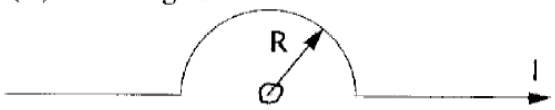
Physics 110A Midterm 1

Given a charge distribution: $\rho = \frac{\rho_0}{r}$ for $r \leq a$ and zero for $r > a$ (ρ_0 is a constant).

a. Find the electric field inside and outside the sphere.
b. Find the electrostatic potential inside and outside the sphere. Take the potential = 0 as $r \rightarrow$ infinity.
(Be sure your potential is continuous at r = a!)

2. An infinitely long wire carries a current I. It is bent so as to have a semicircular detour around the origin, with radius R. Calculate the magnetic induction (**B**) at the origin.



3. Consider the a completely crazy unphysical vector field given by:

$$\mathbf{C}(\mathbf{r}) = \int_V a(\mathbf{r}')\mathbf{R}dv' \quad \text{Where } \mathbf{R} = \mathbf{r} - \mathbf{r}'$$

a. Show that $\mathbf{C}(r) = \nabla\psi(\mathbf{r})$: where $\psi(\mathbf{r})$ is a scalar field and write an integral relation for $\psi(\mathbf{r})$ in terms of $a(\mathbf{r}')$.

b. Find $\nabla \cdot \mathbf{C}(\mathbf{r})$ (Note: it is a constant (*i.e.* the same at all $\mathbf{r}$) independent of the form of $a(\mathbf{r}')$).

c. For $a(\mathbf{r}) = a_0$ for $r \leq b$ and $a(\mathbf{r}) = 0$ for $r > b$: calculate $\nabla \cdot \mathbf{C}$ at all $\mathbf{r}$.

d. Calculate $\mathbf{C}(\mathbf{r})$ at all $\mathbf{r}$.

4. A sphere of radius a has a uniform volume charge density ρ except for a spherical cavity of radius c, at a distance of b from the center of the sphere, where the charge density is zero. Hint this problem is about superposition.

You may use the standard result for the electric field inside a uniform charged sphere:

$$\mathbf{E} = \frac{\rho}{3\varepsilon_0} r\hat{\mathbf{r}} \text{ and } V(r) = -\frac{\rho}{6\varepsilon_0} r^2 \text{ where V=0 at r=0}$$

Find expressions for the electric field and the potential anywhere in the cavity. It is your choice, which you find first. Just be sure that they are mutually consistent.