Econ 101A – Final exam
Mo 18 May, 2009.

Do not turn the page until instructed to.
Do not forget to write Problems 1 and 2 in the first Blue Book and Problems 3 and 4 in the second Blue Book.

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Do not forget to write Problems 1 and 2 in the first Blue Book and Problems 3 and 4 in the second Blue Book. Good luck on solving the problems!

Problem 1. Prisoner Dilemma Game with Altruism (40 points) Consider the standard Prisoner Dilemma game that we discussed in class. As you remember, the story is one of two prisoners, each of which has to choose between defecting (that is, confessing) and not defecting (keeping the mouth shut). The payoffs indicate the number of years in prison:

1\2	Defection	No Defection
Defection	$-4, -4$	$-1, -5$
No Defection	$-5, -1$	$-2, -2$

1. Write the general definition of Nash Equilibrium. (3 points)
2. Using this definition, compute all the *pure-strategy* Nash equilibria of the game (that is, do not allow for probability distributions). (3 points)
3. Now comes the interesting part of the problem. Unlike in the classroom discussion, the prisoners are altruistic. The utility function of player 1 is a function both of his own payoff in years, π_1, but also of the payoff of player 2, π_1. Thus, player one's utility is: $U_1 = \pi_1 + \alpha\pi_2$, with $\alpha \geq 0$. Similarly for player 2: $U_2 = \pi_2 + \alpha\pi_1$. Briefly explain why the parameter α captures altruism, and discuss the special cases $\alpha = 0$ and $\alpha = 1$. (4 points)
4. This implies that the matrix can be rewritten in terms of utility as

1\2	Defection	No Defection
Defection	$-4(1+\alpha), -4(1+\alpha)$	$-1-5\alpha, -5-\alpha$
No Defection	$-5-\alpha, -1-5\alpha$	$-2(1+\alpha), -2(1+\alpha)$

   Compute all the mixed-strategy equilibria of this new game, as a function of α, assuming $\alpha \geq 0$. Call u (for Up) the probability that player 1 defects and l (for Left) the probability that player 2 defects. Discuss intuitively why altruism makes a difference — or does not make a difference — in this game. (15 points)

5. Under what values of α is (s_1^* =No Defection, s_2^* =No Defection) an equilibrium in dominant strategies for this game? State clearly the definition of Dominant Strategy equilibrium and how it differs from Nash Equilibrium. (7 points)
6. Now let's go back to the standard case with no altruism ($\alpha = 0$), but assume that the Prisoner's Dilemma game is played twice, instead of once. That is, the game is repeated once. Each time, the matrix of payoffs above applies. Debate this assertion: "If the prisoners meet twice, we will observe the No Defection equilibrium even in the absence of altruism because the repetition provides an opportunity for collusion" Use backwards induction and be as precise as you can about your statements. (8 points)

Solution to Problem 1.

1. Strategies $s^* = \left(s_i^*, s_{-i}^*\right)$ are a Nash Equilibrium if

$$U_i\left(s_i^*, s_{-i}^*\right) \geq U_i\left(s_i, s_{-i}^*\right)$$

   for all $s_i \in S_i$ and $i = 1, ..., I$

2. (D,D) is the only Nash Equilibrium. This is simple because no matter what the opponent does, Defecting is better than Not defecting.

3. The parameter α is the weight in the utility function on the payoffs of the other player, and as such captures altruism. The case $\alpha = 0$ is the case with no altruism (the standard case), the case $\alpha = 1$ is the case with perfect altruism, the person cares as much about the other as about oneself.

4. We consider the best response player-by-player. For player 1, D is better than ND if

$$-4(1+\alpha)l + (-1-5\alpha)(1-l) \geq (-5-\alpha)l - 2(1+\alpha)(1-l)$$

   or

$$\begin{aligned} -1-5\alpha + (-3+\alpha)l &\geq -2(1+\alpha) + (-3+\alpha)l \text{ or} \\ -1-5\alpha &\geq -2(1+\alpha) \\ \frac{1}{3} &\geq \alpha \end{aligned}$$

   The calculations are completely symmetric for player 2.

