Physics 7c
Midterm 2
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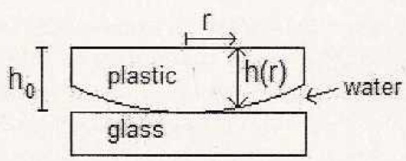
All reasoning should be clear (in particular, in the graphs of problems 3 and 4, everything should be clearly labeled) or credit will not be given.

1. A plastic object with a curved surface (with $n_p = 1.2$) is placed in contact with a flat glass slab (with $n_g = 1.5$) and water ($n_w = 1.33$) fills the area in between, as shown below. The thickness of the plastic object as a function of r (the distance from the center) is ~~$h(r) = r^2/2R$~~ $h(r) = h_0 - \frac{r^2}{2R}$ and the total height from the top of the plastic object to the top of the glass slab is h_0. Monochromatic light of wavelength λ shines on the plastic object from directly above. Neglect reflection from both the top of the plastic object and bottom of the glass slab, and neglect all refraction.



   a) For a beam of light entering at a distance from the center r, what is the difference in path length for the relevant interfering beams?

   b) How far from the center will the first minimum in reflected light occur? (If there is one at the center, write r = 0.)

   c) How far from the center will the first minimum in transmitted light occur? (If there is one at the center, write r = 0.)

2. Coherent light of wavelength λ is incident on a screen with 3 point-like slits, with adjacent slits separated by a distance d. The light passing through these slits then hits a screen. Focus on a point P on the screen at an angle θ away from the slits, and answer the following in terms of λ, d, and θ:

   (i.) Write down the phase difference δ for a pair of neighboring slits.

   (ii.) Draw a phasor diagram representing the electric field contributions from the 3 slits at the point P for an arbitrary δ.

   (iii.) Draw phasor diagrams for the particular values of δ corresponding to

      a) principal (i.e., global) maxima
      b) secondary (i.e., local) maxima
      c) minima

   (**Note**: There are 2 minima and one secondary maxima for $0 \leq \delta < 2\pi$.)

(iv.) What are the values of I/I_0 for the situations in a), b), and c) of part (iii.) above? (I_0 is defined to be the intensity at point P from a single slit, that is, the intensity if two of the point-like slits are blocked.) (*Hint*: look at the phasor diagrams.)

(v.) Sketch a graph of the intensity on the screen given your results from above.

(vi.) Sketch a graph of the intensity if instead the slits are wide (i.e., are not point-like). (The slit width a is still less than the spacing between slits d.)

3. A light source B passes a detector A with speed v and then travels away from A in the +x direction. B emits two short pulses of light, the first when A and B coincide (exactly when B passes A) and the second a time $\Delta t'_B$ later (as measured in B's rest frame, S') sent in the -x direction (toward A).

   a) Draw a spacetime diagram for A's rest frame, S. Include and label A's worldline, B's worldline, the worldline of the second light pulse, and the events of the second light pulse being emitted by B and of being received by A.

   b) Find the time interval Δt_B between the emission of the two pulses as measured in A's rest frame, S (in terms of $\Delta t'_B$ and v). Label Δt_B on your diagram from part a).

   c) Find the spatial separation Δx_A of the emission of the two pulses as measured in A's rest frame, S (in terms of $\Delta t'_B$ and v). Label Δx_A on your diagram from part a).

   d) Find the time interval Δt_A between the arrival of the two light pulses at A, again in A's rest frame, S. (*Hint*: the second pulse travels with speed c.) Write Δt_A in terms of $\Delta t'_B$ and v. Label Δt_A on your diagram from part a).

   e) By interpreting the 2 pulses as successive crests of an electromagnetic wave, use your result from part d) to derive the relation between the frequency of light received and the frequency of light emitted (i.e., the relativistic Doppler effect).

   (The phenomenon that the frequency of emitted and absorbed light are not the same, the relativistic Doppler effect, is fundamentally different from that of sound. For sound waves, one can use the Doppler effect to determine whether it is the source or observer which is moving relative to the medium, air. On the other hand, the relatvitistic effect, which you just derived, cannot be used to determine absolute motion.)

4. A spaceship passes by the Earth with speed v in the +x-direction. The proper length of the ship is L_p. The spaceship fires three rockets simultaneously in its rest frame: one from the front (event 1), one from the middle (event 2) and one from the back (event 3) of the ship.

   i.) Draw a spacetime diagram for the rest frame of the earth. Include and label the worldlines of the front, the middle and the back of the ship. Label the three events of the rockets being fired.

   ii.) Draw the position of the ship at one instant on Earth. What is the ship's length as measured from Earth?

   iii.) In the Earths rest frame, are events 1, 2, and 3 simultaneous? If not, in which order do they occur? Answer this question using only the spacetime diagram in i).

   iv.) Using Lorentz transformations, compute the time interval between events 1 and 2 and between events 2 and 3 (in terms of L_p and v) in Earth's rest frame. Compare to your answer in iii.).