Name ______________________________

1. An isothermal solid sphere is placed in a steady uniform fluid flow with velocity U. We are trying to investigate the steady heat flux of the system. We expect that the heat flux, q, to be a function of the following parameters

Variable	Description	Dimension
D	Diameter	L
η	Fluid Viscosity	M/LT
ρ	Fluid Density	M/L^3
U	Fluid Velocity	L/T
k	Fluid Thermal Conductivity	$ML/T^3\Theta$
c_p	Specific Heat Capacity (per mass)	$L^2/T^2\Theta$
ΔT	Temperature difference between sphere and bulk fluid	Θ
q	Heat flux = heat/(time area)	M/T^3

(a) Fill out the dimension column for each variable. Use Θ for temperature, T for time, L for length and M for mass.

Hint 1: Fourier's law (in one dimension): $q = -k\frac{dT}{dx}$

Hint 2: For a constant pressure process $dQ = c_p \Delta T$ where dQ is heat/mass

(b) How many dimensionless groups do we need to describe the system if we follow Buckingham Pi theorem?

8 variables − 4 dimensions = $\boxed{\text{4 groups}}$

(c) Which of the following groups are valid choices for core variables? **Circle the correct answer or answers (there may be more than one correct answer).**

a. D, η, c_p
b. U, η, q
c. D, U, k, c_p
d. q, ΔT, k, D
e. D, U, η, q

(d) Using core variables D, U, k, c_p, form a non-dimensional group containing η. **Show your work.**

$$\eta D^a U^b k^c c_p^d \; [=] \; \left(\frac{M}{LT}\right)(L)^a\left(\frac{L}{T}\right)^b\left(\frac{ML}{T^3\theta}\right)^c\left(\frac{L^2}{T^2\theta}\right)^d$$

$$\left.\begin{aligned} M &: 1 + c = 0 \\ L &: -1 + a + b + c + 2d = 0 \\ T &: -1 - b - 3c - 2d = 0 \\ \theta &: -c - d = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} c &= -1 \quad a = 0 \\ d &= 1 \quad b = 0 \end{aligned}$$

$$\boxed{N = \frac{\eta c_p}{k}}$$

(e) Ultimately, physical arguments and more sophisticated analysis reveals that the dimensionless relationship below governs the problem:

$$\frac{qD}{k\Delta T} = f\left(\frac{\rho U D}{\eta}, \frac{\eta c_p}{k}\right)$$

You plan to model the heat transfer of silver nanoparticles in a particular organic liquid. The average size of the silver nanoparticles you are interested in have a diameter of D_R = 50 nm (1 nm = 10^{-9} m). However, you only have silver particles with a diameter of D_m = 5 cm (where the subscript m denotes "model") and the organic liquid. How would you design your model experiment to determine the heat flux of silver nanoparticles in the same organic liquid flowing at a velocity U_R?

Same material : $\rho_m = \rho_R$, $\eta_m = \eta_R$.

