end{bmatrix}$$

(c) **(4 pts)** Explain what the nullspace of the A matrix you found in part (a) means in the context of this circuit.

5. Look At That Bling (8 pts)

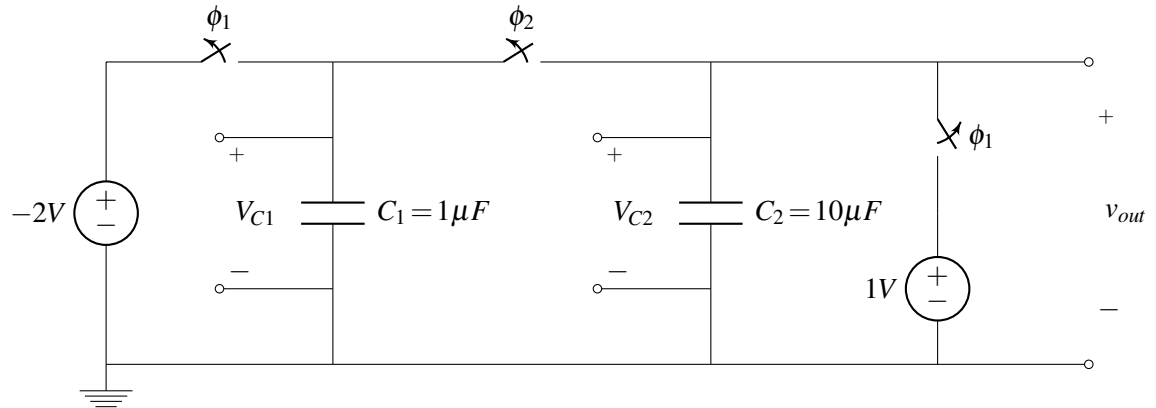Use the Golden Rules to derive V_{out} as a function of V_1 and V_2 for the circuit shown below:
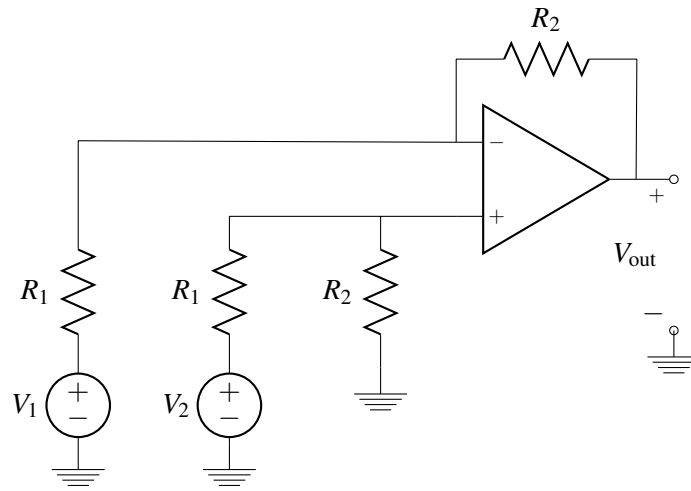
6. Big Caps Reconnected (12 pts)

(a) **(4 pts)** For the circuit shown below, assume that the ϕ_1 switches are initially closed and the ϕ_2 switch is initially open. Calculate V_{out} after the ϕ_1 switches are open and the ϕ_2 switch is closed.
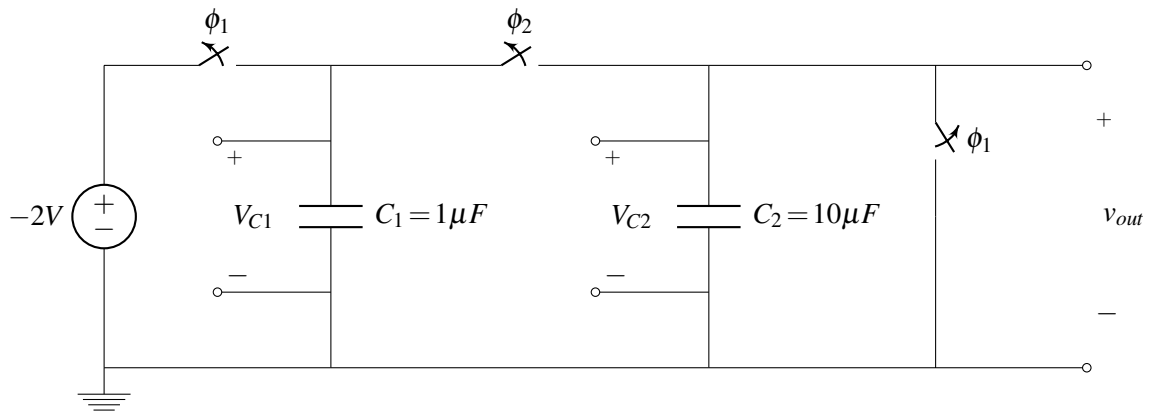
Now consider the following circuit for parts (b) and (c):

