Ch E 150A, Spring 2006
Midterm 1 Solutions

1. This is a falling sphere problem.

To keep spherical catalyst particles from being carried off in the liquid stream, want the particles to be stationary with respect to stream.

This is equivalent to particles falling ~~in a~~ at velocity V in a stationary medium.

⇒ Calculate maximum settling velocity for a falling sphere.

Force balance on falling sphere:

$$C_D = \frac{4}{3} \frac{g D_p (\rho_p - \rho)}{\rho V_p^2}$$

2 possible approaches:

(A) Graphical:

Eliminate unknown V_p from RHS of force balance by multiplying both sides by Re^2

$$C_D Re^2 = \frac{4}{3} \frac{g D_p (\rho_p - \rho)}{\rho V_p^2} \cdot \frac{\rho^2 V_p^2 D_p^2}{\eta^2} = \frac{4}{3} \frac{g D_p^3 (\rho_p - \rho)\rho}{\eta^2}$$

Need to know viscosity of Liquid A (η) to solve.

η can be obtained from capillary viscometry — i.e. we know Δp, L, and Q for stream going to Quality Control lab

using Hagen-Poiseville equation (assuming $Re < 2100$)

$$Q = \frac{\pi}{128} \frac{\Delta p D^4}{L \eta}$$

re-arranging

$$\eta = \frac{\pi}{128} \frac{\Delta p}{L} \frac{D^4}{Q} = 0.29 \text{ Pa-s}$$

(much more viscous than water!)

Check Re for this part of flow (stream to QC lab):

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} = 3.8 \times 10^{-4} \ \frac{m}{s}$$

$$Re = \rho \frac{VD}{\eta} = 1.3 \times 10^{-2}$$

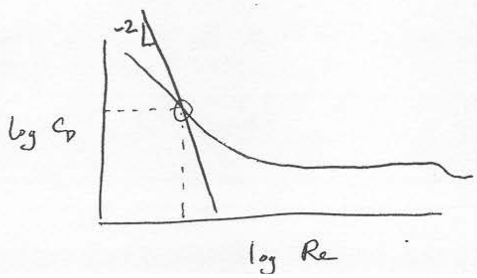
which is < 2100 so Hagen-Poiseuille eqn is okay to use.

Now, can substitute this viscosity into equation above

$$C_D Re^2 = \underbrace{\frac{4}{3} \frac{g D_p^3 (\rho_p - \rho)\rho}{\eta^2}}_{\text{can calculate} = K \text{ (a constant)}}$$

$$C_D Re^2 = K$$

$$C_D = K Re^{-2} \quad \rightarrow \quad \log C_D = \log K - 2 \log Re$$



plot line, read off Re or C_D at intersection, calc. velocity V_p

<u>Alternate Approach</u>:

Assume $Re < 1$ for flow in reactor (Stokes flow)

$$C_D = \frac{24}{Re}$$

substitute into force balance

$$\frac{24}{Re} = \frac{4}{3} \frac{g D_p (\rho_p - \rho)}{\rho V_p^2} = \frac{24 \eta}{\rho V_p D_p}$$

$$V_p = \frac{1}{18} \frac{g D_p^2 (\rho_p - \rho)}{\eta}$$

$$= 1.5 \times 10^{-2} \text{ m/s}$$

Check Re:

$$Re = \frac{\rho V_p D_p}{\eta} = 0.16$$ ✓ okay to use Stokes flow eqn.

<u>Note</u>: If you assumed $\eta = 0.001$ Pa-s (viscosity of water)

then above calc. gives $V_p = 4.42$ m/s

& $Re = 13,200$

2. This is similar to U-bend problem. Start w/ s.s. momentum balance. Be careful to note signs of $\underline{V}_2$, $\underline{A}_2$!

$$0 = \beta_1 w_1 \langle \underline{v} \rangle_1 - \beta_2 w_2 \langle \underline{v} \rangle_2 + p_1 \underline{A}_1 - p_2 \underline{A}_2 - \underline{F} + \int \rho A dz \, \underline{g}$$

a) y-comp

$$0 = 0 - 0 + 0 - 0 - F_y + 0$$

$$\boxed{F_y = 0}$$

x-comp. assume $\beta = 1$ (uniform velocity at plane 1 & plane 2)

$$0 = \rho V_1^2 A_1 - \rho V_2 A_2 (-V_2) + p_1 A_1 - p_2(-A_2) - F_x + 0$$

$$0 = \rho \left(V_1^2 \frac{\pi D_1^2}{4} + V_2^2 \frac{\pi D_2^2}{4} \right) + p_1 \frac{\pi D_1^2}{4} + p_2 \frac{\pi D_2^2}{4} - F_x$$

$$F_x = \frac{\pi}{4} \left[\rho \left(V_1^2 D_1^2 + V_2^2 D_2^2 \right) + p_1 D_1^2 + p_2 D_2^2 \right]$$