$$\left(\frac{\eta c_p}{k}\right)_m = \left(\frac{\eta c_p}{k}\right)_R.$$

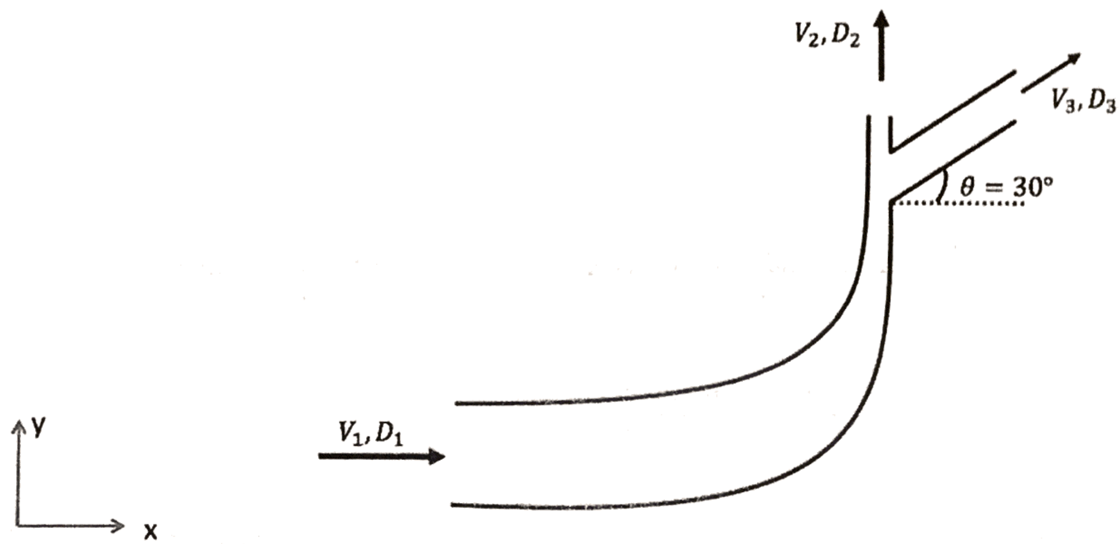
Need $\left(\frac{\rho U D}{\eta}\right)_m = \left(\frac{\rho U D}{\eta}\right)_R \Rightarrow U_m D_m = U_R D_R.$

$$\therefore U_m = \left(\frac{D_R}{D_m}\right) U_R = \boxed{10^{-6}\, U_R}$$

Name ____________________

2. While designing a chemical plant, you are tasked with determining the force exerted on the section of piping sketched below (everything is horizontal, so that gravity acts into the page). Assume an incompressible fluid, turbulent flow, and neglect heat transfer and temperature effects. The fluid is water, and you are given the information below:

Gauge pressures at 1 and 2: $P_1 = 500\ kPa$, $P_2 = 350\ kPa$
$V_1 = 1\ m/s$, $V_2 = 10\ m/s$, $D_1 = 1\ m$, $D_2 = 0.1\ m$, $D_3 = 0.3\ m$



Assume Steady State

a) Determine V_3. **Circle your answer.**

$$0 = \sum_{\text{inlet}} \langle \rho V \rangle_i A_i - \sum_{\text{exit}} \langle \rho V \rangle_i A_i$$

– incompressible, $\langle V \rangle = V$ (turbulent, uniform)

$$0 = \rho V_1 A_1 - \rho V_2 A_2 - \rho V_3 A_3$$

$$V_3 = \frac{V_1 A_1 - V_2 A_2}{A_3} = \boxed{10\ m/s}$$

b) Determine P_3. **Circle your answer.**

- must use conservation of energy
- note 1 inlet 2 exits ⇒ Bernoulli and simplified version w/ Δ do not apply

$$0 = \sum_{inlets} \langle \rho(\cancel{\hat{U}} + \tfrac{1}{2}v^2 + g\cancel{h})V\rangle_i A_i - \sum_{exit} \langle \rho(\cancel{\hat{U}} + \tfrac{1}{2}v^2 + g\cancel{h})V\rangle_i A_i + \sum_{inlet} \langle pV\rangle_i A_i$$
$$- \sum_{exit} \langle PV\rangle_i A_i + \cancel{\dot{Q}_H} + \cancel{\dot{W}_s}$$

$$0 = \tfrac{1}{2}\rho V_1^3 A_1 - \tfrac{1}{2}\rho V_2^3 A_2 - \tfrac{1}{2}\rho V_3^3 A_3 + P_1 V_1 A_1 - P_2 V_2 A_2 - P_3 V_3 A_3$$

$$P_3 = \frac{\rho/2\,[V_1^3 A_1 - V_2^3 A_2 - V_3^3 A_3] + P_1 V_1 A_1 - P_2 V_2 A_2}{V_3 A_3} = \boxed{462 \text{ kPa}}$$

c) Determine the force necessary to hold the piping in place. Use the axes shown in the figure. **Circle your answer.**

Force ⇒ use linear momentum

$$0 = \sum_{in} \langle \rho \underline{v} V_N \rangle_i A_i - \sum_{out} \langle \rho \underline{v} V_N \rangle_i A_i + \sum \underline{F}$$ → includes $P_i \underline{A}_i$ acting into CV

$$x: 0 = \rho_1 V_1 V_1 A_1 - \rho_3 V_3 \overbrace{V_3 \cos\theta}^{\text{x-component}} + P_1 A_1 - P_3 \overbrace{A_3 \cos\theta}^{\text{x-component}} - F_x$$

