Astronomy 7A Midterm #1

October 1, 2015

Name: ______________________________

Section: ______________________________

There are 3 problems.

Write your answers on these sheets showing all of your work. It is better to show some work without an answer than to give an answer without any work. Please clearly label which work corresponds to which problem.

Calculators are allowed to perform arithmetic. Please turn off all cellphones.

If you have any questions while taking the midterm, get the attention of one of the GSIs.

Budget your time; you will have from 11:10 am to 12:30 pm to complete the exam. Of course, you are free to hand in your exam before 12:30 pm. Make sure that you have time to at least briefly think about every required question on the midterm.

You do not need to work on the questions in order, so it is OK to skip a question and come back to it later.

On my honor, I have neither given nor received any assistance in the taking of this exam

Signed: ______________________________

Constants and Some Userful Formulae

c = 3.00×10^{10} cm/s = 3.00×10^{8} m/s
h = 6.626×10^{-27} erg s = 6.626×10^{-34} J s
$\sigma_{\rm SB} = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} = 5.67×10^{-5} erg s^{-1} cm^{-2} K^{-4}
$k_{\rm B} = 1.38 \times 10^{-23}$ J/K = 1.38×10^{-16} erg/K
$m_p = 1.673 \times 10^{-24}$ g = 1.673×10^{-27} kg
$m_e = 9.11 \times 10^{-28}$ g = 9.11×10^{-31} kg
$L_\odot = 3.90 \times 10^{26}$ W = 3.90×10^{33} erg s^{-1}
Solar mass: $M_\odot = 2.0 \times 10^{33}$ g = 2.0×10^{30} kg
$R_\odot = 7.0 \times 10^{10}$ cm = 7.0×10^{8} m
Absolute Magnitude of Sun: $M_\odot = +4.74$
$R_\oplus = 6.4 \times 10^{8}$ cm = 6.4×10^{6} m
1 AU = 1.5×10^{13} cm = 1.5×10^{11} m
1 pc = 3.09×10^{18} cm = 3.09×10^{16} m
1 radian = 206265 arcsec
1 Angstrom = 10^{-8} cm = 10^{-10} m
1 year = 12 months = 365 days = 3.15×10^{7} s

Classical Doppler Shift ($v_r \ll c$):

$$\frac{(\lambda_{\rm observed} - \lambda_{\rm rest})}{\lambda_{\rm rest}} = v_r/c$$

Apparent magnitudes difference:

$$m_1 - m_2 = -2.5 \log \frac{F_1}{F_2}$$

Distance modulus:

$$m - M = 5\log_{10}(d) - 5 = 5\log_{10}\left(\frac{d}{10{\rm pc}}\right)$$

Stefan Boltzmann equation (A=area):

$$L = A\sigma_{\rm SB}T^4$$

Photon energy:

$$E = \frac{hc}{\lambda}$$

Planck Function:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/(\lambda k_B T)} - 1}$$

Wien Peak Law for B_λ:

$$\lambda_{\rm peak} T = 2.8977721 \times 10^{-3} \ {\rm m\,K}$$

Diffraction Limit:

$$\theta_{\rm diff} = 1.22 \, \frac{\lambda}{D}$$

Plate scale

$$\frac{d\theta}{dy} = \frac{1}{f}$$

Energy Levels of Hydrogen:

$$E_n = \frac{-13.606 \ {\rm eV}}{n^2}$$

Optical Depth:

$$\tau = \int n \sigma dx$$

Mean Free Path:

