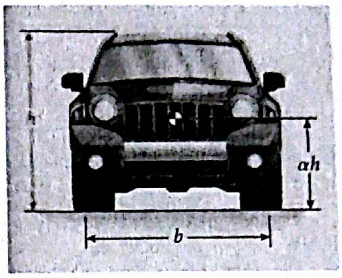
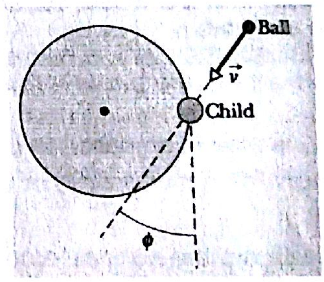
Fall 2015 Physics 7A Lec 002 Yildiz
Final Exam

1. **(20 points)**
An SUV has a height h and a wheelbase of length b. Its center of mass is midway between the wheels and at a distance αh above the ground, where $0 < \alpha < 1$. The SUV enters a turn at a dangerously high speed, v. The radius of the turn is R (R>>b), and the road is flat. The coefficient of static friction between the road and the properly inflated tires is μ_s. After entering the turn, the SUV will either skid out of the turn or begin to tip.



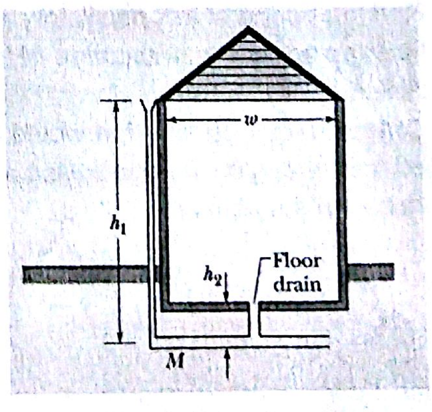
   a) The SUV will skid out of the turn if the friction of the force reaches its maximum value, $F_{fr} \rightarrow \mu_s N$, in which N is the entire normal force acts on the SUV. Determine the speed, v_{skid}, for which this will occur. Assume no tipping occurs.
   b) Right before tipping of the car, the inner wheel is about to lose contact with the road and the entire normal force is sensed by the outside wheel. During tipping, the outside wheel is the point at which the car pivots about. Therefore, the torque keeping the SUV from tipping acts on the outside wheel. Determine the speed, v_{tip}, at which tipping will occur. Assume no skidding occurs.
   c) It is safer if the SUV skids out before it tips. This will occur as long as $v_{skid} < v_{tip}$. Apply this condition and determine the maximum value for α in terms of b, h and μ_s.

2. **(15 points)**
A child of mass m_1 stands on the edge of a stationary merry-go-round of mass m_2 and radius R. The child catches a ball of mass m_3 thrown by a friend. Just before the ball is caught, it has a horizontal velocity of v at an angle $\varphi = 37^\circ$ with a line tangent to the outer edge of the merry-go-round.



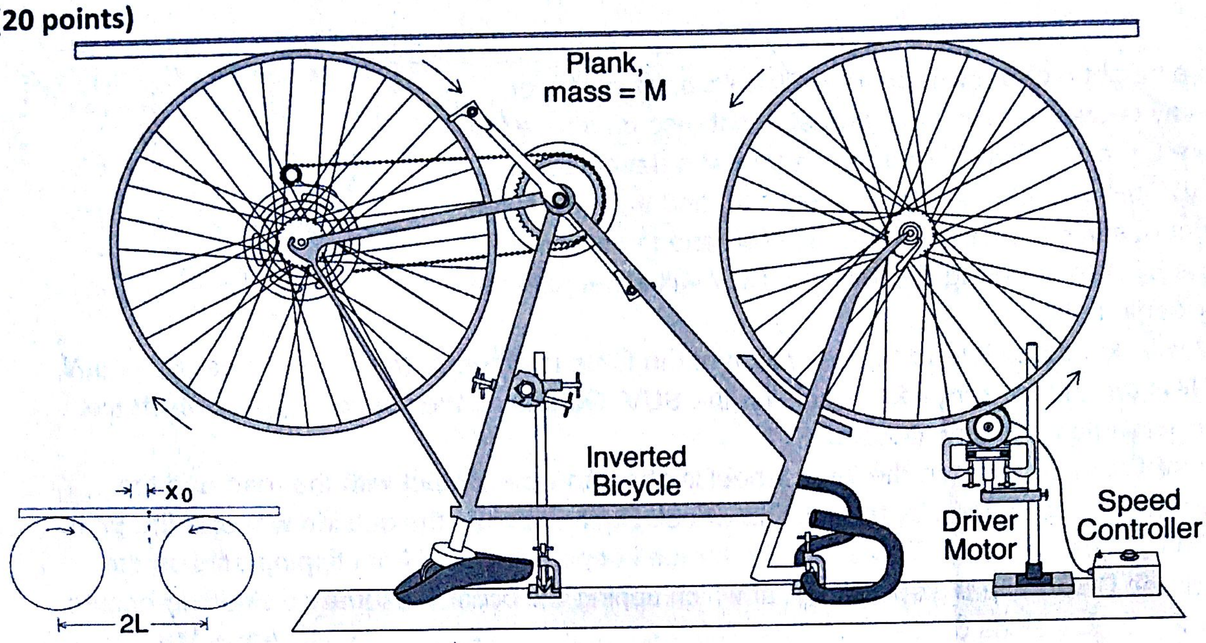
a) What is the rotational inertia of the merry-go-round and the child before and after the child catches the ball with respect to the axis of rotation. Assume that the merry-go-round is a uniform disk, the child and the ball are point-like objects. The system rotates about the center of mass of the merry-go-round.
b) What is the angular velocity of the merry-go-round just after the ball is caught.