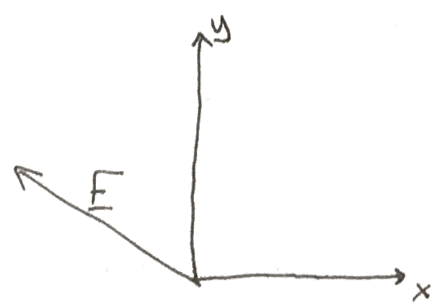
$$\Rightarrow \boxed{F_x = 3.59 \times 10^5 \text{ N}}$$ (by fluid)

$$y: 0 = -\rho_2 V_2 V_2 A_2 - \rho_3 V_3 V_3 A_3 \sin\theta - P_2 A_2 - P_3 A_3 \sin\theta - F_y$$

$$\Rightarrow \boxed{F_y = -2.34 \times 10^4 \text{ N}}$$

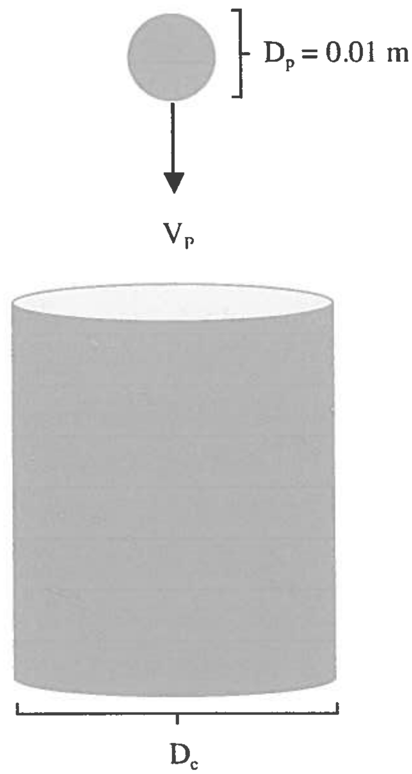
$$|\underline{F}| = \sqrt{F_x^2 + F_y^2} = 3.60 \times 10^5 \text{ N}$$

d) Without doing any additional calculations, **sketch** below the direction of the force to hold the piping in place. Use the coordinate axes indicated on the sketch.



Name Solutions key

3. An aluminum ball ($\varrho_p = 2500$ kg/m^3) is falling through an infinitely large pool of **glycerin** ($\varrho = 1260$ kg/m^3, $\eta = 0.95$ Pa·s) towards the center of a long cylinder with diameter D_c that is also filled with glycerin. After falling for a while, the ball falls through the center of the cylinder, causing a decrease in velocity due to wall effects. Assume the ball instantly reaches a new terminal velocity upon entering the cylinder.



a) Calculate the initial terminal velocity of the ball before it reaches the cylinder. Check any assumptions you make. **Circle your answer.**

Assume Stokes' Flow, Re <1

Equation 4.11 applies

$$V_p = \frac{g D_p^2 (\rho_p - \rho)}{18 \eta} = \frac{(9.8 \text{ m/s}^2)(0.01 \text{ m})^2 (2500 - 1260 \text{ kg/m}^3)}{18(0.95 \text{ Pa}\cdot\text{s})}$$

$$\boxed{V_p = 0.0711 \text{ m/s}}$$

Check Re,

$$Re = \frac{\rho V_p D_p}{\eta} = \frac{(1260\ kg/m^3)(0.0711\ m/s)(0.01\ m)}{0.95\ Pa\cdot s}$$

$$\underline{Re = 0.943 < 1} \quad \checkmark$$

b) What cylinder diameter D_c will result in a 50% reduction in V_p? Check any assumptions you make. **Circle your answer.**

$$V_p' = 0.5\ V_p = 0.035\ m/s$$

$$Re = \frac{\rho V_p' D_p}{\eta} = \frac{(1260\ kg/m^3)(0.035\ m/s)(0.01\ m)}{0.95\ Pa\cdot s}$$