(b) **(4 pts)** What is the charge on each capacitor when the ϕ_1 switches are closed and the ϕ_2 switch is open? *Hint*: Be sure to measure the voltage across each of the capacitors in the way we have indicated on the diagram, and then pay careful attention to the signs of the charges.

(c) **(4 pts)** If the ϕ_2 switch is closed and the ϕ_1 switches are opened after the initial configuration from part (b), what is v_{out} ?

[If you are want the work on this page be graded, please state CLEARLY which problem(s) this space is for.]

Section 2: Free-form Problems

7. Black Boxes (11 pts)

(a) **(6 pts)** Derive the Thévenin equivalent of the circuit shown below; the equivalence should be found from the + and - terminals used to measure V_{out}. Be sure to provide numerical values for all the components of the equivalent circuit.
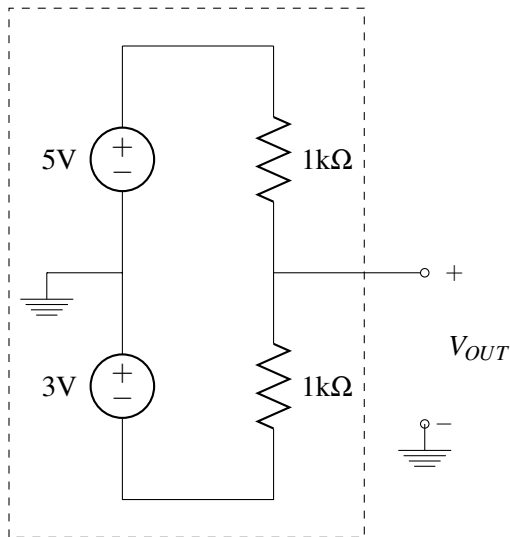
(b) **(5 pts)** Your colleague from Stanford approaches you with the circuit shown below and claims that since they used ideal op-amps in their circuit, the circuit will always provide the same V_L for any positive value of R_L:
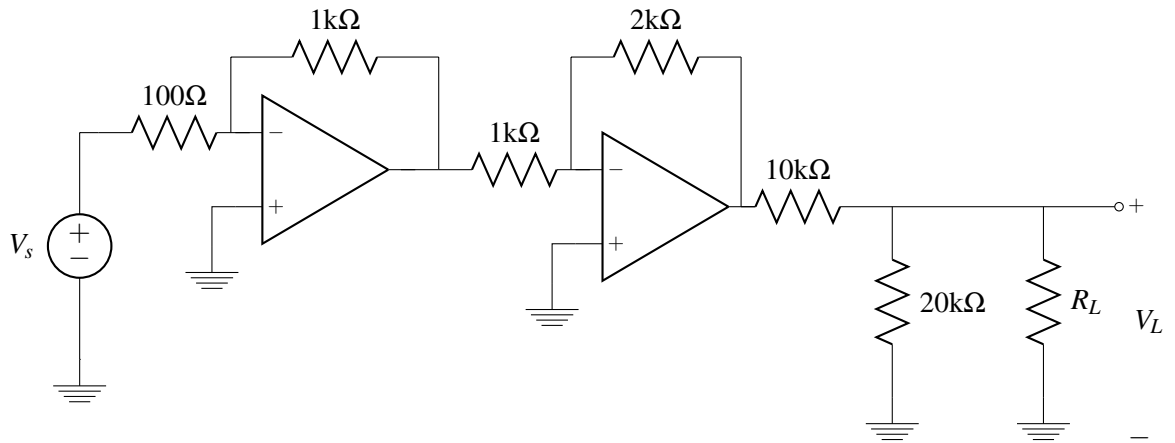

Is your colleague's claim correct? Briefly explain why or why not.

