Astronomy 7A Midterm #1

October 8, 2013

Name: ______________________________

Section: ______________________________

There are 3 problems. The second part of the 3rd problem is optional and will give you bonus points.

Write your answers on these sheets showing all of your work. It is better to show some work without an answer than to give an answer without any work. Feel free to use the backs of pages as well, but please clearly label which work corresponds to which problem.

If you do not know the answer to a particular question, but you need the answer for the next question, use a variable instead.

Calculators are allowed to perform arithmetic. Please turn off all cellphones.

If you have any questions while taking the midterm, get the attention of one of the GSIs or the instructor.

Budget your time; you will have from 12:40 pm to 2:00 pm to complete the exam. Of course, you are free to hand in your exam before 2:00 pm. Make sure that you have time to at least briefly think about every required question on the midterm.

You do not need to work on the questions in order, so it is OK to skip a question and come back to it later.

On my honor, I have neither given nor received any assistance in the taking of this exam

Signed: ______________________________

Constants

c = 3.00×10^{10} cm/s = 3.00×10^{8} m/s
h = 6.623×10^{-27} erg s = 6.63×10^{-34} m^2 kg s^{-1}
$\sigma_{\rm SB} = 5.67 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4} = 5.67×10^{-8} W m^{-2} K^{-4}
$k_{\rm B} = 1.38 \times 10^{-16}$ erg/K = 1.38×10^{-23} J/K
$G = 6.67 \times 10^{-8}$ dyne cm^2/g^2 = 6.67×10^{-11} m^3 kg^{-1} s^{-2}
$m_p = 1.673 \times 10^{-24}$ g = 1.673×10^{-27} kg
$m_n = 1.675 \times 10^{-24}$ g = 1.675×10^{-27} kg
$m_e = 9.11 \times 10^{-28}$ g = 9.11×10^{-31} kg
$L_\odot = 3.90 \times 10^{33}$ erg/s = 3.90×10^{26} W
$M_\odot = 2.0 \times 10^{33}$ g = 2.0×10^{30} kg
$R_\odot = 7.0 \times 10^{10}$ cm = 7.0×10^{8} m
$T_\odot \approx 5800$ K (surface temperature)
1 AU = 1.5×10^{13} cm = 1.5×10^{11} m
1 pc = 3.09×10^{18} cm = 3.09×10^{16} m
1 eV = 1.602×10^{-12} erg = 1.602×10^{-19} J
1 radian = 206265 arcsec
1 Angstrom = 10^{-8} cm = 10^{-10} m
1 year = 12 months = 365 days = 3.15×10^{7} s

Some Useful Formulae

Classical Doppler Shift ($v_r \ll c$):

$$\frac{(\lambda_{\text{observed}} - \lambda_{\text{rest}})}{\lambda_{\text{rest}}} = v_r/c$$

Apparent magnitudes difference:

$$m_1 - m_2 = -2.5 \log \frac{F_1}{F_2}$$

Stefan Boltzmann equation (A=area):

$$L = A\sigma_{\text{SB}} T^4$$

Photon energy:

$$E = \frac{hc}{\lambda}$$

Planck Function:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/(\lambda k_{\text{B}} T)} - 1}$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/(k_{\text{B}} T)} - 1}$$

Rayleigh-Jeans law:

$$B_\lambda(T) \approx \frac{2ck_{\text{B}}T}{\lambda^4}$$

Diffraction Limit:

$$\theta_{\text{diff}} = 1.22 \, \frac{\lambda}{D}$$

Plate scale

$$\frac{d\theta}{dy} = \frac{1}{f}$$

Energy Levels of Hydrogen:

$$E_n = \frac{-13.606 \text{ eV}}{n^2}$$

1 Celestial motions [24 points]

A telescope is monitoring the celestial motion of star HIP-7A2013, as shown in Figure 1. HIP-7A2013 shows a spiraling movement due to the velocity of the star with respect to the sun, and the motion of the earth around our sun.

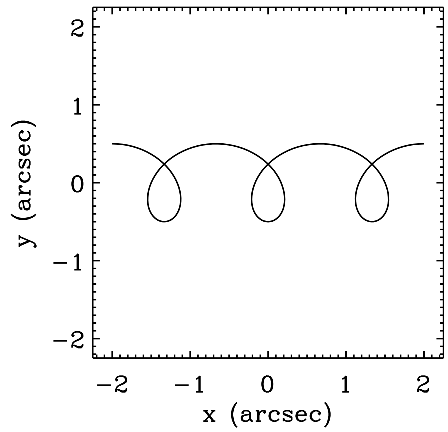

Figure 1: 3-yr path of HIP-7A2013 on the sky. (Note: these are not RA and DEC, so no $\cos\delta$ correction is needed.)

(a) [2 points] What is the parallax angle of HIP-7A2013 in arcsec?

(b) [2 points] What is the distance to HIP-7A2013 in parsec?

(c) [2 points] What is the proper motion of HIP-7A2013 in arcsec per year?

(d) [4 points] What is the tangential velocity (velocity in the plane of the sky) in km/s?

Another telescope took a spectrum of the same star, as show in Figure 2. The absorption line that is shown is Hα, which indicates the transition of an electron from the n=2 to the n=3 level for a hydrogen atom.

(e) [4 points] What is the rest-frame wavelength that corresponds to the Hα line?

