Mike DeWeese Spring 2016 MT1 solutions

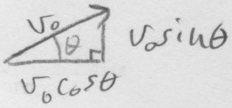
Prob. 1. a.) projectile motion

const. a_y: $y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$

$$\cancel{h_0} = \cancel{h_0} + v_0 \sin\theta\, \cancel{t} + \frac{1}{2}(-g) t^{\cancel{2}}$$

$$v_0 \sin\theta = \frac{1}{2} g t$$

$$\boxed{t = \frac{2 v_0 \sin\theta}{g}}$$



b.) const. a_x: $x = \cancel{x_0}^{\,0} + v_{0x} t + \frac{1}{2} \cancel{a_x}^{\,0} t^2$

$a_x = 0$

$$x = v_0 \cos\theta \, \frac{2 v_0 \sin\theta}{g} = \boxed{\frac{2 v_0^2}{g} \cos(\theta) \sin(\theta)}$$

c.) const. $a_{x_{w.r.}}$: $x_{wr} = x_{0_{wr}} + \cancel{v_{0x_{wr}}}^{\,0} t + \frac{1}{2} a_{x_{wr}} t^2$

$\Delta x_{wc} = \frac{1}{2} v_0 \cos\theta\, t \Rightarrow \cancel{\tfrac{1}{2}} \cancel{t}\, v_0 \cos\theta = \cancel{\tfrac{1}{2}} a_{wr_x} t^{\cancel{2}}$

$$a_{wr_x} = \frac{v_0 \cos\theta}{t} = \frac{\cancel{v_0} \cos\theta\, g}{2 \cancel{v_0} \sin\theta} = \boxed{\frac{1}{2} \cot(\theta)\, g}$$

d.) projectile motion $\Rightarrow \vec{a}_{ball} = (0, -g) \Rightarrow \boxed{a_{ball} = g}$

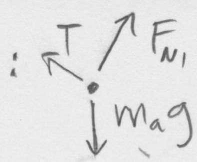
e.) $\vec{v}_{\substack{ball \\ w/ground}} = v_0(\cos\theta, -\sin\theta)$ right befor it is caught

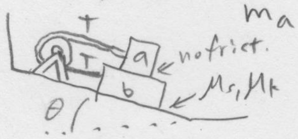
$$\vec{v}_{\substack{ball \\ w/r\ w.r.}} = \vec{v}_{\substack{ball \\ w/r\ ground}} - \vec{v}_{\substack{w.r. \\ w/r\ ground}} = v_0(\cos\theta, -\sin\theta) - (v_0 \cos\theta, 0)$$

const $a_{x_{w.r.}}$ $\Rightarrow$ $v_{\substack{w.r. \\ w/r \\ ground}}(t) = \cancel{v_{\substack{0 \\ w/r \\ w/g}}}^{\,0} + \left(\frac{1}{2} \cot(\theta)\, \cancel{g}\right)\left(\frac{2 v_0 \sin\theta}{\cancel{g}}\right) = v_0 \cos(\theta)$

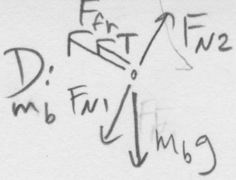
$$\text{so } \vec{v}_{\substack{ball \\ w/r \\ ground}} = v_0(0, -\sin\theta) = \boxed{-v_0 \sin(\theta)\, \hat{j}}$$

2.a) FBD: m_a



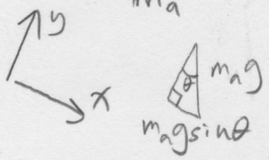


FBD: m_b



← (F_{fr} could be kinetic or static at this point)

b.) N2L m_a y: $F_{N1y} - m_a g\cos\theta = 0 \Rightarrow$ $\boxed{\vec{F}_{N1} = m_a g\cos(\theta)\,\hat{j}}$



c.) N2L m_a x: $-T + m_a g\sin\theta = m_a a_{xa} \overset{0}{} \Rightarrow T = m_a g\sin\theta$ — ①

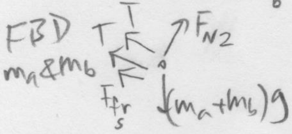
N2L m_b x: $-T - F_{fr_s} + m_b g\sin\theta = m_b a_{bx} \overset{0}{}$ — ②

① in ② $\Rightarrow -m_a g\sin\theta + m_b g\sin\theta - F_{fr_s} = 0$

$F_{fr_s} = (m_b - m_a)g\sin\theta$ — ③

N2L $m_b \& m_a$ y: $(m_a + m_b)g\cos\theta = F_{N2}$ — ④

FBD $m_a \& m_b$



$F_{fr_s} \le \mu_s F_{N2}$ — ⑤

so: ③ & ④ & ⑤ $\Rightarrow \mu_s g\cos(\theta)(m_a + m_b) \ge F_{fr_s} = (m_b - m_a)g\sin\theta$

