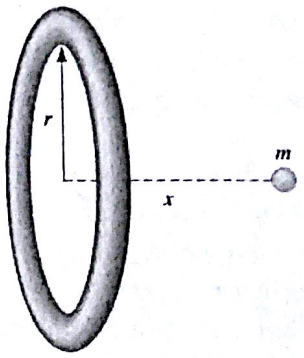
Fall 2015 Physics 7A Lec 002 Yildiz Midterm II

1. **(20 points)**

Several planets (Jupiter, Saturn, Uranus) are encircled by rings, perhaps composed of material that failed to form a satellite. In addition, many galaxies contain ring-like structures. Consider a homogenous thin ring (e.g. assume no thickness) of mass M and outer radius of r.



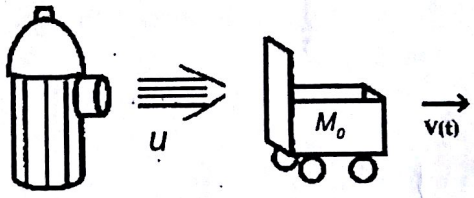
a) Find the direction and magnitude of the gravitational attraction force the ring exerts on a particle of mass m located on the ring's central axis a distance x from the ring center?

b) Suppose the particle falls from rest as a result of the attraction of the ring. What is the speed with which the particle passes through the center of the ring?

2. **(30 points)**

A cart of mass M_0 initially remains at rest on a horizontal surface. At $t = 0$, water is shot from a fire hydrant with horizontal velocity of u and mass per length $\lambda = dm/dl$ towards the cart. In *Case 1*, water that hits the surface of the cart and drips down the ground (e.g. the mass of the cart does not change over time).

Case 1:

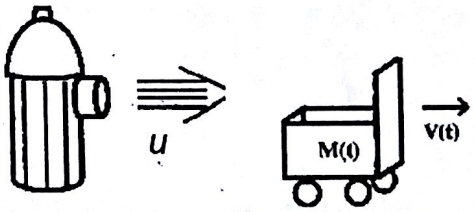


a) What is the mass of water that collides with the cart per unit time (dm/dt) as a function of the velocity v of the cart?

b) Find the velocity $v(t)$ of the cart as a function of time.

c) What is terminal velocity of the cart? (**Note**: You will get partial credit by providing a conceptual answer for terminal velocity, even if you cannot extract it from $v(t)$ in part a).

Case 2:



In *Case 2*, the water collides inelastically, so it sticks to the side of the cart and drips down inside the cart (initial mass and velocity of the cart; and speed and mass per length of the water are identical to that of *Case 1*). As a result, mass of the cart increases as it collects water from the hydrant.

d) **Conceptual Question**: What can you say about the acceleration of the cart in case 2 compared to case 1? Explain your reasoning? Which case has a higher terminal velocity? Which case reaches one half of the terminal velocity more quickly?

e) In *Case 2*, derive an expression for mass $M(t)$ of the cart as a function of time.

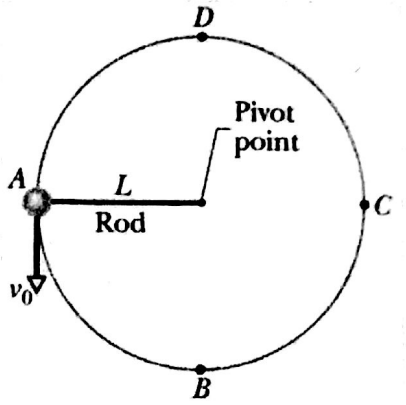
f) In Case 2, find $v(t)$.

g) What is the terminal velocity of the cart? (**Note**: You will get partial credit by providing a conceptual answer for terminal velocity, even if you cannot extract it from $v(t)$ in part e)

(You may ignore gravity and friction in this problem. Assume that water follows a straight path with constant velocity and mass density between the fire hydrant and the cart independent of the distance between them.)

3. **(20 points)**

A massless rigid rod of length L has a ball of mass m attached to one end. The other end is pivoted in such a way that the ball will move in a vertical circle.



First, assume that there is no friction at the pivot. The system is launched downward from the horizontal position A with initial speed v_0. The ball just barely reaches point D and then stops.

a) Derive an expression for v_0 in terms of L, m and g.

b) What is the tension in the rod when the ball passes through point B?

Second, a little grit is placed on the pivot to increases the friction there. Then the ball just barely reaches point C.

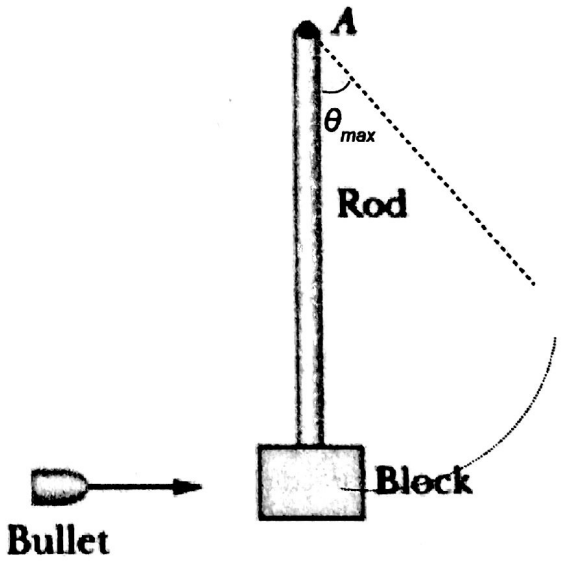
c) What is the decrease in mechanical energy as the ball moves from point A to point C?

d) What is the total distance travelled by the ball before it comes to rest at point B after several oscillations?

e) What is the frictional torque τ_f of the pivot?

4. **(30 points)**

A block of mass m_B is attached to the end of a uniform rod of mass m_R and length of L. The rod is hung down from the ceiling at point A. A bullet with mass m_b is fired into block with a horizontal velocity of v. The bullet collides with the block completely inelastically (meaning that they stick together). The rod-block-bullet system then rotates in the vertical plane, about a fixed axis at point A. Treat the bullet and the block as a particle.



a) What is the distance between the center of mass of the rod-block-bullet system and point A?

b) What is the moment of inertia of the of the rod-block-bullet system about point A?

c) What is the angular velocity (ω_0) of the block right after the collision?

d) After the collision, what is the maximum angle (θ_{max}) the rod-block-bullet system travels in the vertical direction before it stops and starts moving back towards the opposite direction?

(Neglect air resistance and the friction of the pivot point.)