   Unlike other examples of mixed strategy equilibria, we find that the best-response of each player does not depend on the other player's probability of defecting; rather, it depends on how much each player cares about the other. This makes sense for this game. Recall that without altruism, a prisoner's best response does not depend on the other prisoner's action: Defecting is a dominant strategy for both players. However, since Defecting always hurts the other player more than Not Defecting, then if the players care sufficiently about each other, Not Defect becomes the dominant strategy.

   Thus, the full set of mixed strategy equilibria is:

$$\begin{cases} (D, D), \text{ if } \alpha < \frac{1}{3} \\ ((l, 1-l), (u, 1-u)), l \in [0,1], u \in [0,1] \text{ if } \alpha = \frac{1}{3} \\ (ND, ND), \text{ if } \alpha > \frac{1}{3} \end{cases} \tag{1}$$

5. Strategies $s^* = \left(s_i^*, s_{-i}^*\right)$ are an equilibrium in dominant strategies if

$$U_i\left(s_i^*, s_{-i}\right) \geq U_i\left(s_i, s_{-i}\right)$$

   for all $s_i \in S_i$, $s_{-i} \in S_{-i}$, and $i = 1, ..., I$

   The difference with Nash Equilibrium is that for each player i, s_i^* is a best-response *no matter what strategy is chosen by the opponent.*

   It follows from the solution to part 4 that if $\alpha > \frac{1}{3}$, then (ND,ND) is an equilibrium in dominant strategies. This is evident since ND is a best-response for each player regardless of the other player's probability of defecting.

6. Even though it would appear that repetition would make it possible to sustain collusion, this is not the case. The game is repeated once. In the last period, we know that both prisoners will defect because there is no further future. Anticipating this (remember the backward induction!), in the previous period the prisoners know that no matter what they promise they will both end up defecting in the next period. Hence, they defect in period 1.

Problem 2. Wisdom tooth removal and asymmetric information (70 points) Wisdom teeth are usually removed early in life as a preventive measure, to avoid problems that may happen in the future. It is more costly to remove the teeth once the problems have begun.

Suppose there are two types of wisdom teeth: those that are more likely to lead to problems in the future ("bad" teeth) and those that are less likely to lead to problems ("good" teeth). Specifically, assume a good set of teeth has a 10% chance of problems and a bad set of teeth has a 60% chance of problems. All future problems will cost \$500 to solve. However, if the wisdom teeth are removed early in life the probability of future problems is 0. There are N patients with wisdom teeth in Berkeley.

Assume the consumer does not discount between periods and has utility directly equal to his wealth: $u(\$x) = x$. Notice that this means he is risk-neutral.

Suppose there is only one oral surgeon in Berkeley, and she operates as a monopolist in wisdom tooth removal. She faces marginal cost \$100 of removing any set of wisdom teeth, whether they are good or bad teeth.

1. How much is an individual willing to pay for the removal of a good set of wisdom teeth? And for the removal of a bad set of wisdom teeth? (4 points)
2. First-best scenario: If consumers can identify whether their wisdom teeth are good or bad, what will the oral surgeon monopolist set as the price for removing wisdom teeth (p_{FB}), and which type(s) of teeth will be removed? (6 points)
3. How much profit will the oral surgeon monopolist make per set of wisdom teeth? (5 points)
4. Hidden type scenario: Now, suppose individuals cannot tell whether their teeth are good or bad, but they know that across the population, $\frac{2}{5}$ of all peoples teeth are "bad". Solve for the expected benefit to an individual of removing his wisdom teeth, given his (correct) beliefs about the likelihood of each type. (5 points)
5. Given the expected benefit you found above, what price (p_{HT}) will the monopolist charge to remove a set of wisdom teeth when teeth have hidden type? Which type(s) of teeth will be removed in equilibrium? (4 points)
6. Calculate the profit of the monopolist in this case (hidden type) and compare it to his profits in the case above (first best). (Remember that there are N patients, and $\frac{2}{5}$ of them have bad wisdom teeth.) Which case does she prefer? Interpret. (6 points)
7. What about the consumers? compare the aggregate expected utility of all N consumers in the hidden type case with the aggregate in the first-best case. Which case is better, in aggregate? (5 points)
8. Assume now that there is perfect competition in the market for wisdom tooth removal. Specifically, assume that there is an infinite supply of doctors, all of which face marginal cost \$100 of removing a set of wisdom teeth, and that they compete on price p_{PC}. What is the price p_{PC} charged for wisdom teeth removal? Compute it for both the first-best scenario and the hidden-type scenario. (10 points)
9. Still on perfect competition: What is the aggregate expected utility of consumers in the two scenarios (first-best versus hidden-type)? Explain the differences with the monopoly case (5 points)
10. (Harder) Now let's go back to the monopoly case. Without solving mathematically, explain whether the difference $p_{FB} - p_{HT}$ would increase or decrease if consumers were risk-averse. (10 points)
11. Lastly, still in the monopoly case, but without risk-aversion, suppose the share of bad teeth is not $\frac{2}{5}$ but some fraction $q \in (0, 1)$. Determine the range of values q such that in equilibrium, no wisdom teeth are removed. (10 points)

