Astronomy 7A Midterm #2

November 12, 2013

Name: ______________________________

Section: ______________________________

There are 4 problems. The 4b is optional and will give you bonus points.

Write your answers on these sheets showing all of your work. It is better to show some work without an answer than to give an answer without any work. Feel free to use the backs of pages as well, but please clearly label which work corresponds to which problem.

If you do not know the answer to a particular question, but you need the answer for the next question, use a variable instead.

Calculators are allowed to perform arithmetic. Please turn off all cellphones.

If you have any questions while taking the midterm, get the attention of one of the GSIs or the instructor.

Budget your time; you will have from 12:40 pm to 2:00 pm to complete the exam. Of course, you are free to hand in your exam before 2:00 pm. Make sure that you have time to at least briefly think about every required question on the midterm.

You do not need to work on the questions in order, so it is OK to skip a question and come back to it later.

On my honor, I have neither given nor received any assistance in the taking of this exam

Signed: ______________________________

Constants

c $= 3.00 \times 10^8$ m/s
h $= 6.63 \times 10^{-34}$ m^2 kg s^{-1}
$\sigma_{\rm SB} = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
$k_{\rm B} = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
$G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
$m_{\rm p} = 1.673 \times 10^{-27}$ kg
$m_{\rm n} = 1.675 \times 10^{-27}$ kg
$m_{\rm e} = 9.11 \times 10^{-31}$ kg
$m_{\odot} = 1.90 \times 10^{30}$ kg
$m_{\oplus} = 5.97 \times 10^{24}$ kg
1 eV $= 1.602 \times 10^{-19}$ J (kg m^2 s^{-2})
1 eV/$c^2 = 1.783 \times 10^{-36}$ kg
1 AU $= 1.5 \times 10^{11}$ m
1 pc $= 3.09 \times 10^{16}$ m

Some Useful Formulae

Kepler's Third Law:

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

Time-averaged graviational potential energy for the relative 2-body orbit

$$U = -\frac{G m_1 m_2}{a}$$

Virial theorem:

$$-2\langle K \rangle = \langle U \rangle$$

Mass Function:

$$f \equiv \frac{m_2^3}{(m_2 + m_1)^2} \sin^3 i = \frac{P}{2\pi G} v_{r,1}^3$$

Photon energy:

$$E = \frac{hc}{\lambda}$$

Planck Function:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/(\lambda k_\mathrm{B} T)} - 1}$$

Wien Peak Law for B_λ:

$$h\nu_\mathrm{peak} \approx 5k_\mathrm{B}T$$

Energy Levels of Hydrogen:

$$E_n = \frac{-13.606 \text{ eV}}{n^2}$$

Degeneracy of Neutral Hydrogen:

$$g_n = 2n^2$$

Maxwell-Boltzmann Distribution of Velocities:

$$dn/dv = n \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/(2kT)} 4\pi v^2$$

Relative Boltzmann Probabilities:

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

Partition Function For Use in Saha Equation:

$$Z = \sum_i g_i e^{-(E_i - E_1)/(kT)}$$

Saha Equation (for state I vs. II; but generalizable):

$$\frac{n_\mathrm{II} n_e}{n_\mathrm{I}} = \frac{2Z_\mathrm{II}}{Z_\mathrm{I}} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_1/kT}$$

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

Ideal gas law

$$P = nkT$$

1 Super-Earth [12 points]

A telescope is monitoring the brightness of a star-planet system over the time period of a few months. Figure 1 presents the brightness profile, which follows a sinusoidal shape with several dips. Please note that the planet does not radiate by itself (i.e., no fusion), thus any light we see from the planet is reflected light from the star.