3. **(15 points)**
Rain falling on the slanted roof runs off into gutters around the roof edge; it then drains through downspouts (only one is shown) of height h_1 into a main drainage pipe M with a radius R below the basement, which carries water to an even larger pipe below the street (water flow in pipe M is laminar, so it does not cause any turbulence). A floor drain with height h_2 (from pipe M) in the basement is also connect to drainage pipe M. Suppose that all the water striking the roof goes through pipe M and the initial speed of the water at the top of the downspout is negligible. If the house has side width ω and front length L, at what rainfall rate, k



(volume of water that falls per area per time), will water from pipe M reach the height of the floor drain and threaten to go to the basement (R is small compared to h_1 and h_2)?

4. (20 points)



A heavy, uniform, horizontal plank of mass M rests on top of two identical bicycle wheels which are continuously turned rapidly in opposite directions, as shown. (The plank edge just fits within the curve of the rim of each wheel).

The centers of the wheels are a distance 2L apart. The coefficient of sliding friction between the bar and the wheel surfaces is μ (mu), a constant independent of the relative speed of the two surfaces.

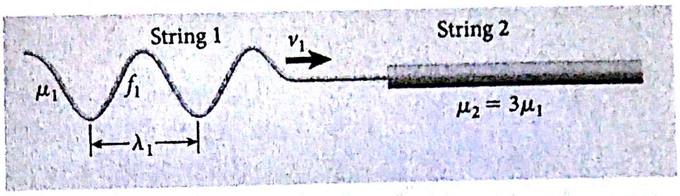
Initially the plank is held at rest with its center at distance x_0 from the midpoint of the wheels. At time t = 0 the plank is released. Because of friction, the plank oscillates back and forth

(x_0 is small compared to L so the plank does not tip. Plank rests within the wheel rim such that there is no motion into or out of the page.)

a) Calculate the normal force on each wheel as a function of distance x from the midpoint of the wheels.
b) Calculate the frictional force acting on the contact point between each wheel and the plank.
c) Prove that the plank will perform simple harmonic oscillation (e.g. write down the differential equation that leads to simple harmonic oscillation for this particular case).
d) Find the period of this oscillatory motion.
e) Find the amplitude and phase of this oscillatory motion.
f) Write $x(t)$.
g) Conceptual problem: What would happen if the tires were rotating in opposite directions (e.g. left tire rotates counterclockwise and right tire rotates clockwise, what would happen to the motion of the plank?

5. **(20 points)**

A sinusoidal wave with an amplitude of A travels to the right at a speed of v_1 along string 1, which has linear mass density μ_1. This wave has frequency f_1 and wavelength λ_1. Since string 1 is attached to string 2 (which has linear mass density $\mu_2 = 3\mu_1$), the first wave will excite a new wave in string 2.



a) What is the direction of the wave in string 2? Explain your reasoning.
b) What is the frequency f_2 of the wave in string 2? Explain your reasoning.
c) What is the velocity v_2 of the wave in string 2?
d) What is the wavelength λ_2 of the wave in string 2?
e) What is the amplitude of the wave in string 2?
f) What percentage of the wave energy of the incoming wave is reflected back from the knot between strings 1 and 2?

6. **(10 points)**

a) If you blow across the opening of a partially filled soda bottle, you hear a tone. If you take a big sip of soda and then blow across the opening again, how will the frequency of the tone change?
b) You blow into an open pipe and produce a tone. By what factor the frequency of the tone changes if you close the end of the pipe and blow into it again?