Solutions

EECS 20, Section 2, Fall 2015

Midterm #2, November 5, 2015

Name: Shyam Parekh SID: ____________

Problem 1

Select one or more correct choices for each question. NO PARTIAL CREDITS.

1.1 (5 Points) A system represented by a set of LCCDEs is causal and LTI if and only if

a. All auxiliary conditions are set to 0.
b. The set of LCCDEs have the same number of terms corresponding to the inputs and the outputs.
c. The condition of the initial rest is satisfied.
d. The LCCDEs have removable poles.

1.2 (5 Points) Suppose a set of LCCDEs represent an LTI system, and the Frequency Response is given by $H(\omega) = \frac{1+2e^{-i2\omega}}{4+e^{-i\omega}-3e^{-i3\omega}}$. Then the Difference Equation satisfied by the system is

a. *y(n) + 2y(n-2) = 4x(n) + x(n-1) -3x(n-3).*
b. *4y(n) + y(n-1) – 3y(n-3) = x(n) + 2x(n-2).*
c. *y(n) + 4y(n-1) + 2y(n-3) = x(n) + 2x(n-2).*
d. None of the above.

1.3 (5 Points) For a given discrete-time system we know that input of δ(n) produces the output of δ(n+1) + δ(n). Then,

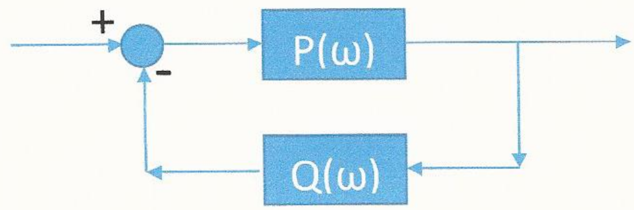
a. The system cannot be causal.
b. The system cannot be memoryless.
c. The system can be causal, but not necessarily.
d. The system can be memoryless, but not necessarily.

1.4 (5 Points) A system is defined to be BIBO stable if and only if

a. The system output is always bounded.
b. The system only accepts bounded input.
c. The system has unbounded output only when the input is unbounded.
d. None of the above.

Problem 2

Consider the discrete-time feedback system shown below:



2.1 (5 Points) Corresponding to the Frequency Response $P(\omega)$, the Impulse Response is given by $p(n) = (0.2)^n u(n), n \in \mathbf{Z}$. Find $P(\omega)$.

$$P(w) = \sum_{n\in Z} p(n) e^{-inw} = \sum_{n \in Z} (0.2)^n u(n) e^{-inw}$$

$$= \sum_{n=0}^{\infty} (0.2)^n e^{-inw} = \sum_{n=0}^{\infty} (0.2 e^{-iw})^n$$

$$= \frac{1}{1 - 0.2 e^{-iw}}$$

2.2 (5 Points) The subsystem with the Frequency Response $Q(\omega)$ satisfies the input-output relationship $y(n) = 0.5 * (x(n) + x(n-1)), n \in \mathbf{Z}$. Find $Q(\omega)$.

Let $x(n) = e^{inw}$. Then,

$$y(n) = 0.5 (e^{inw} + e^{i(n-1)w})$$

$$= 0.5 e^{inw} (1 + e^{-iw})$$

Since we expect $y(n) = e^{inw} Q(w)$,

$$Q(w) = 0.5(1 + e^{-iw})$$

2.3 (10 Points) Let $h(n), n \in Z$ be the Impulse Response of the overall feedback system. Find $h(n)$.

By Black's Equation, $H(\omega) = \dfrac{\dfrac{1}{1-0.2e^{-i\omega}}}{1 + \dfrac{0.5(1+e^{-i\omega})}{1-0.2e^{-i\omega}}}$

Simplifying, $H(\omega) = \dfrac{1}{1-0.2e^{-i\omega} + 0.5 + 0.5e^{-i\omega}}$

$= \dfrac{1}{1.5+0.3e^{-i\omega}} = \dfrac{2}{3} \dfrac{1}{1+0.2e^{-i\omega}}$

Since $\sum_{n=0}^{\infty} \beta^n = \dfrac{1}{1-\beta}$, $h(n) = \frac{2}{3}(-0.2)^n u(n)$.