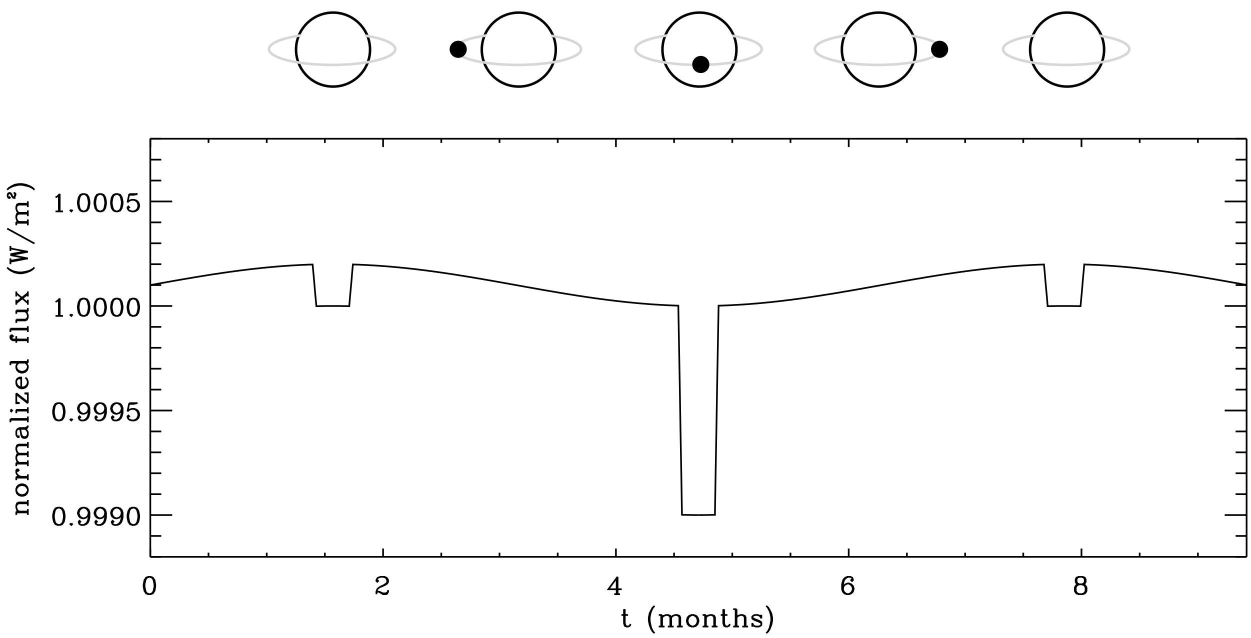

Figure 1: Flux of a star-planet system vs. time. The pictures above the diagram are shown only as an illustration, the scaling is incorrect.

(a) [3 points] Explain the behavior of the brightness curve, including the origin of the sinusoidal shape. Describe the different phases. You may use drawings to illustrate your explanation.

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(b) [2 points] How much smaller is the radius of the planet compared to the radius of the star?

(c) [4 points] From radial velocity measurements we know that the massive star wobbels, with a maximum radial velocity of 10 m/s. Furthermore, from spectral classification we know that the star has a mass of 2 $M_{\odot}$. What is the mass of the companion in solar masses? And in earth masses? (tip: do your calculation in solar masses instead of kg.)

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(d) [3 points] What is the semi-major axis of the planet's orbit in AU?

2 Vega [8 points]

Vega is a A0 V star with a surface temperature of ∼10 000 K. During class we have seen that Balmer lines are strongest around this temperature. Now let's take a look at the Paschen lines in Vega. Paschen lines are created when hydrogen electrons are in the n=3 state. If photons with Paschen wavelengths encounter an hydrogen atom with an electron in the n=3 orbital, they can excite the electrons to even higher orbits, thus creating absorption lines at these specific wavelengths.

(a) [3 points] What is the minimum velocity needed in order to excite an electron from the $n = 2$ to the $n = 3$ orbital in a collision between 2 hydrogen atoms? Assume that one of the two particles is at rest.