Solution to Problem 2.

1. Good tooth: Expected outcome is $.10 \cdot 500 = 50$, so she is willing to pay at most \$50 to remove a good tooth. For future reference, let's denote this value $V_g = 50$
   Bad tooth: Expected outcome is $.60 \cdot 500 = 300$, so she is willing to pay at most \$300 to remove a bad tooth. Let's denote this value $V_b = 300$

2. The monopolist will not remove a *set of* wisdom teeth if she gets paid less than her marginal cost for doing so, so no good teeth will be removed. Since she is only removing bad teeth, she can charge the consumers' full willingness to pay for bad tooth removal: \$300 per tooth.

3. Profit will be \$200 per *set of* wisdom teeth, but only bad teeth will be removed.

4. Expected benefit of removing a tooth of unknown quality is $\pi_g \cdot V_g + \pi_b \cdot V_b$, where π_g is the probability the tooth is good, and π_b is the probability the tooth is bad. With the value given, $\pi_b = 2/5$, this works out to be: $\frac{3}{5}50 + \frac{2}{5}300 = 150$.

5. Since this value is greater than the marginal cost of removing a tooth, the monopolist will charge \$150 (again extracting all the surplus) and all wisdom teeth will be removed.

6. Hidden type: Profit is \$50 per tooth, but all teeth are removed. If total number of people is N, monopolist makes profit $50N$.
   First-best: Profit is \$200 per tooth, but only the bad teeth are removed, so total profit is $200 \cdot \frac{2}{5}N = 80N$.
   The monopolist prefers the first-best case, because even though less people have their teeth removed, she is able to charge a much higher price and makes higher profits.

7. Hidden type: $EU = w_0 - 150$.
   First best: $EU = \frac{2}{5} \cdot (w_0 - 300) + \frac{3}{5} \cdot (\frac{1}{10}(w_0 - 500) + \frac{9}{10}w_0) = w_0 - 150$
   It should not surprise us that the consumer is equally well off in both cases, since the monopolist has been able to charge him exactly his maximum willingness to pay in both cases.

8. If there was perfect competition, however, then the price of wisdom tooth removal would be driven down to its marginal cost (\$100) in *both* scenarios.

9. Then, the consumer would certainly prefer the perfect information (first best) case, because he would only pay to have the tooth removed if it was truly worth it (i.e. only if the tooth is "bad") while in the hidden type case, he pays to have both good and bad teeth removed (this is because the benefit of having bad teeth removed is high enough to compensate for the lost utility of removing good teeth as well).

10. If the consumer is risk-averse, both p_{FB} and p_{HT} will go up since removal of *any* type of tooth eliminates uncertainty (even good teeth have a 1/10 chance of causing harm.) Thus, in both scenarios, the consumer is willing to pay a higher price, given the value she gets from eliminating uncertainty. However, as long as the first best case still results in only bad teeth being removed, the consumer still faces more uncertainty in that case than in the hidden type case. One could say that the benefit of removing all wisdom teeth–even those that are not actually "worth" removing based on their probability of causing harm, is that this eliminates *all* uncertainty with regard to future outcomes. Thus, the difference in p_{FB} and p_{HT} will decrease, since p_{HT} increases more than p_{FB} when risk aversion is introduced.