$m_a(\mu_s g\cos\theta + g\sin\theta) \ge m_b(-\mu_s g\cos\theta + g\sin\theta)$

$m_a \ge m_b \dfrac{\sin\theta - \mu_s\cos\theta}{\sin\theta + \mu_s\cos\theta}$

$$\boxed{m_{a_{min}} = \begin{cases} m_b \dfrac{\sin\theta - \mu_s\cos\theta}{\sin\theta + \mu_s\cos\theta}, & \text{for } \sin\theta \ge \mu_s\cos\theta \\ 0, & \text{otherwize} \end{cases}}$$

← this makes sense if $\sin\theta \ge \mu_s\cos\theta$ but otherwise m_a can't be negative, so F_{fr_k} points uphill

d.) assume m_a sliding uphill, so m_b slids downhill, so sliding $\Rightarrow F_{fr_k} = \mu_k F_{N2} \overset{④}{=} \mu_k(m_a + m_b)g\cos\theta$ — ⑥

N2L m_a x: $-T + m_a g\sin\theta = m_a a_{ax}$ — ⑦

$a_{xa} = -a_{xb}$

N2L m_b x: $-T - F_{fr_k} + m_b g\sin\theta = m_b a_{bx} = -m_b a_{ax}$ — ⑧

⑥ & ⑦ in ⑧ $\Rightarrow m_a a_{ax} - m_a g\sin\theta - \mu_k(m_a + m_b)g\cos\theta + m_b g\sin\theta = -m_b a_{ax}$

$a_{ax}(m_a + m_b) = (m_a - m_b)g\sin\theta + \mu_k(m_a + m_b)g\cos\theta$

(see next p.)

prob 2 cont.

$$a_{ax} = \frac{m_a - m_b}{m_a + m_b} g\sin\theta + \mu_k g\cos\theta \quad \text{— (9)}$$

↳ assuming m_a slides <u>uphill</u>

if m_a is sliding <u>downhill</u>, then F_{fr_k} points <u>downhill</u> since m_b is now sliding uphill

eq. (6) & (7) are unchanged, but eq (8) becomes:

$$-T \underline{+} F_{fr_k} + m_b g\sin\theta = m_b a_{bx} = -m_b a_{ax} \quad \text{— (8')}$$

(6) & (7) & (8') ⇒ $m_a a_{ax} - m_a g\sin\theta \underline{+} \mu_k (m_a + m_b) g\cos\theta + m_b g\sin\theta = -m_b a_{ax}$

so $$a_{ax} = \frac{m_a - m_b}{m_a + m_b} g\sin\theta - \mu_k g\cos\theta \quad \text{— (9')}$$

if m_a slides <u>downhill</u>

e.) if m_a slides uphill:

(9) in (7) $T = m_a(g\sin\theta - a_{ax}) = m_a\left(g\sin\theta - \frac{m_b - m_a}{m_a + m_b} g\sin\theta - \mu_k g\cos\theta\right)$

$$T = m_a g\left(\frac{2m_b}{m_a + m_b}\sin\theta - \mu_k\cos\theta\right)$$

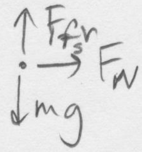
assuming m_a slides uphill

assuming m_a slides downhill:

(9') in (7) ⇒ $$T = m_a g\left(\frac{2m_b}{m_a + m_b}\sin\theta + \mu_k\cos\theta\right)$$

3\. a.) $T = \frac{2\pi R}{v}$ — ①   uniform circ. motion $\Rightarrow a = \frac{v^2}{R}$ towards center — ②

FBD rider



N2L rider x: $F_N = m a_x \overset{②}{=} \frac{m v^2}{R}$ — ③

N2L rider y: $-mg + F_{fr_s} = m a_y^{\nearrow 0}$

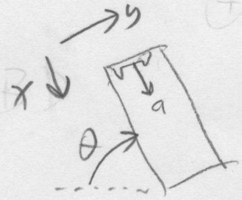
$F_{fr_s} = mg$ — ④

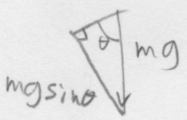
$F_{fr_s} \geq \mu_s F_N \underset{③}{=} \mu_s \frac{m v^2}{R} \underset{①}{=} \mu_s \frac{m 4\pi^2 R^2}{R\, T^2}$

④ $\Rightarrow$ $\cancel{m}g \geq \mu_s \frac{\cancel{m} 4\pi^2 R}{T^2}$

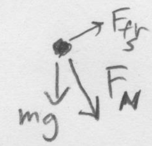
$T^2 \geq \frac{\mu_s 4\pi^2 R}{g}$ $\Rightarrow$ $\boxed{T \geq \sqrt{\frac{\mu_s 4\pi^2 R}{g}}}$

b.)