$$\lambda_{\rm mfp} = 1/(n\sigma) = 1/(\rho\kappa)$$

1 The James Webb Space Telescope [42 points]

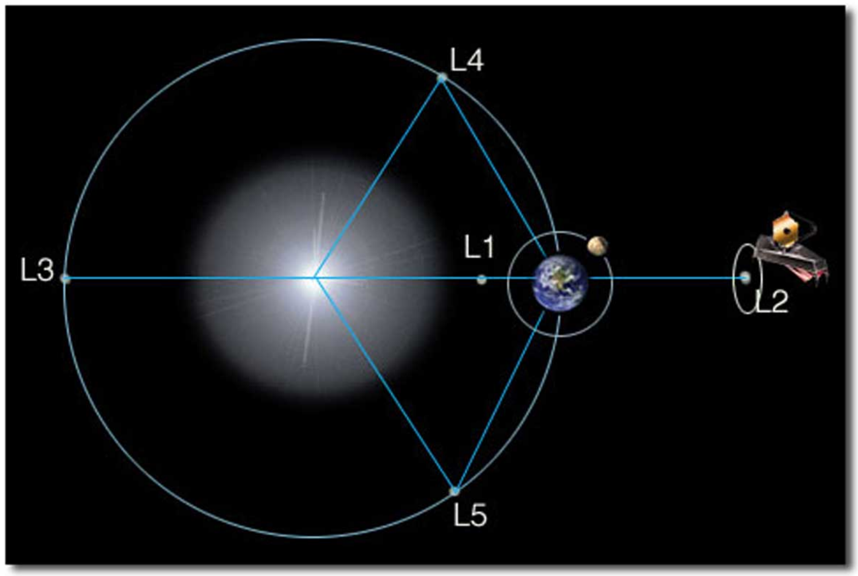

Figure 1: Location of the JWST (not on scale). JWST will be located in Lagrangian point L2, 1.5 million km from Earth.

The James Webb Space Telescope (JWST) is NASA's next space observatory and the successor to the Hubble Space Telescope. JWST features a segmented 6.5-meter diameter primary mirror and will be located near the Earth-Sun 2nd Lagrange point (L2 in Figure 1), 1.5 million km from Earth. A large sunshield of 21 by 14 meters will protect the telescope from light and heat from the Sun and Earth and keep its mirror and four science instruments below 50 K. JWST will operate at wavelengths from 0.6 to 28.5 μm.

(a) [6 points] What is the flux from the Sun received on the sunshield?

The JWST is 1.5×10^9 m from the Earth and thus the distance to the Sun is:

$$d = 1.5 \times 10^9\,\mathrm{m} + 1.5 \times 10^{11}\,\mathrm{m} \approx 1.5 \times 10^{11}\,\mathrm{m}$$

The the flux of the Sun at the distance of the JWST is:

$$F = \frac{L}{4\pi d^2} = \frac{3.90 \times 10^{26}\,\mathrm{W}}{4\pi(1.5 \times 10^{11}\,\mathrm{m})^2} = 1.35 \times 10^3\,\mathrm{W\,m^{-2}}$$

(b) [8 points] The average temperature on Earth is 288 K. Estimate the flux that the sunshield receives from Earth. Can this be ignored compared to the flux of the Sun?

The lumonosity of Earth can be estimate using the Stefan-Boltzmann law, assuming it radiates like a blackbody:

$$L_\oplus = A\sigma_{\mathrm{sb}}T^4 = 4\pi R_\oplus^2\sigma_{\mathrm{sb}}T^4 = 4\pi(6.4 \times 10^6\,\mathrm{m})^2\sigma_{\mathrm{sb}}(288\,\mathrm{K})^4 = 2.0 \times 10^{17}\,\mathrm{W}$$

We can calculate the flux using the same method as in 1a, but now d is the distance between the JWST and Earth:

$$F = \frac{L_\oplus}{4\pi d^2} = \frac{2.0 \times 10^{17}\,\mathrm{W}}{4\pi(1.5 \times 10^9\,\mathrm{m})^2} = 7.1 \times 10^{-3}\,\mathrm{W\,m^{-2}}$$

Thus, it is safe to ignore the flux from Earth compared to the flux of the Sun.

(c) [6 points] The telescope is powered by a solar panel, which has a power of 2000 watts. Assuming an effiency of 50% (so half of the energy is lost), what is the size of the solar panel?

As we can ignore the flux from Earth, we can use the flux from the Sun at the JWST to calculate the size of the solar panel using:

$$F = \frac{dE}{dt\,dA}$$

dE/dt is the energy per unit time received on the solar panel, which is 2000 / 0.5 = 4000 Watt. Thus, the area is

$$dA = \frac{dE}{Fdt} = \frac{2000/0.5}{1.35 \times 10^3} = 3\,\mathrm{m}^2$$

(d) [8 points] What is the highest resolution at which the JWST can observe in arcsec? At which wavelength do we achieve this resolution?

The shorter the wavelength, the higher the resolution. This means that the highest resolution will be achieved at the shortest wavelength, 0.6 micron, where the diffraction limit is

$$\theta_{\rm diff} = 1.22\,\frac{\lambda}{D} = 1.22\,\frac{0.6 \times 10^{-6}}{6.5} = 1.13 \times 10^{-7}\,{\rm radian} = 0.023''$$

(e) [6 points] What is the farthest star for which we can measure a distance using the parallax method with the JWST?

1 parsec means 1 parallax arcsecond. Thus $d = \frac{1}{p''}$ with p'' in arcsec and d in parsec. By definition, p'' is half the maximum angle. The minimum angle we can resolve is 0.023”, thus the minimum parallax angle is 0.0116”. Thus

$$p'' = \frac{1}{d} > 0.0116$$
$$d < 86\,{\rm pc}$$

(f) [8 points] The focal length of the JWST is 131.4 meter. Which pixel scale would you choose for this parallax measurement? 2 or 20 μm? Motivate your answer.