11. As in part 4, the expected benefit of removing a tooth of unknown type is still $\pi_g \cdot V_g + \pi_b \cdot V_b$. In this case, that equals $(1-q)50 + q300 = 250q + 50$, and that is the maximum price the consumer will be willing to pay. The monopolist would accept any price higher than the marginal cost \$100, so the condition in which no wisdom teeth are removed is:

$$\begin{aligned} 250q + 50 &< 100 \\ q &< \frac{1}{5} \end{aligned}$$

Problem 3. Search effort (45 points.) We consider in this problem the decision by an unemployed worker of how much effort to put into searching for a job. The unemployed worker chooses the search effort $e \in [0,1]$. With probability e, the worker receives a job offer that provides utility (from wages and other benefits, etc) of $w > 0$, while with probability $1-e$ the worker receives no job offer and receives only unemployment benefits, which provides utility of b (with $w > b > 0$). In other words, the probability of receiving a job offer is directly proportional to the job-search effort of the worker. The disutility of searching is $-c(e)$, with $c(0) = 0$ and $c'(e) > 0$ for all e.

1. The worker is an expected utility maximizer. Write the expected utility taking into account that the expected utility is the sum of the expected utility of money (wage w or benefit b) plus the disutility cost of searching. (4 points)
2. Under what conditions is the expected utility (strictly) concave in e? Discuss the economic interpretation. Maintain this assumption in what follows. (5 points)
3. The worker maximizes the expected utility with respect to effort e. Derive the first order condition with respect to e. (5 points)
4. Using the implicit function theorem, show that the optimal effort e^* (i) is increasing in the wage w; (ii) is decreasing in the unemployment benefits b. Discuss the intuition for each results. (16 points)
5. Now assume that the agent is risk averse with utility function $x^{1-\rho}/(1-\rho)$. Remember that ρ is the risk-aversion parameter, the higher ρ, the more risk averse the agent is (Assume $\rho > 0$) In our context, this implies that the utility of finding a job is now $w^{1-\rho}/(1-\rho)$ instead of w and similarly the utility of being unemployed is now $b^{1-\rho}/(1-\rho)$ instead of b. Nothing else changes. Write the new expected utility that the consumer maximizes and derive the first order condition (5 points)
6. Again using the implicit function theorem, show how the optimal effort e^* changes as the risk aversion level ρ increases. Provide an explanation. [Notice: This is a somewhat counter-intuitive result] (10 points)

Solution to Problem 3.

1. The expected utility is $EU = ew + (1-e)b - c(e)$.
2. Since the expected utility is a univariate function of e, it is enough to check that the second derivative with respect to e is negative for all levels of e. The first derivative is
$$w - b - c'(e),$$
the second derivative is
$$-c''(e).$$
Hence, the required condition is $c''(e) > 0$ for all e, that is, the marginal cost of search is increasing in the search effort (the first CV sent out is the easiest)
3. The maximization problem is
$$\max_e ew + (1-e)b - c(e)$$
with f.o.c.
$$w - b - c'(e) = 0.$$
4. The implicit function theorem implies
$$\frac{de^*}{dw} = -\frac{1}{-c''(e^*)} > 0$$

and

$$\frac{de^*}{db} = -\frac{-1}{-c''(e^*)} < 0.$$

Intuitively, a higher wage increase the incentive to get a job, and hence the search intensity, Conversely, a higher benefit decreases the incentive to get a job, hence lowering the search effort.

5. The expected utility is now

$$EU = ew^{1-\rho}/(1-\rho) + (1-e)b^{1-\rho}/(1-\rho) - c(e).$$

The first order condition is

$$w^{1-\rho}/(1-\rho) - b^{1-\rho}/(1-\rho) - c'(e) = 0.$$

6. The implicit function theorem implies

$$\frac{de^*}{d\rho} = -\frac{\left(w^{1-\rho} - b^{1-\rho}\right)/(1-\rho)^2 + \left(-w^{1-\rho}\log w + b^{1-\rho}\log b\right)/(1-\rho)}{-c''(e^*)}$$

Unlike what was stated, the solution was not obvious here.

