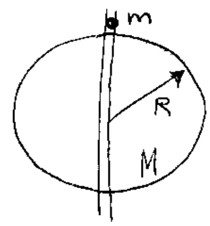
1.

(a)



@ radius r, particle feels force only due to mass at radius $\leq r$

$$F(r) = -G\frac{m M(r)}{r^2}$$

where $M(r) = M\left(\frac{r}{R}\right)^3$

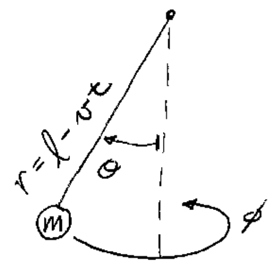
Thus $F(r) = -\frac{GmM}{R^3}\, r = m\ddot{r}$

This is simple harmonic motion w/ $\omega = \sqrt{\frac{GM}{R^3}}$

(b) Initially, $E = -G\frac{mM}{R}$

Since E is conserved, this holds for all time

2.



spherical coordinates

$$T = \frac{m}{2}\left[\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2\right]$$

$$= \frac{m}{2}\left(v^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2\right)$$

$$V = -mgr\cos\theta$$

(a) $L = T - V = \frac{m}{2}\left(v^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2\right) + mgr\cos\theta$

where $r = l - vt$

(b) $\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta}$ $\qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = 2mr\dot{r}\dot{\theta} + mr^2\ddot{\theta}$

$$= -2mvr\dot{\theta} + mr^2\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = mr^2\sin\theta\cos\theta\,\dot{\phi}^2 - mgr\sin\theta$$

$$\Longrightarrow \boxed{\ddot{\theta} = \frac{2v}{r}\dot{\theta} + \sin\theta\cos\theta\,\dot{\phi}^2 - \frac{g}{r}\sin\theta}$$

Since $\frac{\partial L}{\partial \phi} = 0$ we also have $p_\phi = \frac{\partial L}{\partial \dot{\phi}} = \text{const}$

$$\Longrightarrow \boxed{mr^2\sin^2\theta\,\dot{\phi} = \text{const}}$$

(c) From above, $p_\theta = mr^2\dot{\theta}$ and $p_\phi = mr^2\sin^2\theta\,\dot{\phi}$

$$H = p_\theta\dot{\theta} + p_\phi\dot{\phi} - L = \frac{m}{2}r^2\dot{\theta}^2 + \frac{m}{2}r^2\sin^2\theta\,\dot{\phi}^2 - mgr\cos\theta - \frac{m}{2}v^2$$

$$= \frac{p_\theta^2}{2mr^2} + \frac{p_\phi^2}{2mr^2\sin^2\theta} - mgr\cos\theta - \frac{mv^2}{2}$$

(d) $H \neq E$ This is clear by inspection since H has term "$-\frac{mv^2}{2}$" which is [redacted] positive in E

(e) $\frac{dH}{dt} = \frac{\partial H}{\partial t} \neq 0$ since $\frac{\partial r}{\partial t} = -v$

3.

(a) $F = m\ddot{x} = -m\omega_0^2 x - \gamma m v + qE_0 \cos\omega t$

(b) $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \frac{qE_0}{m} \cos\omega t$