$$\underline{Re = 0.472} \quad \text{Stokes Regime}$$

$$C_D = \frac{8}{\pi}\frac{F_D}{\rho V_p^2 D_p^2} = \frac{24}{Re}\phi,$$

$$\phi = \frac{8}{24\pi}\frac{F_D}{\rho V_p^2 D_p^2}\left(\frac{\rho V_p D_p}{\eta}\right)$$

$$\phi = \frac{F_D}{3\pi D_p V_p' \eta}$$

$$F_D = F_G - F_B$$

$$F_D = \frac{\pi}{6} D_p^3 g(\rho_p - \rho)$$

$$F_D = \frac{\pi}{6}(0.01\ m)^3(9.8\ m/s^2)(2500 - 1260\ kg/m^3)$$

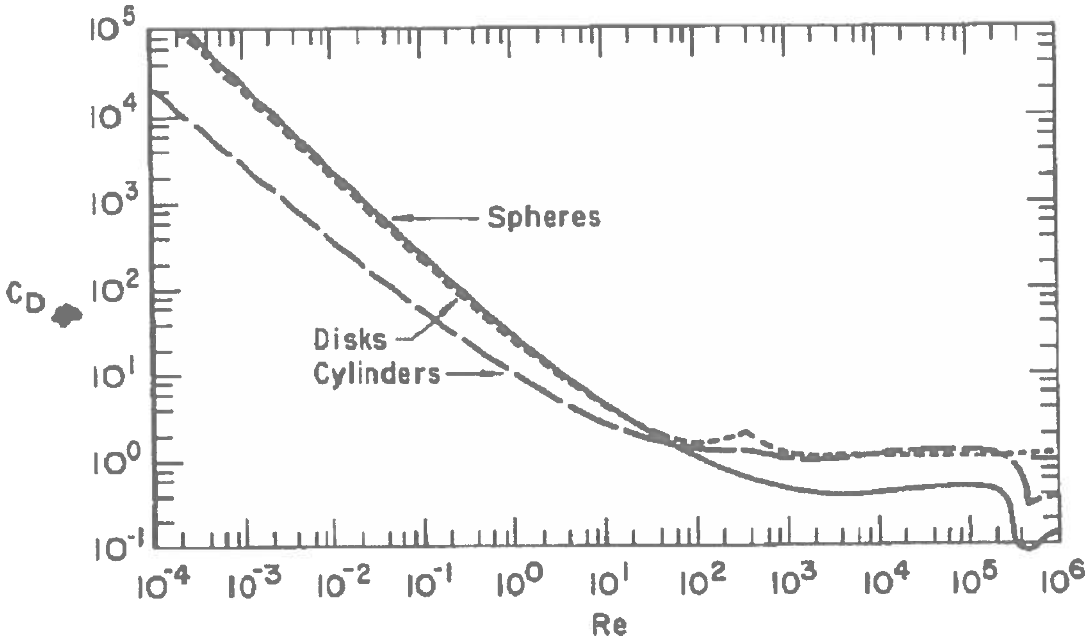
$$F_D = 6.36 \times 10^{-3}\ N$$

$$\phi = \frac{6.36 \times 10^{-3}\ N}{3\pi(0.01\ m)(0.035\ m/s)(0.95\ Pa\cdot s)}$$