(b) [3 points] What is the ratio of hydrogen atoms with electrons in n=3 to n=2 orbitals in the photosphere of the star (i.e., what is N_3/N_2)?

(c) [2 point] How much longer is the mean free path of a Paschen-α photon compared to a Balmer-α photon in the photosphere of the star? Thus, for which of the two wavelengths do we look deeper into the photosphere of the star? Assume that the cross section is the same for a hydrogen n=2 and n=3 atom.

3 A stellar core [7 points]

The temperature in the center of a star is high enough to allow for fusion of Hydrogen. In the initial state, the hydrogen was all ionized, with one electron available for each proton. In the final state, all hydrogen in the core has been burned into (ionized) helium-4 through the PPI chain. The net product of the PPI chain is:

$$4\,^1_1\mathrm{H} \rightarrow \,^4_2\mathrm{He} + 2e^+ + \nu_e$$

Positrons will immediately react with free electrons and release 2 photons.

$$e^+ + e^- \rightarrow 2\gamma$$

a. [3 points] What is the initial mean molecular mass in the core? What is the mean molecular mass, after all hydrogen has been burned into helium? Do not include photons and neutrinos in your calculation.

b. [2 points] Given that the temperature and density stay the same in the core, by how much will the pressure change?

c. [2 points] What will happen to the the size of the core? Please give a qualitative answer and use the hydrostatic equilibrium in your reasoning.

4 Where is Bruce Willis when you need him?! [5 points + 4 bonus points]

Last week a Berkeley undergrad discovered an asteroid on a collision course with the earth! We must act fast if we are going to prevent the total destruction of the Earth. New observations indicate that the asteroid has a semi-major axis of $a = 3$ AU, and an eccentricity of $e = 0.8$.

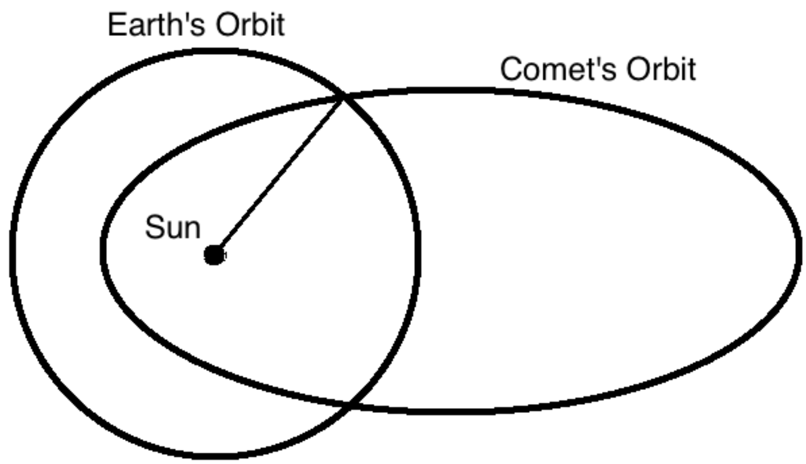

Figure 2: The orbit of the comet and the earth, showing where the collision will happen.

(a) [5 points] Use the virial theorem to show how fast the asteroid will be moving when its orbit intersects the Earth's orbit ? Assume that the earth's orbit is a perfect circle with a radius of 1 AU, and that the asteroid is only gravitationally interacting with the Sun.

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(b) [BONUS: 4 points] Now things get explosive! We are going to use a hydrogen bomb (which fuses hydrogen by the PPI chain) to stop the asteroid in its tracks just before it hits Earth. The asteroid is a nearly perfect sphere made entirely of iron, which has a density of 8000 kg/m^3 and it has a radius of 50 m. Calculate the mass of hydrogen that our bomb must fuse into helium-4 in order to completely balance the kinetic energy of the asteroid. When 4 hydrogen nuclei are converted into one helium-4 nucleus, the energy output is 26.22 MeV. Assume that 100% of the hydrogen bomb's energy goes into the kinetic energy of the asteroid.