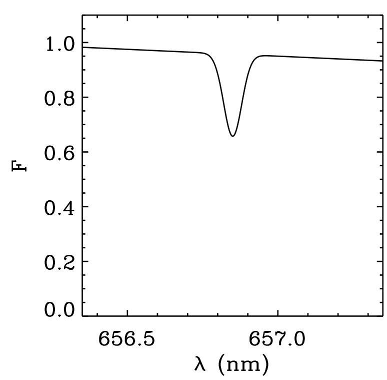

Figure 2: The Hα absorption line of HIP-7A2013

(f) [2 points] Is HIP-7A2013 moving away from us or toward us?

(g) [4 points] What is the radial velocity from the classical Doppler shift of HIP-7A2013 in km/s?

(h) [4 points] What is the total velocity of HIP-7A2013? Account for both tangential and radial components.

2 Observing a binary star [26 points]

We observe a binary star system, for which the two stars are separated by $1.5''$ on the sky. Star A has a temperature of 30,000 K and star B has a temperature of 40,000 K. Assume that the stars have the same size and thus the same surface area.

(a) [1 point] Which star will be brighter at 500 nm?

(b) [3 points] Assuming that the spectra can be approximated by black body curves, what is the ratio of the bolometric luminosities of the two stars.

(c) [2 points] What is the difference in bolometric magnitude? Which star has a larger magnitude?

(d) [4 points] We are now observing the star with a filter between 20 and 25 μm. At these long wavelengths we can use the Rayleigh-Jeans law to approximate the Planck Function. Show how we get from the Planck function to the Rayleigh-Jeans law.

(e) [5 points] What is the approximate relative intensity of the two stars in B_λ for the used filter? Use the Rayleigh-Jeans law and assume that the filter has a constant response between 20 and 25 μm.

(f) [2 points] What is the relative intensity of the two stars in B_ν for the same wavelength interval?

(g) [3 points] For the observations we use a space telescope, which means that our observations are diffraction limited. How large should the telescope mirror be in order to resolve the two stars separately using the 20-25 μm filter?

(h) [2 points] We actually use a telescope with a diameter of 5 meters and the pixel scale of the detector is $0.25''$/pixel. What is the diffraction limit of the telescope for the used filter in pixels?

(j) [1 point] How many pixels are the stars apart?

(i) [3 points] Given that one pixel on the detector is 9 μm in size, what is the focal length of the telescope?

3 A proto-planetary disk [12 points + 6 BONUS points]

A young star has a rotating hydrogen disk around it. Take the star to have a flat spectrum, $F_\lambda = F_0$ for all λ, and ignore any spectral lines the star may have intrinsically. Assume the star is stationary relative to the observer at Earth.

Consider the 3-2 Hα transition of hydrogen in the disk. A fraction of the hydrogen is in the n = 3 state. Another fraction is in the n = 2 state. Take the total density in the disk to be constant, and so low that optical depths in the disk are less than 1 at all wavelengths and along all possible directions.

Assume that whatever intrinsic luminosity the disk has in Hα is much less than the continuum luminosity from the star at and near the Hα wavelength. In other words, the star is a lot brighter than the disk at wavelengths at and near Hα.

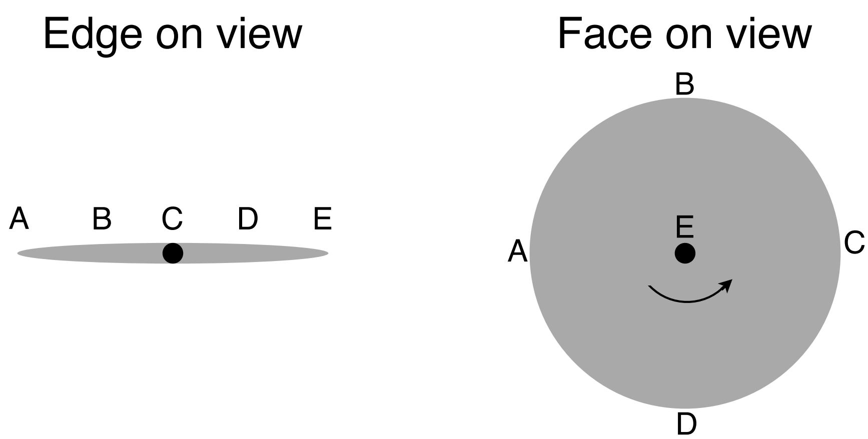

Figure 3: A proto-planetery disk observed edge on and face on. For the edge on view, the left side moves toward us. This is the view of the observer on earth, thus the light we detect travels perpendicular to this image toward us.

(a) [12 points] An observer takes the spectrum of the entire system, in an angularly unresolved observation, with the disk being observed edge-on (see Fig. 3). How would the spectrum around the Hα line look? The axes of your plot should be flux density F_λ vs. wavelength λ. Aside from marking clearly where the rest wavelength λ_0 lie and stellar flux F_0 lie, your sketch need not be quantitative. Your sketch should, however, indicate clearly whether absorption and/or emission lines due to the disk are present. Consider carefully how the spectrum behaves both redward and blueward of λ_0.

(b) [BONUS; 6 points] Next we observe the same system, but now it has a face-on orientation with respect to the earth (see Fig. 3). How would the spectrum around Hα look now?