# Math 121A Spring 2015, Midterm 1

February 18, 2015, 9:10am–10:00am.

Do 4 out of 5 of the following questions (indicate which ones). Each is worth 10 points.

1. Determine whether the following series converge or diverge, using appropriate tests:

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{n^2 + (2n)!}, \qquad \sum_{n=1}^{\infty} (-1)^n \cos(1/n), \qquad \sum_{n=1}^{\infty} \frac{\log n}{n^2}.$$

2. Consider the approximation:

$$\sqrt[5]{1+x} \approx 1 + \frac{x}{5}.$$

   Give a bound for the maximum error of this approximation when $x \in [0, 1/2]$. Justify your reasoning.

3. There are snowboarders ($B$) and skiers ($S$) at Lake Tahoe. Their populations each year are determined by the populations the previous year, according to the formulas:

$$B(n) = 2B(n-1) - S(n-1),$$

$$S(n) = (1/2)B(n-1) + (1/2)S(n-1).$$

   Suppose that initially $B(0) = 100$ and $S(0) = 50$. What will their relative proportions be after a long time? Will the total population stabilize, tend to zero, or tend to infinity as $n$ grows? *Hint: write these equations in matrix form.*

4. Suppose that

$$x^2 + y^3 = \sin(s) + \cos(t)$$

$$\text{and} \quad xy = s - t.$$

   Find $\left(\frac{\partial x}{\partial y}\right)_s$ as a function of $x, y, s, t$.

5. Use Lagrange multipliers to find the point on the sphere

$$x^2 + y^2 + z^2 = 1$$

   which is closest to the point $(0, 3, 4)$.