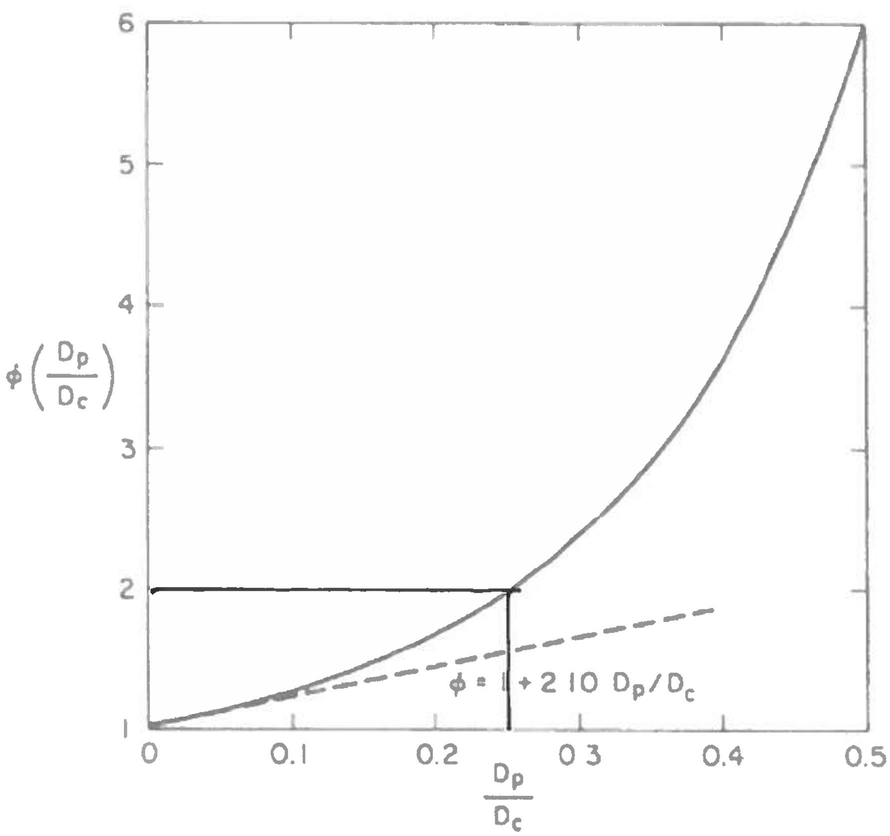
$$\underline{\phi = 2.03}$$

See graph: $\frac{D_p}{D_c} \approx 0.25 \longrightarrow D_c = \frac{D_p}{0.25}$

$$\boxed{D_c = 0.04\ m}$$



10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
C_D
Spheres
Disks
Cylinders
10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5 10^6
Re



6
5
4
3
2
1
$\phi\left(\frac{D_p}{D_c}\right)$
$\phi = 1 + 2.10\ D_p/D_c$
0 0.1 0.2 0.3 0.4 0.5
$\frac{D_p}{D_c}$

Name ____

4. A packed bed reactor consists of a tube of diameter D_1 packed with spherical particles of diameter D_p at a void fraction of ε. The reactor has a viscous, incompressible, Newtonian fluid flowing through it. Two bypass pipes are connected in parallel with the reactor as shown in the figure below. Both pipes are smooth and circular in cross section, the larger pipe has diameter D_2 and the smaller pipe has diameter D_3. Both the reactor and the pipes are horizontal, and the lengths of the packed bed reactor and the two bypass pipes are the same and given by L=1.5 m.

On the smallest bypass pipe, the pressure drop is known over a short section as shown in the figure.

Neglect any pressure drops associated with the flow splits, bends, etc., shown by the thick gray lines in the figure.

Given:

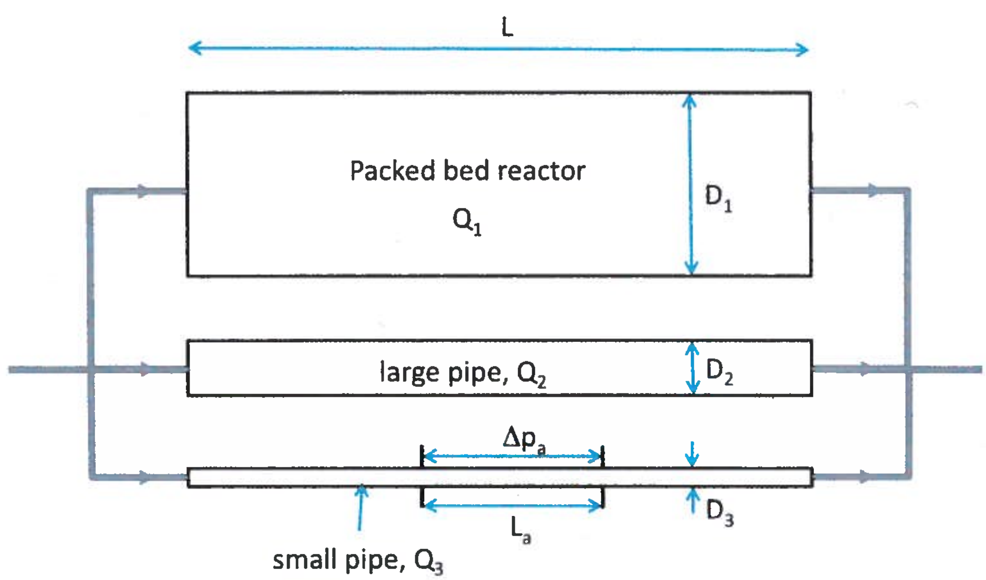
$D_1 = 1.6$ m
$D_p = 6 \times 10^{-4}$ m
$\varepsilon = 0.55$