The plate scale formula is given by:

$$\frac{d\theta}{dy} = \frac{1}{f}$$

In order to achieve the diffraction limit, the angle subtended by each pixel, θ, needs to be smaller than $\theta_{\rm diff}$ (otherwise our resolution would be limited by the angular size of the pixel rather than the diffraction limit). Thus

$$\theta = \frac{dy}{f} < \theta_{\rm diff}$$
$$dy < f\theta_{\rm diff} = 131.4\,{\rm m} \times 1.13 \times 10^{-7}\,{\rm rad} = 15\ \mu{\rm m}$$

Since $dy < 15\mu$m, we will pick dy to be 2μm.

2 HII regions [38 points]

We use the JWST to observe a bright star surrounded by a hydrogen gas cloud. The cloud is at a distance of 500 pc and at rest with regard to us. The star illuminates the surrounding gas cloud, and as a result 99.96% of the hydrogen atoms are ionized. We also call this an HII region. Assume that all of the electrons belonging to the neutral hydrogen atoms are in the $n = 1$ *orbital. The electron density is* $200\,\mathrm{cm}^{-3}$*.*

(a) [6 points] What energy and wavelength does a photon need to ionize a hydrogen atom with the electron in the $n = 1$ *level? Why do we call this transition the Lyman limit?*

We have to use the Rydberg equation: $E = 13.6\ eV(\frac{1}{n_1^2} - \frac{1}{n_2^2})$ with $n_1 = 1$ and $n_2 = \infty$ because the electron escapes the atom. This, of course, gives us exactly 13.6 eV. Thus we need a photon with energy $E > 13.6$ eV and wavelength $\lambda < \frac{hc}{E} = 91.2$ nm. This is called the Lyman Limit because a Lyman transition (which is any transition ending in the $n = 1$ state) can create no larger energies than this (but photons with larger energies can be absorbed).

(b) [6 points] Not all stars can photo-ionize their surrounding gas cloud. What type of star will be most efficient in photo-ionizing surrounding medium, a cold or a hot star? Motivate your answer.

We have to think about the Stefan-Boltzmann equation, which was given on the equations list: $L = A\sigma T^4$. If we have stars of the same size, the hotter one will have a greater luminosity and will therefore give off higher energy photons. We can also think about the blackbody equation, which tells us that hotter stars have higher peak energies and, relatively, give off more high energy photons that could photo-ionize hydrogen.

(c) [8 points] Hydrogen in the $n = 1$ *orbital has a cross section of* $6.3 \times 10^{-18}\,\mathrm{cm}^2$ *near the Lyman limit. What is the mean free path of a photon at the Lyman limit in pc? What is the opacity?*

We will use the equation $\lambda_{\rm mfp} = 1/n\sigma$. In this question, we're thinking about the absorption of photons by neutral hydrogen. Therefore, we want to use the density of neutral hydrogen for n, and the cross-section for interaction between neutral hydrogen and a photon for σ. Since the question tells us that the electron density is 200 cm^{-3} and the gas is 99.96% ionized, then the neutral hydrogen density is (0.04/99.96)*200 = 0.08 cm^{-3}. Therefore, the mean free path is 1.98×10^{18} cm or 0.64 pc.

We find the opacity by recalling that $n\sigma = \kappa\rho$, where ρ is the mass density. To calculate ρ, we multiply the number density of electrons by the electron mass, the number density of protons by the proton mass, and the number density of neutral hydrogen atoms by the proton mass:

$\rho = 200\text{cm}^{-3} * (9.11 \times 10^{-28}\text{g}) + 200\text{cm}^{-3} * (1.67 \times 10^{-24}\text{g}) + 0.08\text{cm}^{-3} * (1.67 \times 10^{-24}\text{g}) = 3.34 \times 10^{-22}$ g cm^{-3} $= 3.34 \times 10^{-19}$ kg m^{-3}

Therefore, since $\kappa = \frac{n\sigma}{\rho}$, where n is the neutral hydrogen density and σ is its cross-section, $\kappa = 1.6 \times 10^3$ cm^2 g^{-1}, or in SI, $\kappa = 1.6 \times 10^2$ m^2 kg^{-1}.

For this half of the question, we gave full credit (4 points) to any solution that used the correct method but arrived at the wrong answer because of an incorrect answer for $n\sigma$ in the first half of the question.