FBD rider



a_x points to center of circle ✓

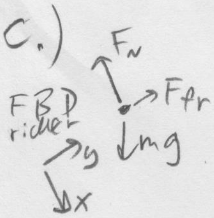
N2L rider y: $F_N + mg\sin\theta = m a_x = \frac{m v^2}{R}$

$F_N = \frac{m v^2}{R} - mg\sin\theta$

$= \frac{m 4\pi^2 R^{\cancel{2}}}{\cancel{R} T^2} - mg\sin\theta$

$\boxed{F_N = \frac{4\pi^2 m R}{T^2} - mg\sin\theta}$

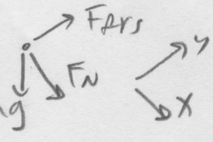
c.)



N2L rider x: $-F_N + mg\sin\theta = -\frac{m 4\pi^2 R}{T^2}$

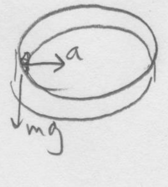
$\boxed{F_N = mg\sin\theta + \frac{4\pi^2 m R}{T^2}}$

d.) N2L y: $F_{fr_s} - mg\cos\theta = m a_y^{\nearrow 0}$

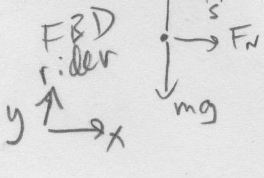


$\boxed{F_{fr_s} = mg\cos\theta}$

e.)



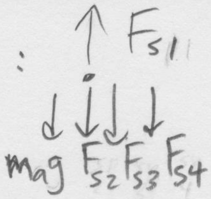
FBD rider



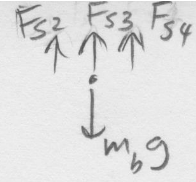
N2L y: $F_{fr_s} - mg = m a_y^{\nearrow 0} \Rightarrow \boxed{F_{fr_s} = mg}$

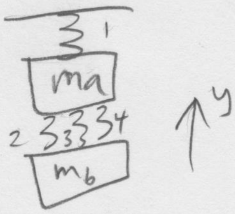
N2L x: $F_N = m a_x = \frac{m v^2}{R} = \frac{m 4\pi^2 R}{T^2}$ (don't even need this)

4.a.) FBD_a:

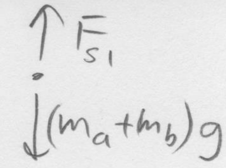


FBD_b:





FBD $m_a \& m_b$



$\Downarrow$

N2L $m_a \& m_b$ y: $\boxed{F_{s1} = (m_a+m_b)g}$

Force is upwards

b.) all 3 lower springs have same length & same spring constant and they are stretched by the same amount. so:

$F_{s2} = F_{s3} = F_{s4}$

N2L y m_a: $(\cancel{m_a}+m_b)g + 3F_{s2} - \cancel{m_a g} = m_a a_y \rightarrow 0$

$\boxed{F_{s2} = \frac{m_b g}{3}}$ points downwards

c.) $\Delta F_{s1} = k\Delta l_1$ —③ $\Delta F_{s2} = \Delta F_{s3} = \Delta F_{s4} = 2k\Delta l_2$ —④

$\Delta l_1 + \Delta l_2 = D$ —⑤

initially:

N2L m_a y: $F_{s1} - m_a g - 3F_{s2} = 0$ —①

after raising m_a:

$F_{s1} - \Delta F_{s1,y} - m_a g - 3(F_{s2} - \Delta F_{s2,y}) = 0$ ②

① in ② ⇒ $\Delta F_{s1,y} = -3\Delta F_{s2,y}$

$\Delta F_{s1} = 3\Delta F_{s2}$

③ & ④ ⇒ $\cancel{k}\Delta l_1 = 3\cdot 2\cdot \cancel{k}\cdot \Delta l_2$

$\Delta l_1 = 6\Delta l_2$

⑤ ⇒ $\Delta l_1 = 6(D - \Delta l_1)$

$\Delta l_1 = \frac{6}{7}D$

so m_a is raised up by $\boxed{\frac{6}{7}D}$

d.) N2L m_b y is unchanged so $\Delta l_2 = 0$ ⇒ M_b is lower by $\boxed{d}$

e.) N2L m_a y: $F_{s1} - m_a g = m_a a_y \rightarrow 0$

so $\Delta l_1 = \boxed{\frac{m_b g}{k}}$ is how high m_a is raised