8. IoTatron (13 pts)

You are an engineer who has designed a new device called the "IoTatron" that has three electrical contacts. It has a symbol and an equivalent circuit that are pictured below. *Reminder*: A dependent current source acts just like a normal current source, but (in this case) with the value of the current being set by a control voltage.
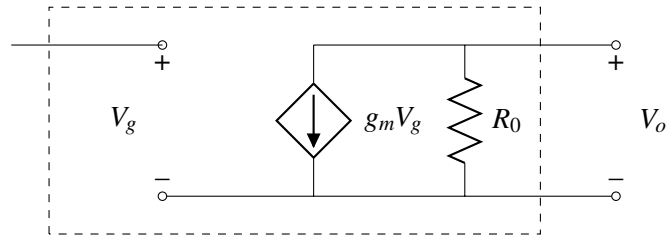
(a) **(4 pts)** For an IoTatron with an externally applied voltage of V_{in} as shown below, write an equation for V_{out} in terms of V_{in}, g_m, and R_0.
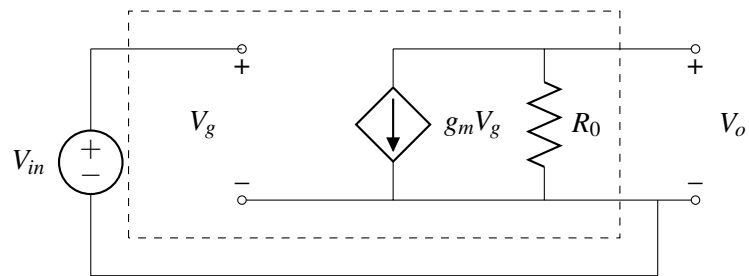
(b) **(4 pts)** You have an IoTatron with $R_o = \infty$, and have connected it in the circuit pictured below (note that R_o has been removed since it is now an open-circuit). Solve for V_c in terms of g_m and I_{in}.
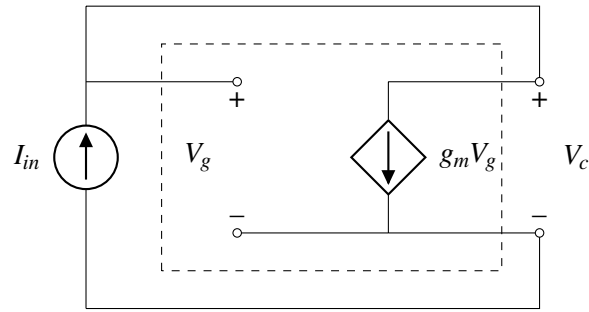
(c) **(5 pts)** You attach another IoTatron with $R_o = \infty$ to the one in part (b), but this one also has a finite R_L, resulting in the circuit shown below. Solve for I_L in terms of g_m, I_{in}, and R_L.
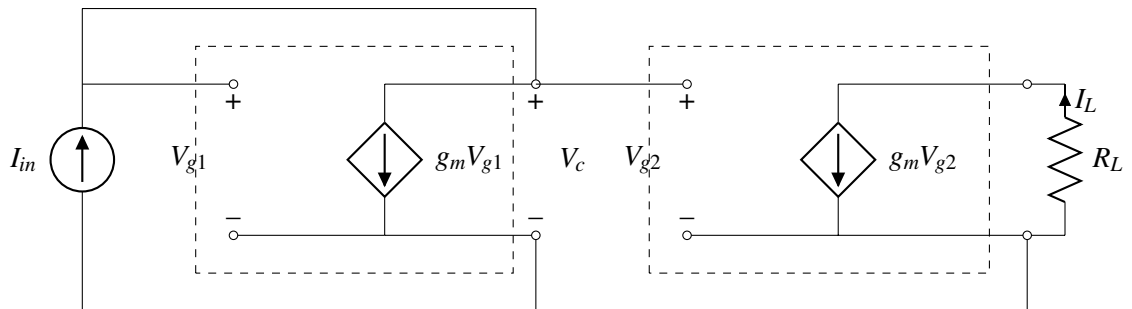
## 9. Fruity Fred (12 pts + BONUS 10 pts)

Fruity Fred just got back from Berkeley Bowl with a bunch of mangoes, pineapples, and coconuts. He wants to sort his mangoes in order of weight, so he decides to use his knowledge from EE 16A to build a scale.

He finds two identical bars of material (M_1 and M_2) of length L (meters) and cross-sectional area A (meters^2), which are made of a material with resistivity ρ. He knows that the length of these bars decreases by k meters per Newton of force applied, while the cross-sectional area remains constant.