This is the driven oscillator of §3.6 with
$\beta = \gamma/2$; $A = qE_0/m$

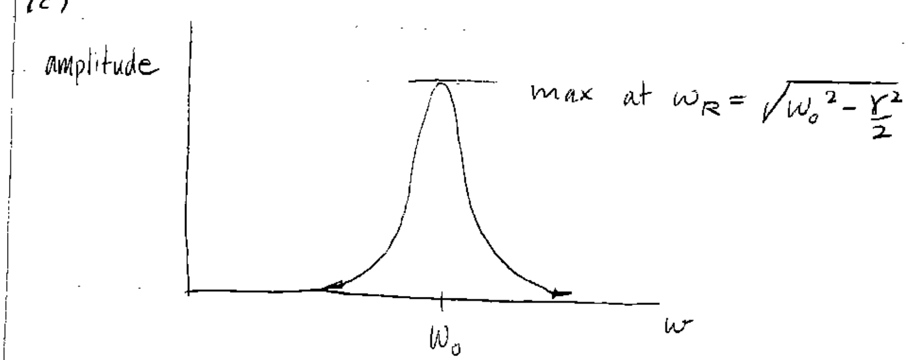
Steady state solution given in text as

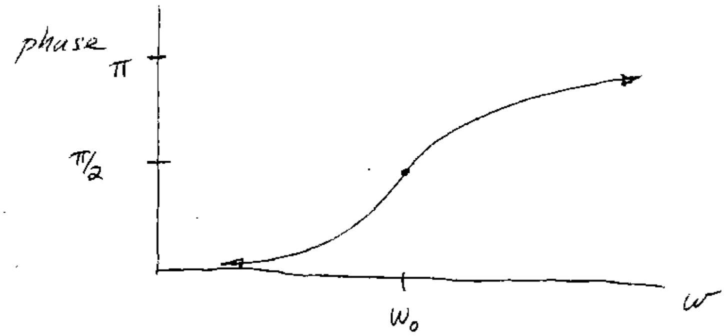
$$x(t) = \frac{A}{\sqrt{(\omega_0^2-\omega^2)^2+4\omega^2\beta^2}} \cos(\omega t - \delta)$$

$$= \frac{qE_0/m}{\sqrt{(\omega_0^2-\omega^2)^2+\omega^2\gamma^2}} \cos(\omega t - \delta)$$

where $\delta = \tan^{-1}\left(\frac{\omega\gamma}{\omega_0^2-\omega^2}\right)$

(c)





(d) $\langle T \rangle = \langle \frac{1}{2} m \dot{x}^2 \rangle = \frac{m}{2} \langle \dot{x}^2 \rangle$

$$\dot{x} = \frac{qE_0/m}{\sqrt{(\omega_0^2-\omega^2)^2+\omega^2\gamma^2}} (-\omega) \sin(\omega t - \delta)$$

Since $\langle \sin^2 \rangle = \frac{1}{2}$, $\langle T \rangle = \frac{m}{2} \left(\frac{qE_0}{m} \right)^2 \frac{\omega^2}{(\omega_0^2-\omega^2)^2+\omega^2\gamma^2} \left(\frac{1}{2}\right)$

$$\langle T \rangle = \frac{q^2 E_0^2}{4m} \cdot \frac{\omega^2}{(\omega_0^2-\omega^2)^2+\omega^2\gamma^2}$$

(e) $\langle T \rangle$ is maximized when $\frac{\partial}{\partial \omega} \left[\frac{\omega^2}{(\omega_0^2-\omega^2)^2+\omega^2\gamma^2} \right] = 0$

$$\Rightarrow \left[(\omega_0^2-\omega^2)^2 + \cancel{\omega^2\gamma^2} \right] 2\omega - \omega^2 \left[2(\omega_0^2-\omega^2)(-2\omega) + \cancel{2\omega\gamma^2} \right] = 0$$

$$\Rightarrow (\omega_0^2-\omega^2)^2 2\omega = \omega^2 (-4\omega)(\omega_0^2-\omega^2)$$

$$\Rightarrow \omega = 0, \omega_0$$

$\omega = 0 \Rightarrow T = 0$ so max $\langle T \rangle$ occurs when $\boxed{\omega = \omega_0}$

(f) max amplitude occurs at $\omega_R = \sqrt{\omega_0^2 - \frac{\gamma^2}{2}} \neq \omega_0$

If energy were constant for the system, maximum $\langle T \rangle$ would correspond to maximum amplitude (ie, maximum $\langle u \rangle$) Since $\omega_R \neq \omega_0$ we conclude that energy is not conserved.