2.4 Fill in each blank with LPF, HPF, Comb Filter, or Notch Filter.

a. (5 Points) The subsystem $P(\omega)$ is a LPF.

b. (5 Points) The subsystem $Q(\omega)$ is a LPF. (since Q corresponds to two-point moving average)

c. (5 Points) The overall feedback system is a HPF.

For a. & c., recall that impulse response $\alpha^n u(n)$ corresponds to a LPF for $\alpha \in (0,1)$, and a HPF for $\alpha \in (-1,0)$.

2.5 (5 Points) Is the overall feedback system BIBO stable? Justify your answer.

Since the system is LTI, BIBO stable

iff $\sum_{n \in \mathbb{Z}} |h(n)| < \infty$

Here, we have $\sum_{n \in \mathbb{Z}} |h(n)| = \frac{2}{3} \sum_{n \in \mathbb{Z}} |(-0.2)^n|$

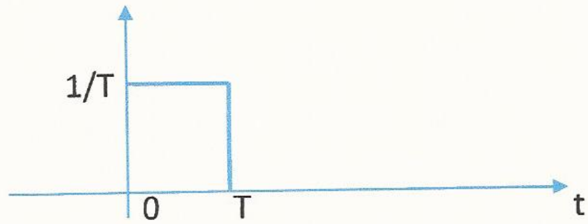
$= \frac{2}{3} \sum_{n \in \mathbb{Z}} (0.2)^n = \frac{2}{3} \cdot \frac{1}{1-0.2}$

$= \frac{5}{6} < \infty$

Hence, BIBO stable.

Problem 3

Suppose the Impulse Response *h(t)* of a continuous-time LTI system is as shown below:



3.1 (10 Points) Find the Frequency Response $H(\omega)$.

$$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-i\omega t}\, dt = \frac{1}{T}\int_0^T e^{-i\omega t}\, dt$$

$$= \frac{1}{T} \left.\frac{e^{-i\omega t}}{-i\omega}\right|_0^T = \frac{1}{T}\cdot\frac{1}{i\omega}\left(1 - e^{-i\omega T}\right)$$

$$= e^{-i\omega T/2} \left(\frac{e^{i\omega T/2} - e^{-i\omega T/2}}{2i}\right)\cdot\frac{1}{\omega T/2}$$

$$= e^{-i\omega T/2}\, \frac{\sin \omega T/2}{\omega T/2} = e^{-i\omega T/2}\cdot \mathrm{sinc}(\omega T/2)$$

3.2 (10 Points) For a general input signal *x(t)*, how is the output *y(t)* is related to the input *x(t)*?

$$y(t) = h * x(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \frac{1}{T}\int_0^T x(t-\tau)\, d\tau$$

Let $t - \tau = u$. Then, $d\tau = -du$, $\tau = 0 \Rightarrow u = t$ &
$\tau = T \Rightarrow u = t - T$

$$y(t) = -\frac{1}{T}\int_t^{t-T} x(u)\, du = \frac{1}{T}\int_{t-T}^{t} x(u)\, du$$

i.e. $y(t)$ is average of $x(t)$ over the previous T units of time (over $t-T$ to t).

3.3 (5 Points) Is the system causal? Justify your answer.

Causal. Since $y(t) = \frac{1}{T}\int_{t-T}^{t} x(u)\,du$, $y(t)$

only depends on the present & past values of

$x(t)$.

3.4 (5 Points) Is the system memoryless? Justify your answer.

Not momoryless. Since $y(t) = \frac{1}{T}\int_{t-T}^{t} x(u)\,du$,

$y(t)$ clearly depends on the past values

of $x(t)$.

3.5 (10 Points) For the input $x(t) = e^{i\omega t}, t \in \mathbf{R}$, we find that the output is always 0 at the frequencies $\omega = \pm k\pi, k = 1, 2, 3, \ldots$. Find the value of T.

· For input $e^{j\omega t}$, output is $e^{j\omega t} H(\omega)$.

So, output will be ϕ at zero's of $H(\omega)$.

Now, $H(\omega) = e^{-i\omega T/2} \operatorname{sinc}(\omega T/2)$

Hence, $|H(\omega)| = 0$ iff $\omega T/2 = \pm m\pi$ for some $m \in \{1, 2, 3, \cdots\}$.

Since we want $|H(\omega)| = 0$ $\forall \pm k\pi$, $k \in \{1, 2, 3, \cdots\}$,

the possible values of T are $\{2, 4, 6, \cdots\}$.

(Smallest possible value of T is 2).