For parts d-f, we gave full credit for any solution that used the correct approach but started with the wrong answer from previous parts (so you were only penalized once for an incorrect answer).

(d) [6 points] Once ions and electrons recombine, they will cascade to the groundstate by emitting photons. Thus, the majority of the re-emitted photons will have wavelengths longer than the Lyman limit. Based on this information, what is the typical diameter of an HII region. Motivate your answer.

"The majority of the re-emitted photons have wavelengths longer than the Lyman limit" means that most of the photons from the recombination transitions will escape from the cloud. This means that the cloud cannot be optically thick. We can approximate this condition by saying that the cloud will have an optical depth of 1 for these photons. Since the distance to an optical depth of 1 is the mean free path, the cloud's radius will be 0.64 pc. This means that the diameter will be 1.28 pc.

(e) [6 points] What is the solid angle of the HII region as seen from Earth?

Recall the formula for solid angle $\Omega = A/r^2$, where A is the cross-sectional area of the HII region and r is the distance of the HII region from earth, 500 pc. If the size of the HII region is 1.28 pc, then the radius is 0.64 pc, so the solid angle is 5.1×10^{-6} sr.

(f) [6 points] Due to the cascading of the electrons, HII regions show clear emission lines in their spectrum. In particular Balmer α is a very strong line, with a luminosity of 10^{30} W. What is the intensity of the Balmer α emission within the HII region as seen from Earth?

Recall that the units of intensity are W m^{-2} sr^{-1}. To find intensity, we can divide the flux received at Earth by the solid angle in part e. The flux received at Earth is simply 10^{30} W/(4 π (500 pc)^2) = 3.4×10^{-10} W m^{-2}. Dividing by the solid angle yields an intensity of 6.6×10^{-5} W m^{-2} sr^{-1}.

3 Heliocentric correction [20 points]

In addition to the Doppler shift from the relative velocity of stars compared to the Sun, there is also a Doppler shift due to the movement of the Earth around the Sun. Thus, Doppler shift measurements need to be corrected for this value.

(a) [8 points] Assume that a star is at rest compared to our Sun. What is the minimum and maximum Doppler shift caused by motion of Earth around the Sun?

We can find the velocity of the earth simply by looking at the length of a year and the astronomical unit.

$$v = \frac{2\pi(1AU)}{1\,\text{year}} = \frac{2\pi 1.5 \times 10^{11}\ \text{m}}{3.15 \times 10^{7}\ \text{s}} = 2.992 \times 10^{4}\ \text{m/s}$$

Now we apply the Doppler shift equation given on the sheet:

$$\frac{v}{c} = \frac{\Delta\lambda}{\lambda} = 9.97 \times 10^{-5}$$

The maximum shift caused by Earth is then about one ten-thousandth of the wavelength. The absolute minimum shift is, of course, zero, which occurs when the Earth is at its closest and furthest approach to the star. However, because of the ambiguous wording of the question, we also accepted -9.97×10^{-5} as the minimum Doppler shift.

(b) [12 points] Now we observe a Balmer β absorption line in the spectrum of a star which is moving with a speed of 200 km/s away from the Sun. What is the maximum absolute difference from the rest-frame Balmer β absorption line in Angstroms? What is the minimum absolute difference? Motivate your answer.

Start by deciding what situation would cause the maximum absolute difference. That would be when the Earth is moving away from the star even as the star is moving away from it. In the classical limit, we can simply add the velocities, which would give us a total velocity of 230 km/s. Applying this to Doppler equation, we get

$$\frac{2.2992 \times 10^5}{3 \times 10^8} = 7.664 \times 10^{-4} = r_{max}$$

For the minimum situation, the Earth would be going toward the star, giving us a total velocity of 170 km/s.

$$\frac{1.7008 \times 10^5}{3 \times 10^8} = 5.669 \times 10^{-4} = r_{min}$$

Now we need to find the actual value of Balmer β so we can apply r_{max} and r_{min} . Balmer β is the transition with energy

$$E = 13.6(\frac{1}{n_1^2} - \frac{1}{n_2^2})$$

where $n_1 = 2$ and $n_2 = 4$. This gives us an energy $E = 13.6(.25 - .\bar{1}) = 1.89\ eV$, which corresponds to a wavelength of $\lambda = \frac{hc}{E} = 6574\ \mathring{A}$. And so:

$$\Delta\lambda_{max} = \lambda r_{max} = 5.04\ \mathring{A}$$

$$\Delta\lambda_{min} = \lambda r_{min} = 3.73\ \mathring{A}$$

Unlike part (a), this question doesn't accept a Doppler shift of 0 as an answer because that doesn't give the "minimum absolute difference" that the question asks for.