$D_2 = 0.1$ m

$D_3 = 0.04$ m
$L_a = 0.3$ m
$\Delta p_a = 2000$ Pa
$Q_3 = 0.0045$ m^3/s

L = 1.5 m for the reactor and the two pipes.

The fluid has a density of $\rho = 10^3$ kg/m^3.



a) Given the information above, what is the pressure drop across the total length L of the small pipe? **Circle your answer.**

$$\Delta p_{total} = L\left(\frac{\Delta p_a}{L_a}\right) = \left(\frac{2000\ Pa}{0.3\ m}\right)(1.5\ m) = 10^4\ Pa$$

b) What is the volumetric flow rate Q_2 in the large pipe? Be sure to indicate, and then check if possible, any assumptions you make in your calculation. **Circle your answer.**

To calculate Q_2, we need η, which we can get from pipe 3, for which we know Q_3 and Δp_3

Assuming $Re < 2100$, Hagen-Poiseuille eqn can be re-arranged to

$$\eta = \frac{\pi}{128}\frac{|\Delta p|}{L}\frac{D^4}{Q} = \frac{\pi}{128}\left(\frac{2000\ Pa}{0.3\ m}\right)\cdot\frac{(0.04\ m)^4}{0.0045\ m^3/s} = 0.0931\ Pa\cdot s$$

Check: $$Re = \frac{\rho V_3 D_3}{\eta} = \frac{4\rho Q_3}{\pi D_3 \eta} = \frac{4\left(10^3\frac{kg}{m^3}\right)\left(0.0045\frac{m^3}{s}\right)}{\pi(0.04\ m)(0.0931\ Pa\cdot s)} = 1540\ \checkmark$$

($Re < 2100$)

Now, for pipe 2, $\Delta p_2 = \Delta p_3 = 10^4\ Pa$ (from a)

Since $D_2 >> D_3$, guess turbulent flow

$$Q_2 = 2.26\left(\frac{\Delta p}{L}\right)^{4/7}(\rho^3\eta)^{-1/7}D_2^{19/7} = 4.86\times10^{-2}\ \frac{m^3}{s}$$

Check Re:

$$Re = \frac{4\rho Q_2}{\pi D_2 \eta} = 6.64\times10^3\ \checkmark$$

In correct range for Blasius eqn.

c) What is the volumetric flow rate Q_1 in the packed bed reactor? Be sure to indicate, and then check if possible, any assumptions you make in your calculation. **Circle your answer.**

$$\Delta P_{PB} = \Delta P_1 = \Delta P_2 = \Delta P_3 = 10^4 \text{ Pa}$$

Assume $Re_p < 10$, from Ergun Eqn: $f_p \approx \frac{150}{Re_p}$

$$\frac{\Delta p}{L} = \frac{150 V_\infty \eta (1-\epsilon)^2}{D_p^2 \epsilon^3} = \frac{150 V_\infty (0.093)(1-0.55)^2}{(6\times10^{-4})^2 (0.55)^3} = \frac{10^4}{1.5}$$

$$\Rightarrow V_\infty = 1.412 \times 10^{-4} \text{ m/s}$$

$$Q_1 = V_\infty A = V_\infty \frac{\pi D_1^2}{4} = 2.84 \times 10^{-4} \frac{m^3}{s}$$

Check: $Re_p = \frac{D_p V_\infty \rho}{(1-\epsilon)\eta} = 2.02 \times 10^{-3}$ ✓ $(Re_p < 10)$