Problem 4. General equilibrium with taxes. (75 points) Now we look at an economy with one consumer and two goods, and consider what happens in general equilibrium when we impose taxes on those goods. In particular, let the demand for goods one and two be defined as $Y_1^D = M - p_1 + p_2$, and $Y_2^D = M - p_2 + p_1$, and let the supply of the two goods be $Y_1^S = p_1$, and $Y_2^S = \overline{Y}$, where $\overline{Y}$ is a constant and $M > \overline{Y}$. Think of M as income. To reiterate, the indices 1 and 2 here denote different goods, not different consumers.

1. Are goods one and two gross complements or gross substitutes? Prove your answer mathematically referring to the definition of gross complements and substitutes. (5 points)
2. Provide economic intuition for the supply of good two: what does it mean when the supply of a good is constant, can you think of an example? (5 points)
3. Still on the supply of good 2: what is the price elasticity of supply for good 2? Is good two elastic or inelastic in supply? (5 points)
4. There are six unknowns in this economic system: $Y_1^D, Y_2^D, Y_1^S, Y_2^S, p_1, p_2$. Impose the equilibrium conditions $Y_1^D = Y_1^S$ and $Y_2^D = Y_2^S$ and solve for equilibrium prices and quantities in both markets. [Your answers should be in terms of M and $\overline{Y}$ since these are the only exogenous parameters in the system.] (4 points)
5. Discuss qualitatively why it makes sense (or not) that demand must equal supply in equilibrium (5 points)
6. Let's do some comparative statics. How does p_2 respond to increase in the fixed supply $\overline{Y}$? Discuss the intuition. Why does p_1 also respond to increase in $\overline{Y}$, despite the fact that good 1 is not in short supply? (6 points)
7. More comparative statics. Why is the price for good 1 different from the price of good 2 despite the demand functions being symmetrical? [Make sure that your answer provides economic intuition rather than just mathematical intuition.] (5 points)
8. Final comparative statics: How does income M affect the equilibrium prices and quantities? Provide intuition (5 points)

Now consider imposing a tax on good one. Instead of the per-unit tax we have considered in the past, this time we consider a per-dollar tax. The tax rate is $t \in [0, 1]$ and the total revenue generated by the tax is $t \cdot p_1 Y_1$. In other words, the tax is a proportion of the amount of money spent on good one. Thus, the price paid by consumers for good one is $(1 + t)\, p_1$ while the price received by producers is just p_1. [This is exactly like the sales tax we have in the US. Thus, in California, $t = .0975$. Do not use this number in your calculations. It is just an example.]

9. Write down the revised expressions for Y_1^D, Y_2^D, Y_1^S, and Y_2^S, taking into account taxes. [Hint: The expressions are the same as in point 4 above, but in the demand functions substitute p_1 with $p_1\,(1 + t)$] Why do taxes not enter the expression of the supply function? (5 points)
10. Solve for the equilibrium prices and quantities for both goods. (4 points)
11. Compare the prices and quantities under this taxation scheme to your results from part 4. What is surprising about these results? Provide economic intuition for this surprising outcome. (10 points)

Now consider, instead, putting the tax t on good two (and no tax on good 1). We will explore the implications of whether the tax is placed on good one or good two.

12. Write down the revised expressions for Y_1^D, Y_2^D, Y_1^S, and Y_2^S and solve for the equilibrium prices and quantities of both goods.(6 points)

13. Compare the prices and quantities under this taxation scheme to your results from part 10. Who is harmed and/or helped by each of the schemes? Who pays the tax in each of the schemes? Provide as much economic intuition as possible for your answers, and in particular for the differences between the results for the two schemes. [Hint: your answers to parts 1 and 2 should play a prominent role in your answer to this part of the problem.] (10 points)

Solution to problem 4.

1. To compute gross complements (substitutes), we compute

$$\begin{aligned}\frac{\partial Y_1^D}{\partial p_2} &= 1 \text{ and} \\ \frac{\partial Y_2^D}{\partial p_1} &= 1\end{aligned}$$

   and hence the two goods are gross substitutes.

2. In this case, the good is perfectly inelastic with respect to price. For example, in the short-run the number of umbrellas in a store is fixed.

3. The price elasticity for the supply of good 2 is

$$\eta_{Y_2,p_2} = \frac{\partial Y_2}{\partial p_2}\frac{p_2}{Y_2} = 0.$$

   As we wrote above, the supply of good 2 is perfectly inelastic.