From Conserv. of mass $\rho V_1 \frac{\pi D_1^2}{4} = \rho V_2 \frac{\pi D_2^2}{4}$ $\quad V_2 = V_1 \frac{D_1^2}{D_2^2}$

so

$$F_x = \frac{\pi}{4} \left[\rho \left(V_1^2 D_1^2 + V_1^2 \frac{D_1^4}{D_2^2} \right) + p_1 D_1^2 + p_2 D_2^2 \right]$$

$$= \frac{\pi D_1^2}{4} \left[\rho V_1^2 \left(1 + \frac{D_1^2}{D_2^2} \right) + p_1 + p_2 \frac{D_2^2}{D_1^2} \right]$$

using gage pressures, $p_1 = 100$ kPa $\quad p_2 = 0$

3. (20 points)

a) Equation K on the equations sheet attached to this exam describes the conservation of some quantity. Name the quantity:

linear momentum

b) Also with regard to equation K, what does F represent (in words)? Be specific.

net force exerted by fluid on the surroundings

c) What is w_1 in equation K? What are it's dimensions?

mass flow rate into control volume at plane 1

it's dimensions are $\frac{\text{mass}}{\text{tim}}$ (e.g. $\frac{kg}{s}$).

d) In the movies we watched in class, we saw fluid contained in a cylinder, with a drop of colored fluid added. The cylinder was then rotated about it's axis. Briefly describe what happens to the drop of colored fluid when the cylinder rotation is stopped at low Re and at high Re.

At low Re, where inertial forces are small, the fluid motion stops instantaneously when the cylinder wall rotation stops.

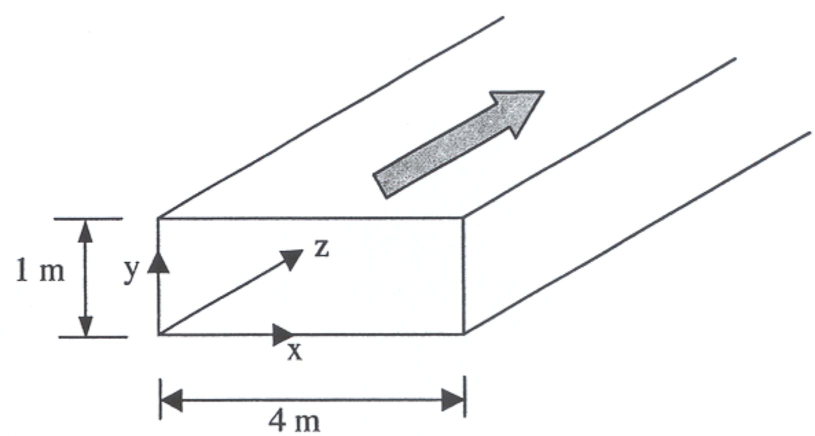
At high Re, inertia causes the dye (& the other fluid) to continue moving or "coasting" even after the wall motion stops.

e) In the conduit shown below, the velocity is independent of y, but depends on x as indicated. What is the average velocity $\langle v_z \rangle$?

$v_z = 1\, m/s \quad for \quad 0 < x < 1$

$v_z = 5\, m/s \quad for \quad 1 < x < 2$

$v_z = 2\, m/s \quad for \quad 2 < x < 4$



$$\langle v_z \rangle = \frac{1}{A} \int_{y=0}^{y=1} \int_{x=0}^{x=4} v_z \, dx\, dy$$

$$= \frac{1}{(1)(4)} \, y\Big|_0^1 \left[\int_{x=0}^{1} (1)\, dx + \int_1^2 (5)\, dx + \int_2^4 (2)\, dx \right] = \frac{1}{4}(1)\left[x\Big|_0^1 + 5x\Big|_1^2 + 2x\Big|_2^4 \right]$$

$$= \frac{1}{4}\left[1 + 5 + 4\right] = \frac{10}{4}\ m/s = 2.5\ \frac{m}{s}$$