He builds his scale as shown below, where the top of the bars are connected with an ideal electrical wire. The left side of the diagram shows the scale at rest (with no object placed on it), and the right side shows it when the applied force is F (Newtons), causing the length to decrease by kF meters. Fred's mangoes are not very heavy, so $L \gg kF$.
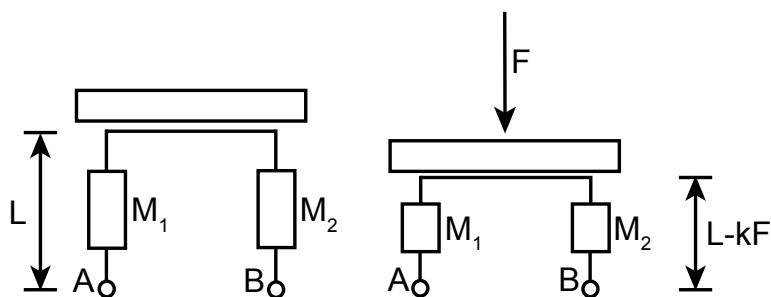
(a) **(4 pts)** Let R_{AB} be the resistance between nodes A and B. Write an expression for R_{AB} as a function of A, L, ρ, F, and k.

(b) **(8 pts)** Fred's scale design is such that the resistance R_{AB} changes depending on how much weight is placed on it. However, he really wants to measure a voltage rather than a resistance.

Design a circuit for Fred with an output voltage $V_{out} = -\alpha R_{AB}$, where $\alpha > 0$ is a constant with units $\frac{V}{\Omega}$. Your circuit should include R_{AB}, and **you may use any number of voltage sources, resistors, and op amps** in your design. Be sure to label your components and provide an expression relating their value to α.

(c) **(BONUS: 10 pts)** Fruity Fred is rather fickle and now wants a circuit whose output voltage is directly proportional to the force F applied on the scale.

Using only voltage sources, resistors, and op amps, design a circuit such that $V_{out} = \beta F$, where $\beta > 0$ is a constant of your choice with units $\frac{V}{\text{Newton}}$. Choose and label values for the components such that $\beta = 1 \frac{V}{\text{Newton}}$. *(Hint: you can do this by extending your circuit from the previous question.)*

10. Spinning Cs (18 pts + BONUS 10 pts)

In this problem, we'll examine a stylized version of a 1D MEMS (Micro-Electro-Mechanical System) gyroscope whose side view is shown below. It consists of a free-spinning non-conductive cylinder in the center with thickness h and radius r. An arched conductive plate is attached to this cylinder, and there is a fixed (i.e., not rotating) arched conducting plate a small distance d away. Note that air (with a dielectric constant of ε_0) separates the two conducting plates.
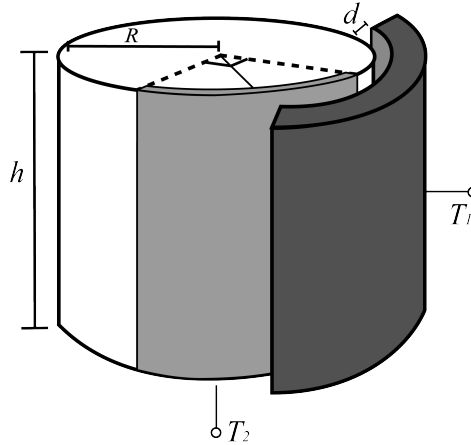

For the following problems, define C_g as the effective capacitance between T_1 and T_2. You can model the capacitors here as parallel plate capacitors. You can also assume that $R >> d$, so that the difference in surface area between the outer and inner plates is negligible. Ignore any capacitance that is not due to direct overlap between the plates (i.e., no fringing capacitance).