4. Imposing $Y_1^D = Y_1^S, Y_2^D = Y_2^S$, the system is

$$\begin{aligned}p_1 &= M - p_1 + p_2 \\ \overline{Y} &= M - p_2 + p_1\end{aligned}$$

   >From the first equation, we get

$$p_1 = \frac{M + p_2}{2}$$

   which plugged into the second equation gives

$$\begin{aligned}\overline{Y} &= M - p_2 + \frac{M + p_2}{2} \text{ or} \\ p_2^* &= 2\left(\frac{3}{2}M - \overline{Y}\right) = 3M - 2\overline{Y} \text{ and hence} \\ p_1^* &= \frac{M + p_2^*}{2} = \frac{M + 3M - 2\overline{Y}}{2} = 2M - \overline{Y}\end{aligned}$$

   In terms of quantities, we get

$$\begin{aligned}Y_1 &= p_1^* = 2M - \overline{Y} \\ Y_2 &= \overline{Y}\end{aligned}$$

5. In equilibrium, if demand is larger than supply, producers will have an incentive to raise price. If demand is smaller than suply, producers will have in incentive to lower prices.

6. As the fixed supply goes up, p_2 goes down. This makes sense, as an increase in supply in general will lower own price given that the demand curve is downward-sloping. An increase in the fixed supply of good 2 also lowers the price of good 1. The intuition here is less straightforward. An increase in the (fixed) supply of good 2 lowers the price of good 2, which lowers the demand of good 1 (see Y_1^D); in turn, the downward shift in demand lowers the price of good 1.

7. The two prices are different in equilibrium because the two goods have a different supply curve, it is fixed for good 2, while it is not fixed for good 1.

8. Finally, an increase in income increase the price of both goods, as well as the supply of good 1. This is to be expected, since an increase in income causes an increase in demand, leading to a price increase, and hence inducing more production of good 1.

9. The system becomes

$$
\begin{aligned}
Y_1^D &= M - (1+t)\,p_1 + p_2, \\
Y_2^D &= M - p_2 + (1+t)\,p_1, \\
Y_1^S &= p_1, \\
Y_2^S &= \overline{Y}
\end{aligned}
$$

10. The solution is

$$
\begin{aligned}
p_1 &= 2M - \overline{Y} \\
Y_1 &= 2M - \overline{Y} \\
p_2 &= (3+2t)\,M - (2+t)\,\overline{Y} \\
&= 3M - 2\overline{Y} + t\left(2M - \overline{Y}\right) \\
Y_2 &= \overline{Y}
\end{aligned}
$$

11. The price and quantity for good one remain the same. The only difference is that the price of good two has gone up, by exactly the amount of the tax on good one. $[p_2 = 3M - 2\overline{Y} + tp_1]$ The surprise is that the price distortion got passed on to good two. This is because the increase in the price paid by consumers for good one, while initially driving down demand for good one, also drives up demand for good two, and since good two is in fixed supply, it's price goes up, which in turn has a rebalancing effect on demand for good one.

12. The system now is

$$
\begin{aligned}
Y_1^D &= M - p_1 + (1+t)\,p_2, \\
Y_2^D &= M - (1+t)\,p_2 + p_1, \\
Y_1^S &= p_1, \\
Y_2^S &= \overline{Y}.
\end{aligned}
$$

and the solution is

$$
\begin{aligned}
p_1 &= 2M - \overline{Y} \\
Y_1 &= 2M - \overline{Y} \\
p_2 &= \frac{3M - 2\overline{Y}}{1+t} \\
Y_2 &= \overline{Y}.
\end{aligned}
$$

13. Again the price and quantity for good one are the same, as is, of course, the quantity of good two. The price of good two, however, has gone down in this case, rather than up, because the demand for good two goes down, but since the distortion falls entirely on producers there is no effect on the demand for good one. Under scheme one, consumers of good one pay more for the same quantity, with no effect on producers, while consumers of good two also pay more for the same quantity, with producers benefiting. Thus, consumers of good one pay the tax, and there is a transfer of surplus from consumers of good two to producers of good two. Under scheme two, consumers and producers of good one are unaffected, consumers of good two are also unaffected, and the only impact is on producers of good two, who pay the whole tax.