Since the outer plate is stationary while the cylinder is free to rotate, when the device turns, the inner and outer plates change alignment, thus changing the capacitance (C_g). For example, when the inner and outer plates are completely misaligned (as shown in the figure below with a top view of the gyroscope), $C_g = 0$.
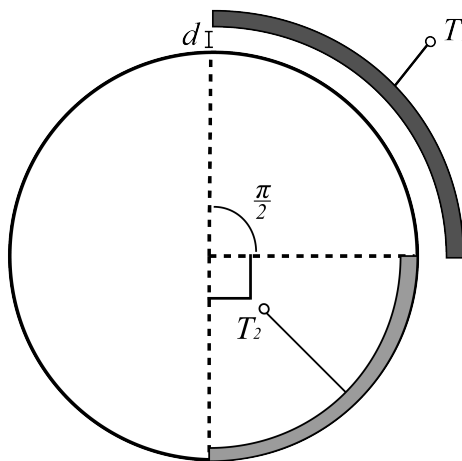
(a) **(4 pts)** The overlapping area between T_1 and T_2 is $R\theta h$, where θ is the angle of overlap in radians. Given that the conductive strips cover $\frac{1}{4}$ of the cylinder, what is the value of C_g as a function of ε_0, R, h, and d for the configuration shown below where $\theta = \frac{\pi}{4}$?
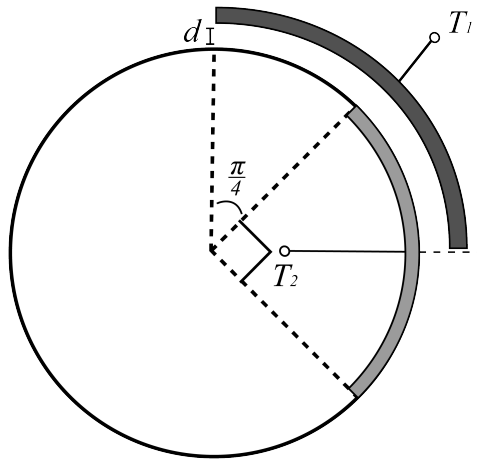
(b) **(4 pts)** Consider ϕ as the clockwise angle change in radians from the starting position shown in part(a), as shown below. Assuming your anwer to part (a) was that $C_g = C_{g,nom}$, write an equation for C_g as a function of ϕ and $C_{g,nom}$.
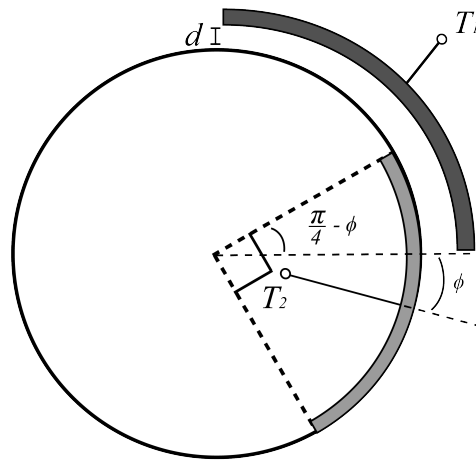
(c) **(5 pts)** Now let's design some circuits that use the value of C_g to extract some information about the gyroscope's rotation (i.e., the current value of ϕ). Using any combination of ideal circuit elements ***except* for current sources**, design a circuit that connects to T_1 and T_2, and that outputs a voltage that decreases in magnitude as the gyroscope cylinder rotates clockwise (i.e., as ϕ increases). You can assume that $-\frac{\pi}{4} \leqslant \phi \leqslant \frac{\pi}{4}$

(d) **(5 pts)** Assuming your design from part (c) produces a $V_{sensor} = (0.5 - \frac{\phi}{\pi}) * 1V$, design a new circuit that takes this V_{sensor} as an input and that outputs $5V$ if the cylinder rotates counterclockwise from the initial position (i.e., $\phi \leq 0$), and $-5V$ if it rotates clockwise (i.e., $\phi > 0$).

(e) **(BONUS: 10 pts)** Instead of letting the cylinder rotate freely, let's connect it to a voltage-controlled motor that tries to keep the cylinder in its original orientation. The motor applies a torque clockwise if fed a positive voltage relative to ground, and applies a torque counterclockwise if fed a negative voltage relative to ground. At the original position, the motor should have 0V applied to it. In order for the motor not to apply more torque than necessary, the magnitude of the input voltage to the motor should only increase as the cylinder gets further from the original position.

In this problem, model the motor as a 10Ω resistor. Design a circuit that drives the motor as described in order to keep the cylinder in the original orientation. You may re-use your design from part(c), but you will not receive full credit for this sub-part if the re-used design from (c) is incorrect.

PRINT your name and student ID: _____

[If you are want the work on this page be graded, please state CLEARLY which problem(s) this space is for. You can also draw us something if you want or give us suggestions or complaints. You can also use this page to report anything suspicious that you might have noticed.]